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Explaining Big Crunch By Friedmann Equations

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Abstract:
A biggest challenge for today's astrophysicist is to solve the mystery of primordial evolution of the universe. This is also known as belongs to Planck's era. In this stage space-time singularity and inflationary phase all occur when universe was just of the size 10^{-5}m. All this evolution occurs in a tiny fraction of a second. This description is made by joining the cosmological principle of isotropy and homogeneity, the Hubble law, and the Einstein's field equations of General Relativity in the Big Bang Theory or Standard Cosmological Model (SCM). By solving equation proposed by Albert Einstein in the general relativity the cosmological singularity proposed by Alexander Friedmann explain well the Big Bang model of origin of our universe. The expansion of universe started some 15 billion year ago with Big Bang. The studies of red shift in the spectrum of galaxies in near vicinity suggest the expansion of our universe. The experimental studies of Cosmic Microwave Background (CMB) also suggest that Universe is originated due to Big Bang. Early of the twentieth century some astrophysicist started to propound that how long this expansion will last and what is going to be happened with this universe. There are three different fate of our universe suggested in popular literature which are: Big Rip, Big Freez and Big Crunch. In the present work we proposed solution of Friedmann equations to find condition of the Big Crunch.

Keywords: *Big bang, Big crunch, Friedmann Equations.*

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I. Introduction:

Einstein gravitational field equations are used to describe the general evolution of time after big bang till today[1,2]. Using field equations proposed by Einstein, Friedmann proposed cosmic singularity by which origin of the universe is explained. In the year of 1965 a remarkable discovery of Cosmic Microwave Background by Arno Penzias and Robert Wilson[3], prove the dynamic nature of universe proposed by Friedmann. Using Robertson-Walker metric with the warp function $R(t)$, which is cosmic scale factor, one can find solution of Friedmann equation to propose Big Bang[4,5]. Using the ideas proposed by Friedmann, Georges Lemaitre said that the universe is originated very hot and close state called primordial atomic state and exploded due to big bang[6]. The Friedmann- Lemaitre universe was homogenous and isotropic in nature. It is given by Robertson-Walker metric (i, j = 1,2,3):

$$ds_{RW}^2 = c^2dt^2 - R^2(t)\gamma_{ij}dx^i dy^j \tag{1}$$

Here γ_{ij} is third spatial metric of constant gaussian curvature parameter k.

Solution of Friedmann Equations: As we know that Einstein's gravitational field equation is

$$G^{\mu\nu} - \Lambda g^{\mu\nu} = \frac{8\pi G}{c^4}T^{\mu\nu} \tag{2}$$

The left-hand side of the equation 2 is depends on $g^{\mu\nu}$ and is geometrical and shows that gravitation is playing significant role in space-time geometry. The right-hand side of equation 2 is depend on energy momentum tensor $T^{\mu\nu}$.The energy momentum tensor can be determined by distribution of mass/energy of universe. If a perfect fluid is considered which is moving with relativistic velocity then

$$T^{\mu\nu} = (\epsilon + p)\frac{u^\mu u^\nu}{c^2} - pg^{\mu\nu} \tag{3}$$

Here ϵ is energy density, p is pressure and u^μ is four velocity and defined as $\frac{dx^\mu}{d\tau}$ where ($d\tau = \frac{1}{c}ds$), obviously

$$G^{\mu\nu} = R^{\mu\nu} - \frac{1}{2}g^{\mu\nu}R = \frac{8\pi G}{c^4}T^{\mu\nu}$$

In right hand side first term is representing Ricci tensor and the R in second term give scalar curvature. By inserting equation 1 and 3 in 2 the Friedmann equation is obtained as

$$\dot{R}^2 + kc^2 = \frac{8\pi G}{3c^2}\epsilon_{tot}R^2 + \frac{\Lambda}{3}c^2R^2 \tag{4}$$

By differentiating again the above equation gives

$$\ddot{R} = -\frac{8\pi G}{3c^2}(\epsilon_{tot} + 3p_{tot})R + \frac{\Lambda}{3}c^2 R \qquad 5$$

Here ϵ_{tot} and p_{tot} are total energy density respect to total pressure by adding up various components. In addition to above the energy conservation is obtained

$$\dot{\epsilon_k} = -3(\epsilon_k + p_k)\frac{\dot{R}}{R} \qquad 6$$

The above is obtained by vanishing $T^{\mu\nu}$. Since equation 4,5 and 6 are not independent thus they are not sufficient to determine the $R_t(t)$, $\epsilon_{tot}(t)$ and $p_{tot}(t)$. Thus a fourth relation is introduced which give pressure p_k of the k^{th} component in the terms of energy density ϵ_k and called equation of state.

$$p_k = \omega_k \epsilon_k \qquad 7$$

Here $k = r, m, \Lambda$ with

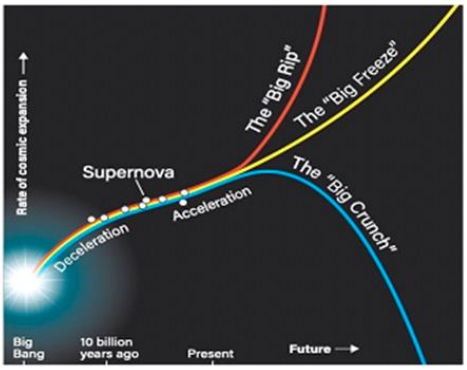
$$\omega_r = \frac{1}{3}, \quad \omega_m = 0, \omega_\Lambda = -1 \qquad 8$$

Thus pressure p_k is linear function of energy density ϵ_k and a constant parameter ω_k. Solution of equation 6 is obtained by keeping ω_k = constant, thus

$$\epsilon_k(t) = \epsilon_{k,0}(\frac{R_0}{R_{(t)}})^{3(1+\omega_k)} \qquad 9$$

By inserting the value of equation 7 and 9 in equation 4 we will get a nonlinear differential equation which determine the cosmic scale factor $R(t)$.

As it is considered that at the time of Big Bang $t = 0$ we get $R(0) = 0$, and hence $\dot{R}(0) > 0$. The interesting results obtained if we put $t = \infty$ by considering that Universe will come to an end at the time which is known as Big Crunch.



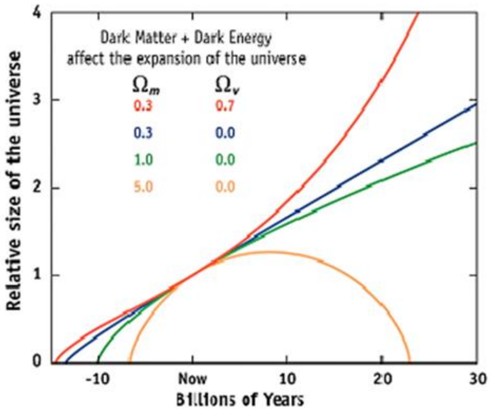


Fig 2: The Beginning to the end of the Universe The Big Crunch vs NASA WMAP Collaboration [7]
Fig 3: Fate of Universe in the view of DM and DE Courtesy Big Freeze by Eric Betz "Astronomy" 31 Jan 2021

II. Conclusion:

The Friedmann equations play very significant role to understand the physical chemistry of our universe. It describes what are the elements by which the universe is made, it also helps to understand how the universe is evolved. In the present work we present an idea how the universe come to an end by giving a nonlinear differential equation. The above solution give light on the understanding about Big Crunch which can be the ultimate fate of the universe and fit well with the idea of cyclic universe proposed by Albert Einstein in the beginning of twentieth century.

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