IOSR Journal of Applied Physics (IOSR-JAP)
e-ISSN: 2278-4861.Volume 16, Issue 4 Ser. 2 (Jul. – Aug. 2024), PP 34-41
www.iosrjournals.org

Measuring the Coefficient of Restitution and Its Changes in Temperature

Soujanyaram Koganti

I. Introduction

Playing squash every evening has been a sacred regimen for me and at first, I believed that the weather outside cannot (directly or indirectly) affect this seemingly simple sport played indoors. Being encompassed by the 3 white walls and a glass back, I assumed the rain or the cold breeze outside could not possibly affect the way I play my sport indoors. Until recently my credence was unshakable, but then I noticed a peculiar behavior of the black squash ball. On cooler days, the ball would bounce less than usual while on warmer days, its bounce increased by a large proportion.

This sparked my curiosity so I went ahead and tried comparing the bounce of other types of balls with a change in temperature. After looking at the behavior of cricket, tennis, ping-pong, and football, I observed that the effect of temperature on these was negligible. The idea that temperature too can play a pivotal role in the way I play my favorite game, I wanted to find out quantitatively how temperature affects the bounce of a squash ball. This led to my research question which is:
How does the temperature of a bouncing squash ball affect its coefficient of restitution?

Purpose of the Investigation:

The investigation aims to discover a, if any, quantitative relationship between the temperature of a squash ball and its coefficient of restitution. Through this data, I and squash players around the world can know accurately the effect of temperature on the bounce of a ball and can use this data to their tactical advantage by adjusting the game style accordingly.

II. Background Information:

A squash ball is made of raw butyl rubber along with other materials (both natural and synthetic). When it is bounced, the gravitational potential energy is converted to kinetic energy as the ball accelerates towards the ground. During the collision, the kinetic energy is converted into elastic potential energy as the ball deforms. The ball consists of air which compresses because of this deformation and almost immediately, the air expands again (restorative force) returning the ball to normalcy. The elastic potential energy is converted back to kinetic energy. This conversion of energy, however – under standard conditions- is not perfectly elastic, which means the kinetic energy before the bounce is not equal to kinetic energy after the bounce, and energy is lost as heat and sound. Conclusively, this means that there will be a difference in the drop height of a ball and the rebound height (In case of a perfectly elastic collision the rebound height would be equal to the drop height). The difference in the heights might be more commonly referred to as the "bounciness of a ball" or the coefficient of restitution.

The coefficient of restitution is defined as the ratio of the final velocity to the initial velocity between two objects after their collision. It can be calculated using the following equation:

$$e = \frac{v_2}{v_1}$$

e = Coefficient of Restitution, v_2 = Final speed, v_1 = Initial speed
However, calculating the final speed of the ball with high accuracy is impractical in a high school laboratory. To avert this I derived another equation that removes the requirement of calculating speed.

$$e = \frac{v_2}{v_1} = \sqrt{\frac{{v_2}^2}{{v_1}^2}}$$

e = Coefficient of Restitution, v_2 = Final speed, v_1 = Initial speed

$$e = \sqrt{\frac{{v_2}^2}{{v_1}^2}} = \sqrt{\frac{\frac{1}{2}m{v_2}^2}{\frac{1}{2}m{v_1}^2}} = \sqrt{\frac{KE_2}{KE_1}}$$

m = mass of Squash ball, KE = $\frac{1}{2}mv^2$, KE_2 = Kinetic Energy after bounce, KE_1 = Kinetic Energy before bounce

$$e = \sqrt{\frac{KE_2}{KE_1}} = \sqrt{\frac{GPE_2}{GPE_1}} = \sqrt{\frac{mgh_2}{mgh_1}}$$

The assumption is made that all gravitational potential energy is being converted to kinetic energy when the ball is travelling towards the surface and that all the kinetic energy after the collision is being converted into gravitational potential energy as the ball is moving away from the surface. Air resistance is considered to be negligible.
g = gravitational constant of Earth

$$e = \sqrt{\frac{h_2}{h_1}}$$

h_2 =Rebound Height, h_1 = Drop Height

Hypothesis:
There is a statistically significant positive relationship between temperature and the coefficient of restitution of a squash ball, and as temperature increases, the coefficient of restitution also increases. The temperature of a ball can be related to its pressure by the Ideal Gas Law.

$$PV = nRT$$

Although this law is only applicable to Ideal Gases, we can assume that there is an ideal gas scenario inside the squash ball such that temperature (T) is directly proportional to pressure (P). This law states that as the temperature goes up, the pressure goes up – considering the change in volume of the ball is negligible. Therefore, I believe that a rise in pressure will cause the air inside the ball to expand more vigorously after a collision, which in turn will cause the ball to bounce more.

Preliminary Work:
A major problem I faced was to devise a way through which the temperature of the squash ball could be altered and maintained. Ways available to me were: Using a microwave/refrigerator to warm/cool the ball or using a water bath. The former directly heats the ball and it poses a risk of exploding. Furthermore, direct heating might lead to permanent deformation of the ball therefore considering the safety and the inefficiency of this method, it was deemed unsuitable for this experiment. The water bath was used to alter the temperature of the squash ball.

Experimental Variables:
Independent Variable: The temperature of the water bath will be the independent variable in this experiment. Thus, the temperature of the squash ball will be varied by immersion in a water bath. An analog mercury thermometer will be used to measure the temperature of the water bath. The absolute uncertainty in the readings would be half of the least count, i.e. 1/2 = ± 0.5°C. The temperature of the water bath will be the closest experimental simulation to the change in temperature of the surroundings of the squash ball which affects its coefficient of restitution.

Dependent Variable: The dependent variable is the rebound height, H, which is the maximum height reached after the initial bounce of the ball. This is calculated using tracker software and it will then be used to calculate the coefficient of restitution of the squash ball. As the tracker will be using a meter ruler as a reference for the length, the uncertainty of the ruler will be the uncertainty of the rebound height i.e. 0.1cm/2: ± 0.05cm.

Controlled Variables:

Variable	**Reason to control it**	**How to control it**
Height from where the ball is dropped	While increasing the drop height might increase the rebound height, it might not be proportional because a ball dropped from a higher	Have a predetermined height of bounce, using a meter scale, and mark the location. Using the tracker software I made sure that the height was uniform in all the trials
The surface of the floor	A hard surface absorbs less energy when compared with a soft surface. The more energy absorbed by the surface, the less that remains in the ball for it to bounce.	The wooden floors of the Squash courts were used in this experiment
The initial force applied on the ball	When an external force is applied, the ball will fall down with more kinetic energy. This will subsequently increase the bounce.	The ball will be dropped without applying any external force, therefore all resultant force is due to gravity

The type of ball	In squash, there are various types of balls with different levels of bounce. For example, a blue dot ball will bounce higher than a yellow dot ball, even if all the other conditions are kept constant.	I will only be using the yellow double-dot squash ball for this experiment.

Table 1 Controlled Variables

Safety Measures:
The experiment involved using a heating mantle to heat water. Subsequently, the squash ball will be very hot to handle with bare hands; heat resistant gloves were used throughout the experiment and insulated tongs were used to transfer the squash ball to the drop height area.

Apparatus Required:
1. 100-centimetre rule (Absolute uncertainty ± 0.05cm)
2. 1 Alcohol-in-glass Thermometer (Absolute uncertainty ± 0.5°C)
3. 1 Clamp
4. 1 Video Recorder
5. 1 Water Bath (Temperature limitation: 0-100°C)
6. 1 Insulated tongs
7. 1 Stopwatch
8. 1 Pair of heat resistant gloves
9. 5 yellow double-dot squash balls
10. Tracker software (https://physlets.org/tracker/)

Procedure:
1. The video recorder was set up on a tripod in front of the drop site.
2. The water in the water bath was brought to a temperature of 30°C.
3. The temperature was measured using an Alcohol-in-glass thermometer.
4. The thermometer was kept in the water using a clamp so that the bulb does not touch the base of the water bath.
5. One squash ball was placed in the water bath and held under the water using the insulated tongs for 2 minutes.
6. The squash ball was removed from the water bath using insulated tongs
7. The ball was then transported using heat-resistant gloves to the drop site and held approximately 200cm above ground.
8. The ball was dropped from the drop height making sure no external force was exerted on the ball and that the ball followed a near-vertical path.
9. A l00cm ruler was held at the base of the drop site, to provide a point of reference to determine the height of the bounce as shown in Figure ii.
10. The motion of the ball was recorded by a video recorder
11. A tracker was used to calculate drop and bounce height (Figure ii), this was rebound height 1
12. Steps 3-10 were repeated four more times using the four remaining squash balls as rebound height 2, 3, 4, and 5.
13. Steps 2-10 were repeated for temperatures 40°C, 50°C, 60°C, 70°C, 80°C, and 90°C.

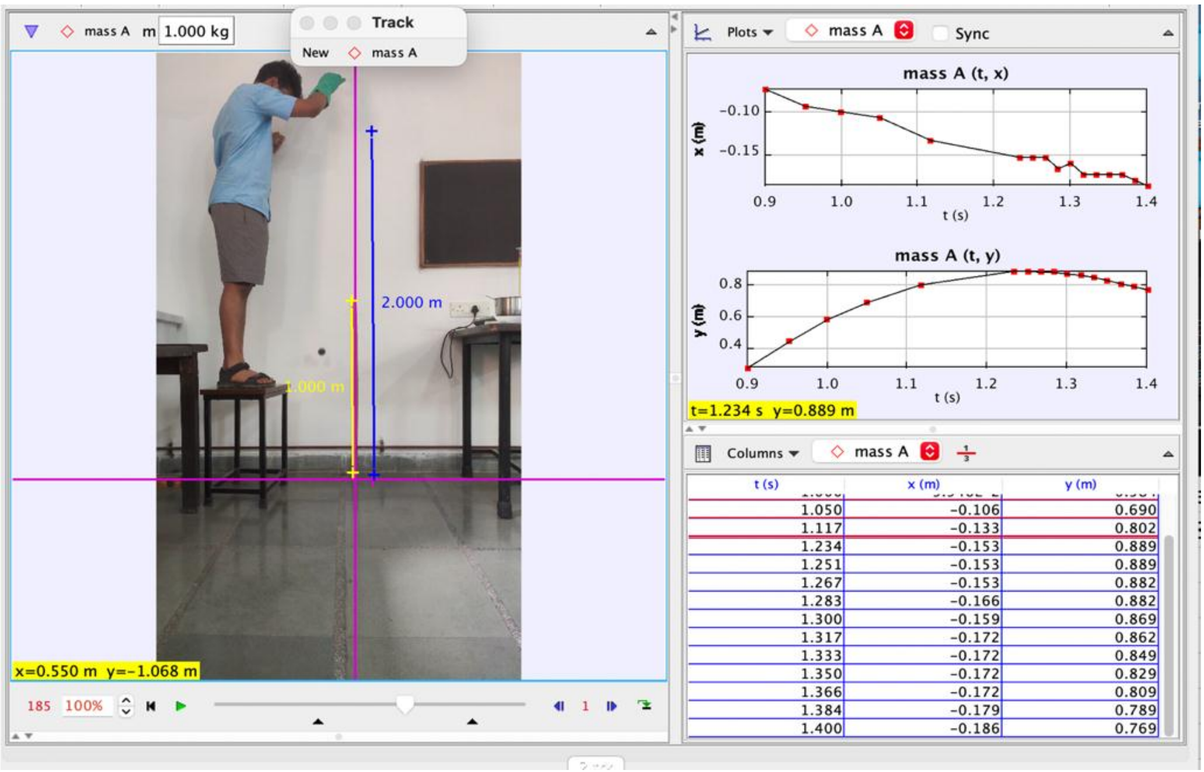


Figure 1 Tracker Analysis for finding rebound height by comparing the image to the known length of the scale beside (In yellow)

Raw Data:
The data collected from the methodology mentioned above is summed up in Table 2. I used Microsoft Excel to compile the data as I can use equations to further process my data:

Temperature of Water Bath (± 0.5°C)	Drop Height (±0.05cm)	Rebound Height (±0.05cm) 1	Rebound Height (±0.05cm) 2	Rebound Height (±0.05cm) 3	Rebound Height (±0.05cm) 4	Rebound Height (±0.05cm) 5
30.0	200.00	47.80	49.80	49.90	48.30	49.30
40.0	200.00	68.40	65.60	66.80	67.40	66.70
50.0	200.00	84.50	86.40	83.20	83.40	84.90
60.0	200.00	98.50	99.80	98.50	98.30	99.30
70.0	200.00	107.80	106.20	109.30	108.20	107.40
80.0	200.00	113.20	111.20	111.70	110.80	111.50
90.0	200.00	115.00	113.90	114.40	114.10	114.30

Table 2 Rebound Height of squash ball at varying temperatures

Processed Data:
An average of the rebound heights was taken to reduce the impact of random uncertainties of the experiment. The following sample calculations show how the average was calculated and how the error uncertainty was propagated:

Calculating the absolute error in the Average Rebound Height

$$\Delta RH_{avg} = \frac{\sqrt{\sum_{i=1}^{n} \quad (RH[uncertainty]_i)^2}}{5}$$

$$= \frac{\sqrt{(0.05)^2 + (0.05)^2 + (0.05)^2 + (0.05)^2 + (0.05)^2}}{5} = \frac{\sqrt{0.0125}}{5} = \frac{\sqrt{5}}{100} = 0.02cm$$

Sample calculation of Average Rebound Height

$$Average\ Rebound\ Height_{30.0°C} = \frac{\sum_{i=1}^{n} \quad RH_i}{n}$$

$$= \frac{47.80 + 49.80 + 49.90 + 48.30 + 49.3}{5} = 49.02cm$$

RH = Rebound Height, n = Total number of trials, RH[uncertainty] = Absolute uncertainty of Rebound Height

Temperature of Water Bath (± 0.5°C)	Drop Height (± 0.05 cm)	Average Rebound Height (± 0.02 cm)
30.0	200.00	49.02
40.0	200.00	66.98
50.0	200.00	84.48
60.0	200.00	98.88
70.0	200.00	107.78
80.0	200.00	111.30
90.0	200.00	114.18

Table 3 Average Rebound Heights at varying temperatures

The coefficient of restitution of the squash ball was then calculated taking the average rebound height as the Rebound Height h_2 and the drop height as the Drop Height h_1 in the equation derived earlier, which was $e = \sqrt{\frac{h_2}{h_1}}$.
The error uncertainty in the coefficient of restitution was calculated using the following method:

$$\Delta e = |p| \left(\frac{h_2}{h_1}\right)^{p-1} \times \left|\frac{h_2}{h_1}\right| \times \sqrt{\left(\frac{\Delta h_2}{h_2}\right)^2 + \left(\frac{\Delta h_1}{h_1}\right)^2}$$

p = power of the division of the rebound heights = 0.5, Δh_2 = Absolute Uncertainty in h_2, , Δh_1 = Absolute Uncertainty in h_1

Sample Calculation of Coefficient of Restitution along with error uncertainty for Temperature 30.0°C

$$e = \sqrt{\frac{49.02}{200.0}} = 0.495 \ \approx 0.50$$

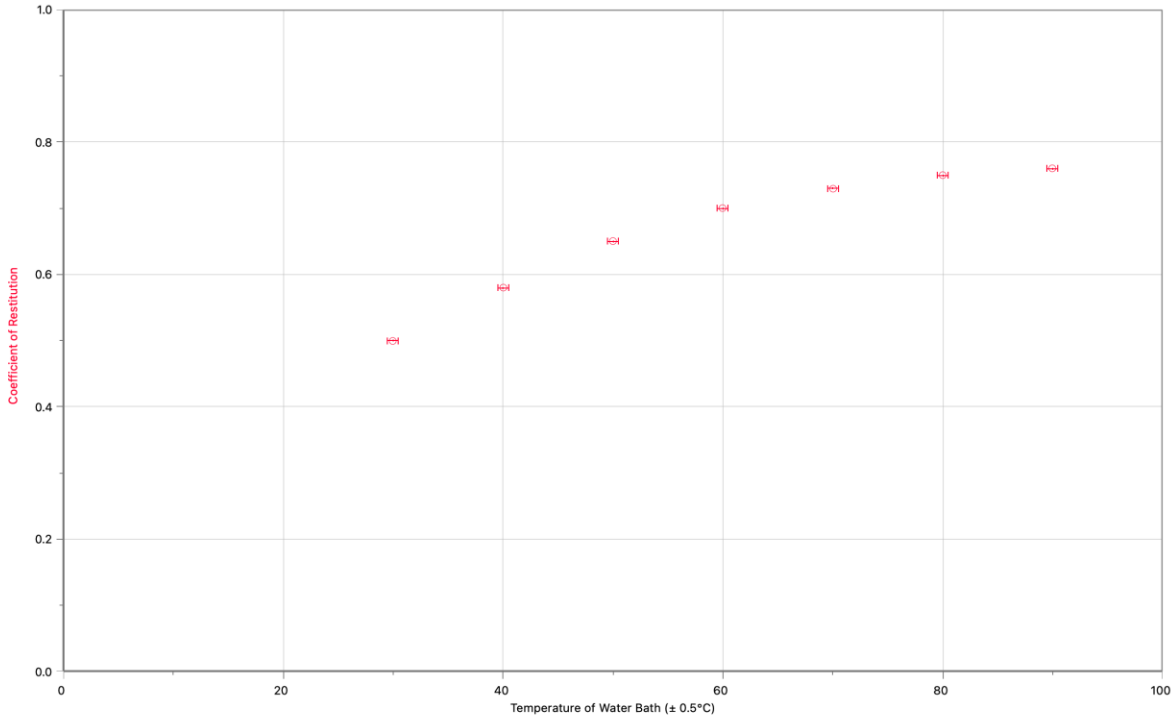
$$\Delta e = 0.5 \times \left(\frac{49.02}{200}\right)^{-0.5} \times \left(\frac{49.02}{200}\right) \times \sqrt{\left(\frac{0.02}{49.02}\right)^2 + \left(\frac{0.05}{200}\right)^2} = 6.27 \times 10^{-4} \approx 0$$

The absolute uncertainty in the coefficient of restitution was negligible under the experimental accuracy and it was therefore considered 0

Temperature of Water Bath (± 0.5°C)	Coefficient of Restitution
30.0	0.50
40.0	0.58
50.0	0.65
60.0	0.70
70.0	0.73
80.0	0.75
90.0	0.76

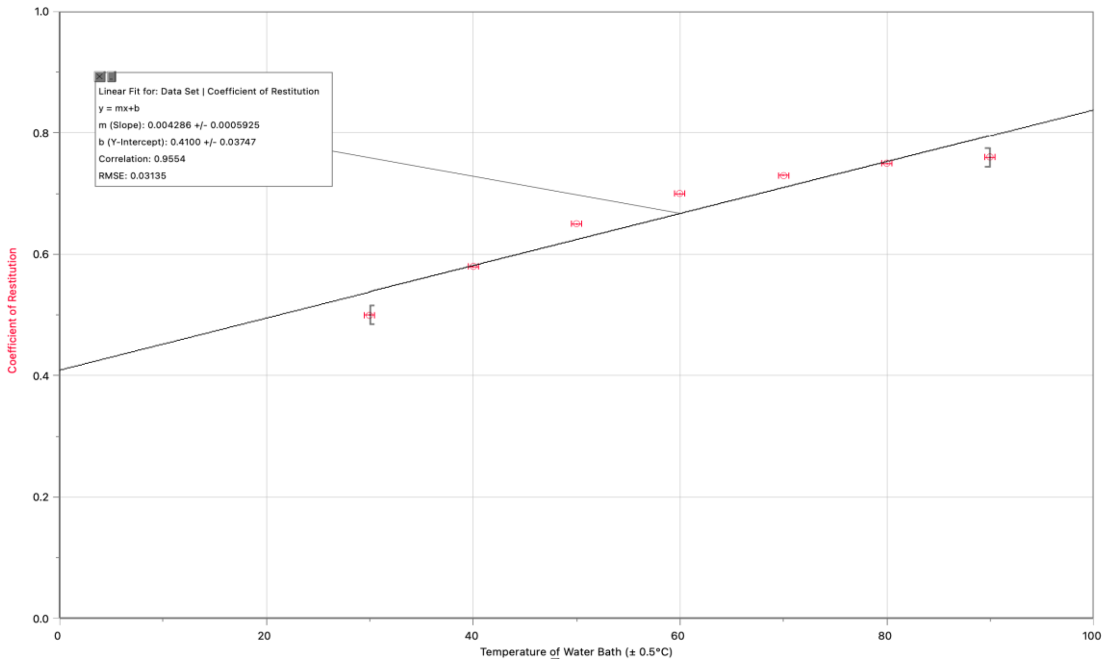
Table 4 Coefficient of Restitution at varying temperatures

Analysis of Data
The calculated coefficient of restitution (y-axis) is plotted against the temperature (x-axis) in Graph 1



Graph 1 Plotted points of Coefficient of Restitution against Temperature

To deduce the relationship between the plots, various equations were analyzed to see the best fit (highest correlation and lowest Root-mean-Square-Deviation). Following is the graph with the best-fit line and its equation :



Graph 2 Linear Function: y = 0.004286x + 0.4100 | Coefficient of restitution against Temperature

As predicted by the hypothesis, there is a statistically significant positive relationship between temperature and as temperature increases, the coefficient of restitution also increases. The best-fit line is highly accurate as the correlation is 0.9554 which is nearly equal to 1 (perfect correlation). The root-mean-square deviation is also small when compared to the data set being used. The equation for the best-fit line is 0.004286x + 0.4100 and from this, we can deduce that the y-intercept is 0.4100 or 0.41. This, according to the graph, is the coefficient of restitution at 0°C and it is a fairly reasonable estimate. Also with each degree increase in temperature the coefficient of restitution increases by 0.004286. This, however, does not mean that we can extrapolate beyond the points already mentioned because the coefficient has a theoretical maximum and minimum value; the maximum being a perfect collision and the minimum being no bounce at all. Therefore, the range is between 0 and 1 and the trend is valid only between these values.

The linear function also correlates to the ideal gas law (PV = nRT), and as the temperature increased, pressure proportionally increased. This increase in pressure caused the bounce of the squash ball to be more elastic and in turn it increased the coefficient of restitution proportionally.

To test the accuracy of the best-fit line, I compared it with logarithmic and cubic equations. In the logarithmic line of best fit (Graph 3), the line touched the x-axis at 3.7°C which is unrealistic because this suggests that at temperatures equal to and below 3.7°C the ball would not bounce at all. In the cubic line of best fit (Graph 4), the graph decreased sharply at temperatures below 0°C, and at temperatures above 100°C the graph shows negligible change of slope. Although the values predicted by it are more accurate than the linear function, justifying the cubic model is more difficult, and providing a just explanation is out of the scope of the experiment.

III. Conclusion & Evaluation:

The experiment proved the hypothesis and provided realistic values based on my physical understanding of the world and bouncing balls. The correlation was 0.9554, which is very close to the ideal value of perfect correlation 1. However, the methodology used was not perfect and it led to errors in the data produced. This might explain why the line of best fit does not pass through all data points. Furthermore, there is a presence of random errors as the data is accurate but not precise. This can be seen as there is a varying difference between the theoretical value predicted by the line of best fit and the actual value derived.

One source of error was that it was necessary to transport the squash balls from the water bath and as the temperature of the ball was higher than the surrounding, some of the heat might have dissipated leading to a decrease in the actual temperature before the drop. Also, the temperature of the water bath was plotted against the coefficient of restitution which suggests that the actual temperature of the squash ball might have been different from the water bath. This might explain the differences observed in graph 2. Another limitation was the use of the Tracker software. Although this method was fairly accurate, it involved using reference points and manually calibrated axes and scales to measure the height. This might have led to an error in judgment and subsequently a decline in accuracy. If there was software to measure the rebound height without human intervention, the accuracy of the experiment could be increased.

An extension to this investigation can be to alter the pressure directly and see its effects on the coefficient of restitution. Through this, we will actually get to know if the Ideal Gas equation being used in the analysis to explain the trend holds true when we directly alter the pressure. I could also use a larger range of temperatures to see if the trend holds for more extreme ranges. This would only be possible if the use of a water bath is eliminated as a water bath can only work for temperatures between 0 and 100°C. Lastly, another extension could be to record at least 2-3 consecutive bounce heights of the ball. The 1st rebound height could be used as the drop height for the 2nd bounce. This can be used to measure the coefficient of restitution for the second bounce. As the coefficient of restitution is the square root of the ratio of rebound height and drop height, even if the drop height of the 2nd bounce is low, the rebound height will decrease proportionally and therefore the coefficient should ideally stay constant.

Bibliography:

[1]. *Queensland Parliament*. documents.parliament.qld.gov.au/committees/EIC/2013/QldAssessment/tp-20Mar2013-Assignment.pdf.

[2]. "Coefficient of Restitution: Definition, Explanation and Formula." *Science ABC*, 6 Jan. 2022, www.scienceabc.com/pure-sciences/coefficient-of-restitution-definition-explanation-and-formula.html.

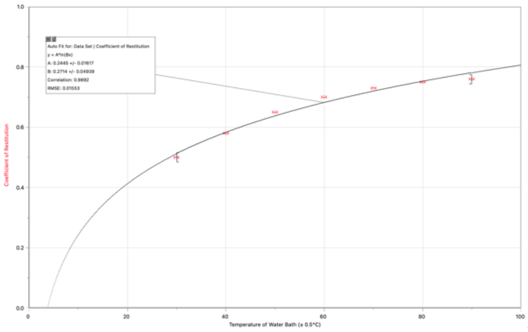
[3]. Boone, Stan. "Everything You Wanted to Know about a Squash Ball." *Racket Sports World*, 3 May 2022, racketsportsworld.com/squash-balls-wiki/.

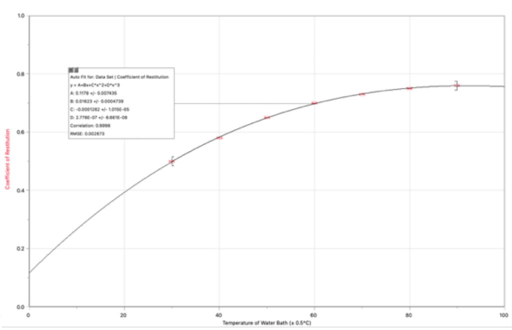
[4]. "American Association of Physics Teachers." *Citation*, aapt.scitation.org/.

[5]. "The Effect of Temperature on a Squash Ball." *Bartleby*, www.bartleby.com/essay/The-Effect-Of-Temperature-On-A-Squash-P3S6UETZA4FP.

[6]. "Offizielle Seite Von Victor Badminton." *VICTOR Europe GmbH*, www.victor-europe.com/.

Appendix :


Auto Fit for: Data Set | Coefficient of Restitution
y = A*ln(Bx)
A: 0.2445 +/- 0.01617
B: 0.2714 +/- 0.04939
Correlation: 0.9892
RMSE: 0.01553
Coefficient of Restitution
Temperature of Water Bath (± 0.5°C)


Auto Fit for: Data Set | Coefficient of Restitution
y = A+Bx+C*x^2+D*x^3
A: 0.1179 +/- 0.007435
B: 0.01623 +/- 0.0004739
C: -0.0001262 +/- 1.015E-05
D: 2.778E-07 +/- 6.661E-08
Correlation: 0.9998
RMSE: 0.002673
Coefficient of Restitution
Temperature of Water Bath (± 0.5°C)