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Level Set Evolution without Re-initialization: A New Approach

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***ABSTRACT**: This paper presents a novel method for implicit active contours, which is completely free of the costly re-initialization procedure in level set evolution (LSE). A diffusion term is introduced into LSE, resulting in a specialized-LSE equation, to which a piecewise constant solution can be derived. In order to have a stable numerical solution of the RD based LSE. I propose a two-step splitting method (TSSM) to iteratively solve the LSE equation: first iterating the LSE equation, and then solving this equation. The second step regularizes the level set function obtained in the first step to ensure stability, and thus the complex and costly re-initialization procedure is completely eliminated from LSE. By successfully applying diffusion to LSE, the LSE model is stable by means of the simple finite difference method, which is very easy to implement. The proposed RD method can be generalized to solve the LSE for both variational level set method and PDE-based level set method. The RD-LSE method shows very good performance on boundary anti-leakage, and it can be readily extended to high dimensional level set method.The extensive and promising experimental results on synthetic and real images validate the effectiveness of the proposed RD-LSE approach.*

***Keywords:** Active contours, image segmentation, Level set, reaction-diffusion, PDE, variational method*

I. INTRODUCTION

In the past two decades, active contour models (ACMs, also called snakes or deformable models) [1] have been widely used in image processing and computer vision applications, especially for image segmentation [5][9][12-13][17-18][30][34][41-42][52-53][59].

The original ACM proposed by Kass et al. [1] moves the explicit parametric curves to extract objects in images. However, the parametric ACM has some intrinsic drawbacks, such as its difficulty in handling topological changes and its dependency of parameterization [2]. The level set method later proposed by Osher and Sethian [2] implicitly represents the curve by the zero level of a high dimensional function, and it significantly improves ACM by being free of these drawbacks [2-5]. The level set methods (LSM) can be categorized into partial differential equation (PDE) based ones [8] and variational ones [9]. The level set evolution (LSE) of PDE-based LSM is directly derived from the geometric consideration of the motion equations [6], which can be used to implement most of the parametric ACMs, such as Kass et al.'s snakes [1], region competition snakes [12], and geodesic active contours [5], etc.

The LSE of variational LSM is derived via minimizing a certain energy functional defined on the level set [9], such as Chan-Vese ACM [18], Vese and Chan's piecewise smoothing ACM [42], local binary fitting ACM [30][41], etc. Moreover, the variational LSM can be easily converted into PDE-based LSM by changing slightly the LSE equation while keeping the final steady state solution unchanged [7]. In implementing the traditional LSMs [4-5][8][10][18], the upwind schemes are often used to keep numerical stability, and the level set function (LSF) is initialized to be a signed distance function (SDF). Since the LSF often becomes very flat or steep near the zero level set in the LSE process and this will affect much the numerical stability [8][14], a remedy procedure called re-initialization is applied periodically to enforce the degraded LSF being an SDF [14]. The first re-initialization method was proposed by Chopp [35] and it directly computes the SDF. However, this method is very time-consuming. In [35], Chopp also proposed a more efficient method by restricting the front movement and the re-initialization within a band of points near the zero level set. However, it is difficult to locate and discretize the interface by Chopp's methods [10].

The method proposed by Sussman et al. [14] iteratively solves a re-initialization equation. Nonetheless, when the LSF is far away from an SDF, this method fails to yield a desirable SDF. The re-initialization method in [8] addresses this problem by using a new signed function, but it will shift the interface to some degree [10]. In order to make the interface stationary during re-initialization, a specific method for the two-phase incompressible flow was proposed in [39], which focuses on preserving the amount of material in each cell. The method in [38] uses a true upwind discretization near the interface to make the interface localization accurate, and it can keep the interface stationary.

All the above mentioned re-initialization methods, however, have the risk of preventing new zero contours from emerging [16], which may cause undesirable results for image segmentation, such as failures to detecting the interior boundary.

In recent years, some variational level set formulations [9][34][59] have been proposed to regularize the LSF during evolution, and hence the re-initialization procedure can be eliminated. These variational LSMs without re-initialization have many advantages over the traditional methods [4-5][8][10][18], including higher efficiency and easier implementation, etc [9]. Some global minimization methods [61][62] eliminate the re-initialization procedure by combining the total variational model with the Chan-Vese model [18] or the Vese-Chan's piecewise smoothing model [42]. However, these global minimization methods [61][62] can only be applied to some variational LSF with specific forms.

This paper proposes a new LSM, namely the reaction-diffusion (RD) method, which is completely free of the costly re-initialization procedure. The RD equation was originally used to model the chemical mechanism of animal coats [58]. It includes two processes: reaction, in which the substances are transformed into each other, and diffusion, which causes the substances to spread out over a surface in space. The RD equation was also used to describe the dynamic process in fields such as texture analysis [55-57], natural image modeling [54] and phase transition modeling [21][25-28][31]. In particular, the RD equation in phase transition modeling is based on the Van der Waals-Cahn-Hilliard theory [26], which is widely used in mechanics for stability analysis of systems with unstable components (e.g., density distributions of a fluid confined to a container [31]). It has been proved that the stable configurations of the components are

piecewise constant in the whole domain, and the interfaces between the segmented areas have minimal length [21][26]. These conclusions have been used in image classification with promising results [33]. However, the phase transition method cannot be directly applied to image segmentation because of the inaccurate representation of interface and the stiff parameter ε in its RD equation [16].

The joint use of phase transition and LSM has been briefly discussed in [60]. However, [60] aims to apply the curvature-related flow in phase transition to analyze the evolution driven by the curvature based force. In fact, the curvature motion based on the phase transition theory has been widely studied [11][13][17][48-51]. For example, the classical Merriman-Bence-Osher (MBO) algorithm [11] applies a linear diffusion process to a binary function to generate the mean curvature motion with a small time step; the methods in [13][48-51] convolve a compactly supported Gaussian kernel (or an arbitrary positive radically symmetric kernel) with a binary function to generate similar motion (often called the convolution-generated curvature motion).

Motivated by the RD based phase transition theory [27], it is proposed to introduce a diffusion term into the conventional LSE equation, constructing a RD-LSE equation to combine the merits of phase transition and LSM. I present the unique and stable equilibrium solution of RD-LSE based on the Van der Waals-Cahn-Hilliard theory, and give the accurate representation of interface. The re-initialization procedure is completely eliminated from the proposed RD-LSM owing to the regularization of the diffusion term. A two-step splitting method (TSSM) is proposed to iteratively solve the RD equation in order to eliminate the side effect of the stiff parameter ε

In the first step of TSSM, the LSE equation is iterated, while in the second step the diffusion equation is solved, ensuring the smoothness of the LSF so that the costly re-initialization procedure is not necessary at all. Though the diffusion method has been widely used in image processing, to the best of my knowledge, my work is the first one to apply diffusion to LSE, making it re-initialization free with a solid theoretical analysis under the RD framework. One salient advantage of the proposed method is that it can be generalized to a unified framework whose LSE equation can be either PDE-based ones or variational ones.Another advantage of RD-LSE is its higher boundary anti-leakage and anti-noise capability compared with state-of-the-art methods [9][34][59].

In addition, due to the diffusion term, the LSE formulation in the proposed method can be simply implemented by finite difference scheme instead of the upwind scheme used in traditional LSMs [5][8][10][18]. The proposed RD-LSE method is applied to representative ACMs such as geodesic active contours (GAC) [5] and Chan-Vese (CV) active contours [18]. The results are very promising, validating the effectiveness of RD-LSE. The rest of the paper is organized as follows. Section 2 presents the Specialized -LSM. Section 3 presents the experimental results of RD-LSM, and analyzes the consistency between theory and implementation followed by Section 4 which presents the conclusion .

II. SPECIALIZED (RD) LEVEL SET EVOLUTION

Since the zero level is used to represent the object contour, we only need to consider the zero level set of the LSF. As pointed out in [8], with the same initial zero level set, different embedded LSFs will give the same final stable interface. Therefore, we can use a function with different phase fields as the LSF. Motivated by the phase transition theory [20][27], we propose to construct a RD equation by adding a diffusion term into the conventional LSE equation. Such an introduction of diffusion to LSE will make LSE stable without re-initialization. The stable solution of the RD equation is piecewise constant with different phase fields in the domain Ω, and it is also the solution of the LSE equation.

By adding a diffusion term "εΔφ" into the LSE equation in Eq. (3) or Eq. (4), we have the following RD equation for LSM:

1.

$$\begin{cases} \phi_t = \varepsilon\Delta\phi - \dfrac{1}{\varepsilon}L(\phi), \mathrm{x} \in \Omega \subset R^n \\ \text{subject to } \phi(\mathrm{x}, t=0, \varepsilon) = \phi_0(\mathrm{x}) \end{cases}$$

where ε is a small positive constant, L(φ) = –F|∇φ| for PDE-based LSM or L(φ) = –Fδ(φ) for variational LSM, Δ is the Laplacian operator defined by ,

$$\Delta(\cdot) \triangleq \sum_{i=1}^{n} \partial^2(\cdot) / \partial x_i^2$$

∂ /∂(.) indicates the partial differential operator, and φ(x) is the initial LSF. Eq. (13) has two dynamic processes: the diffusion term "εΔφ" gradually regularizes the LSF to be piecewise constant in each segment domain Ωi, and the reaction term "–ε–1L(φ)" forces the final stable solution of Eq. (13) to L(φ)=0, which determines Ωi. In the traditional LSMs [4-5][8][10][18], due to the absence of the diffusion term we have to regularize the LSF by an extra procedure, i.e., re-initialization.
In the following, based on the Van der Waals-Cahn Hilliard theory of phase transitions [26], the equilibrium solution of the RD equation is first analyzed when ε→0+ for variational LSM, and then generalize the analysis into a unified framework for both PDE based LSM and variational LSM .

Algorithm 1: RD based level set evolution (RD-LSE)

1.Initialization : $\varphi^n = \varphi_0, n = 0$
2.Compute $\varphi^{n+1/2} = \varphi^n - \Delta t_1 . L(\varphi^n),$
3.Compute $\varphi^{n+1} = \varphi^n - \Delta t_2 . \Delta(\varphi^n),$
Where, $\varphi^n = \varphi^{n+1/2}$

4.If φ^{n+1}, **satisfies stationary condition stop; otherwise n=n+1 and return to step 2.**

III. EXPERIMENTAL RESULTS

The following figure show the output of the proposed Specialized level set method and other level set methods which involve re-initialization. It is evident from the output figures that the proposed method exhibits higher performance over the existing methods. Fig.1 shows the output of the segmented image using the proposed Level Set method. Fig. 2 depicts the level set which is created by using the proposed method. It can be observed clearly that the evolving contour is an exact 3d representation of the input figure.

Whereas in the existing method the evolving level set does not perform well with the same number of iterations and unpredictable shrinking of the contour occurs resulting in un-predictable results.

The experiments on synthetic and real images demonstrated the promising performance of the approach. Motivated by the convolution-generated curvature motion [48-51] in phase transitions, the diffusion procedure in our TSSM algorithm can also be replaced by convolving any positive, radically symmetric kernel with a small enough width. A Gaussian kernel is used for regularization .

This implies that the RD method can be readily extended to multiphase level set method based on the theory of phase transitions in mixtures of Cahn-Hilliard fluids [26]. The figures given below show the superior performance of the proposed method.

Result of Proposed Specialized LSE method:

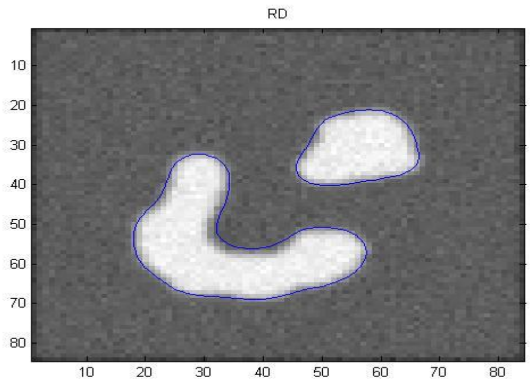

Fig.1:Segmented image using proposed Specialized LSE

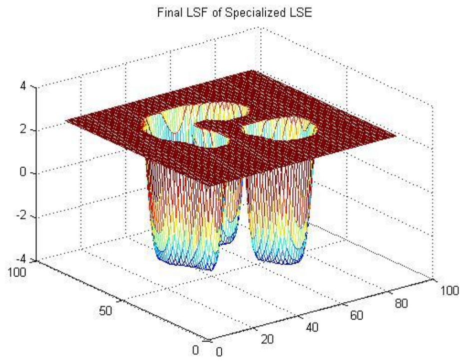

Fig.2: Level Set Function using proposed Specialized LSE

Result of Existing method:

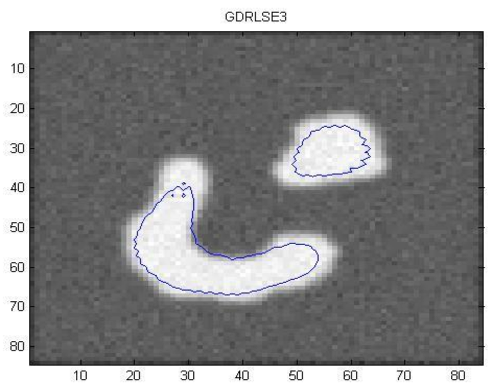

Fig. 3: Segmented image with Re-initialization

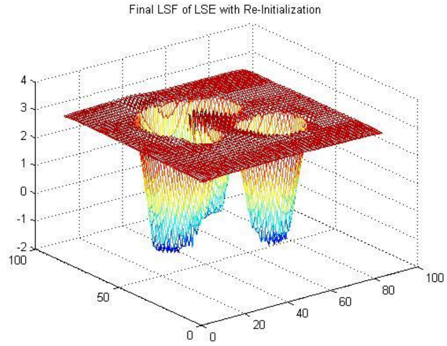

Fig.4: Level Set Function using Re-Initialization

IV. CONCLUSION

This paper proposes a specialized level set evolution (LSE), which is completely free of the re-initialization procedure required by traditional level set methods. A two-step-splitting-method (TSSM) was then proposed to effectively solve the RD based LSE. The proposed RD method can be generally applied to either variational level set methods or PDE-based level set methods. It can be implemented by using the simple finite difference scheme.

The RD method has the following advantages over the traditional level set method and state-of-the-art algorithms [9][59][34]. First, the RD method is general, which can be applied to the PDE-based level set methods and variational ones. Second, the RD method has much better performance on weak boundary anti-leakage. Third, the implementation of the RD equation is very simple and it does not need the upwind scheme at all. Fourth, the RD method is robust to noise.

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