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EXPANSION OF THE GENERALIZED HYPERGEOMETRIC POLYNOMIAL SET $B_n(x_1, x_2, x_3)$ IN TERMS OF $G_1^{\lambda_1}\left(X_1^r, S^1, P^1\right)$

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Abstract : In the present paper an attempt has been to express the polynomial set $B_n(x_1, x_2, x_3)$ in terms of . Many interesting new results may be obtained as particular cases on specializing the parameters. Out of these particular results some of the stand for well known polynomials and some of them are believed to be new. These polynomials are of outmost importance for science and engineers because they occur in the solution of differential equation. integral equation etc. Which describe physical problem. Many orthogonal polynomial have their wide application in quantum mechanics chemical kinetics and electromagnetic theory etc.

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I. INTRODUCTION

Sing and Singh [1] define the generalized hypergeometric polynomial set $B_n(x_1, x_2, x_3)$ by means of generating relation.

$$\left(\upsilon_\lambda + \mu_1 x_1^{(c-d)} t^{r_1}\right)^{-\sigma_1} \times \left[\begin{matrix} (C_p);(E_g);(\alpha_u)(\alpha_m^1) & \\ & \mu x_1^r t, \mu_2 x_2^{r_1} t^{r_1}, \mu_3 x_3^{r_1} t^{r_2} \\ (D_q);(F_h);(\beta_v);(\beta_k^1) & \end{matrix}\right]$$

$$= \sum_{n=0}^{\infty} B_{n,r;r_1;r_2;r_3;(D_q);(F_h);(\beta_v);(\beta_k^1)}^{\mu;\mu_1;\mu_2;\mu_3;\sigma_1;a;c;d;(C_p);(E_g);(\alpha_u)(\alpha_m^1);}\left(x_1, x_2, x_3\right) t^n \qquad \ldots (1.1)$$

Where μ, μ_1, μ_2, μ_3, σ_1, a, c, d, are real numbers and r, r_1 are non-negative integer and r_2, r_3 are natural numbers.

The left hand side of (1.1) contains the product of generalized hypergeometric function and Lauricella function in the notation of Burchanall and Chaundy [2]. The generalized polynomial set contains number of parameters.

For Simplicity we shall denote

$$B_{n,r;r_1;r_2;r_3;(D_q);(F_h);(\beta_v);(\beta_k^1)}^{\mu;\mu_1;\mu_2;\mu_3;\sigma_1;a;c;d;(C_p);(E_g);(\alpha_u)(\alpha_m^1);}\left(x_1, x_2, x_3\right)$$

by $B_n(x_1, x_2, x_3)$

Where n denotes the order of the polynomial set.

After little simplication (1.1), gives

$$B_n(x_1,x_2,x_3)=\sum_{s_1=0}^{\left[\frac{n}{r_1}\right]}\sum_{s_2=0}^{\left[\frac{n-r_1s_1}{r_2}\right]}\sum_{s_3=0}^{\left[\frac{n-r_1s_1-r_2s_2}{r_3}\right]}$$

$$\times\frac{\left[(C_p)\right]_{n-r_1s_1-(r_2-1)s_2-(r_3-1)s_3}\left[(E_g)\right]_{n-r_1s_1-r_2s_2-r_3s_3}}{\left[(D_q)\right]_{n-r_1s_1-(r_2-1)s_2-(r_3-1)s_3}\left[(F_u)\right]_{n-r_1s_1-r_2s_2-r_3s_3}}$$

$$\times\frac{\left[(\alpha_u)\right]_{s_2}\left[(\alpha_m^1)\right]_{s_3}\left[(\sigma_1)\right]_{s_1}\left(\mu x_1^r\right)^{n-r_1s_1-r_2s_2-r_3s_3}\mu_1^{s_1}\left(\mu_2x_2^{r_2}\right)^{s_2}\left(\mu_3x_3^{r_3}\right)^{s_3}}{\left[(\beta_v)\right]_{s_2}\left[(\beta_k^1)\right]_{s_3}(n-r_1s_1-r_2s_2-r_3s_3)!\;\nu_n^{s_1}\;s_2!\;s_3!}\qquad\ldots(1.2)$$

The Polynomial $B_n(x_1, x_2, x_3)$ happens to the generalization of as many thirty eight orthogonal and non-orthogonal polynomials.

II. NOTATIONS

1. $(m) = 1, 2, 3, \ldots\ldots, m.$
2. $(A_p) = A_1, A_2, A_3, \ldots\ldots A_p.$
3. $[(A_p)] = A_1, A_2, A_3, \ldots\ldots A_p.$
4. $[(A_p)]_n = (A_1)_n, (A_2)_n, (A_3)_n \ldots\ldots (A_p)_n.$
5. $\Delta(a,b)=\frac{b}{a},\frac{b+1}{a},\ldots\ldots\frac{b+a-1}{a}.$
6. $R=\frac{\left[(C_p)\right]_n\left[(E_g)\right]_n\left(\mu x_1^r\right)^n}{\left[(D_q)\right]_n\left[(F_h)\right]_n n!}$

III. $B_n(x_1, x_2, x_3)$ IN TERM OF $G_1^{\lambda_1}\left(x_1^r, s^1, p^1\right)$

Theorem : For $r_2 > 1$ and $r_3 > 1$, we achieve

$$B_n(x_1,x_2,x_3)=\nu_n^{\sigma_1}R\frac{1}{(-\lambda^1)_n}\sum_{i=0}^{n}\frac{(-p^1)^{nc^1-i}\,G_i^{\lambda^1}\left(x_1^r,s^1,p^1\right)}{(nc^1-i)!\qquad i!}$$

$$\times F_{p+g:u:m}^{2+g+h:v:k}\left[\begin{array}{l}\left[(-nc^1+i);r,r_1,r_2,r_3\right],\left[(1+\lambda^1-n);r,r_1,r_2,r_3\right],\\ \text{———————},\left[(1-(c_p)-n);r,r_1,r_2-1,r_3-1\right],\end{array}\right.$$

$$\left[\left(1-(D_q)-n\right):r,r_1,r_2-1,r_3-1\right],\left[\left(1-(F_h)-n\right):r,r_1,r_2,r_3\right)\right],\left[(\alpha_u):1\right],\left[(\alpha_m^1):1\left[(\sigma_1):1\right]\right]$$

$$\left[\left(1-(E_g)-n\right):r,r_1,r_2,r_3\right)\right],\left[(\beta_v):1\right],\left[(\beta_k^1):1\right]\text{———————————},$$

$$;\frac{\mu_1x_1^{(c-d)r_1}(-1)^{r_1(p+q+g+h+c+1)}}{\nu_a\;\mu^{r_1}(-p^1)^{r_1c^1}},\frac{\mu_2x_2^{r_2}(-1)^{r_2(p+q+g+h+c^1+1)+p+q}}{\mu^{r_2}(-p^1)^{r_2c^1}},$$

$$\left.\frac{\mu_3x_3^{r_3}(-1)^{r_3(p+q+g+h+c^1+1)+p+q}}{\mu^{r_3}(-p^1)^{r_3c^1}}\right]\qquad\ldots(3.1)$$

Proof : We have from (1.2)

$$\sum_{n=0}^{\infty}B_n(x_1,x_2,x_3)\,t^n=\upsilon_a^{-\sigma_1}\sum_{n=0}^{\infty}\sum_{s_1=0}^{\infty}\sum_{s_2=0}^{\infty}\sum_{s_3=0}^{\infty}\frac{(\sigma_1)_{s_1}\mu_1^{s_1}}{s_1!\quad\upsilon_a^{s_1}}$$

$$\times\frac{x_1^{(c-d)r_1}\left[(C_p)\right]_{n+s_2+s_3}\left[(E_g)\right]_n\left[(\alpha_u)\right]_{s_2}\left[(\alpha_m^1)\right]_{s_3}\left(\mu x_1^r\right)^n}{\left[(D_q)\right]_{n+s_2+s_3}\left[(F_h)\right]_n\left[(\beta_v)\right]_{s_2}\left[(\beta_k^1)\right]_{s_3}\qquad n!}$$

$$\times\frac{\left(\mu_2x_2^{r_2}\right)^{s_2}\quad\left(\mu_3x_3^{r_3}\right)^{s_3}\qquad t^{n+r_1s_1+r_2s_2+r_3s_3}}{s_2!\qquad\quad s_3!}\qquad\ldots(3.2)$$

Also, we have from [3]

$$\left(x_1^r\right)^n=\sum_{i=0}^{n}\frac{(-p^1)^{ns^1-1}\quad n!\quad G_1^{\lambda^1}\left(x_1^r,s^1,p^1\right)}{(nc^1-i)!\qquad i!\qquad(-\lambda^1)_n}$$

Hence, (3.2) can be written as

$$\sum_{n=0}^{\infty} B_n(x_1,x_2,x_3)\, t^n = \upsilon_a^{-\sigma_1} \sum_{n=0}^{\infty} \sum_{s_1,s_2,s_3=0}^{\infty} \sum_{i=0}^{n} \frac{\left[(C_p)\right]_{n+s_2+s_3}}{\left[(D_q)\right]_{n+s_2+s_3}}$$

$$\times \frac{\left[(E_g)\right]_n \left[(\alpha_u)\right]_{s_2} \left[(\alpha_u^1)\right]_{s_3} (\sigma_1)_{s_1} \mu_1^{s_1} \left(\mu_2 x_2^{r_2}\right)^{s_2} \left(\mu_3 x_3^{r_3}\right)^{s_3} x_1^{(c-d)r_1 s_1}}{\left[(F_h)\right]_n \left[(\beta_v)\right]_{s_2} \left[(\beta_k^1)\right]_{s_2} s_1!\ \upsilon_a^{s_1}\ s_2!\ s_3!}$$

$$\times \frac{\mu^n \left(-p^1\right)^{ns^1-1} G_i^{\lambda^1}\left(x_1^r, s^1, p^1\right)\ t^{n+r_1s_1+r_2s_2+r_3s_3}}{(nc^1-\lambda)!\ \ i!\ \ (-\lambda^1)_n}$$

$$= \upsilon_a^{-\sigma_1} \sum_{n=0}^{\infty} \sum_{i=0}^{\infty} \sum_{s_1=0}^{\left[\frac{n}{r_1}\right]} \sum_{s_2=0}^{\left[\frac{n-r_1s_1}{r_2}\right]} \sum_{s_3=0}^{\left[\frac{n-r_1s_1-r_2s_2}{r_3}\right]} \frac{\left[(C_p)\right]_{n-r_1s_1-(r_2-1)s_2-(r_3-1)s_3}}{\left[(D_q)\right]_{n-r_1s_1-(r_2-1)s_2-(r_3-1)s_3}}$$

$$\times \frac{\left[(E_g)\right]_{n-r_1s_1-r_2s_2-r_3s_3} \left[(\alpha_u)\right]_{s_2} \left[(\alpha_m^1)\right]_{s_3} \mu^{n-r_1s_1-r_2s_2-r_3s_3}}{\left[(F_h)\right]_{n-r_1s_1-r_2s_2-r_3s_3} \left[(\beta_v)\right]_{s_2} \left[(\beta_k^1)\right]_{s_3} (-\lambda^1)_{n-r_1s_1-r_2s_2-r_3s_3}\ i!}$$

$$\times \frac{\left(-p^1\right)^{ns^1-i-r_1s_1s^1-r_2s_2s^1-r_3s_3s^1} G_i^{\lambda^1}\left(x_1^r, s^1, p^1\right)(\sigma_1)_{s_1} \mu_1^{s_1} x_1^{(c-d)r_1s_1}}{\left(nc^1-i-r_1s_1c^1-r_2s_2c^1-r_3s_3c^1\right)!\ \ s_1!\ \ \upsilon_a^{s_1}}$$

$$\times \frac{\left(\mu_2 x_2^{r_2}\right)^{s_2} \left(\mu_3 x_3^{r_3}\right)^{s_3} t^n}{s_2!\ \ s_3!} \qquad \ldots (3.3)$$

Equating the co-efficient of t^n from both sides of (3.3) and after little simplification we obtain for $r_2 > 1$ and $r_3 > 1$.

$$B_n(x_1,x_2,x_3) = \upsilon_a^{-\sigma_1} R \frac{1}{(-\lambda^1)_n} \sum_{i=0}^{\infty} \frac{\left(-p^1\right)^{ns^1-1} G_i^{\lambda^1}\left(x_1^r, s^1, p^1\right)}{(nc^1-i)!\ \ i!}$$

$$\times \sum_{s_1,s_2,s_3=0}^{\infty} \frac{\left[1-(D_q)-n\right]_{r_1s_1-(r_2-1)s_2-(r_3-1)s_3}}{\left[1-(C_p)-n\right]_{r_1s_1-(r_2-1)s_2-(r_3-1)s_3}}$$

$$\times \frac{\left[1-(F_h)-n\right]_{r_1s_1-(r_2-1)s_2-(r_3-1)s_3} \left[(1+\lambda^1-n)\right]_{r_1s_1-r_2s_2-r_3s_3}}{\left[1-(E_g)-n\right]_{r_1s_1-(r_2-1)s_2-(r_3-1)s_3} \left(-p^1\right)^{r_1s_1s^1-r_2s_2s^1-r_3s_3s^1}}$$

$$\times \frac{\left(-nc^1+i\right)_{r_1s_1c^1-r_2s_2c^1-r_3s_3c^1} (\sigma_1)_{s_1} \left[(\alpha_u)\right]_{s_2} \left[(\alpha_m^1)\right]_{s_3} \mu_1^{s_1}}{\mu^{r_1s_1-r_2s_2-r_3s_3} \left[(\beta_v)\right]_{s_2} \left[(\beta_k^1)\right]_{s_3} s_1!}$$

$$\times \frac{x_1^{(c-d)r_1s_1} \left(\mu_2 x_2^{r_2}\right)^{s_2} \left(\mu_3 x_3^{r_3}\right)^{s_3} (-1)^{r_1(p+q+g+h+c^1+1)s_1}}{\nu_\alpha^{s_1}\ \ s_2!\ \ s_3!}$$

$$\times \frac{(-1)^{\{r_2(p+q+g+h+c^1+1)+p+q\}s_2} (-1)^{r_3(p+q+g+h+c^1+1)+p+q}}{1} \qquad \ldots (3.4)$$

The single terminating factor for $\left(-nc^1+i\right)_{r_1s_1c^1-r_2s_2c^1-r_3s_3c^1}$ makes all summations in (3.4) runs up to ∞.

Hence the theorem

IV. PARTICULAR CASES

Separating the term corresponding to $s_1 = 0 = s_3 \Rightarrow x_1 = 0 = x_3$ in (3.1), we obtain a number of results on specializing the remaining parameters.

1. **Hermite Polynomials :**
   On taking $p = 0 = q = g = h = \upsilon = u$; $r_2 = 2 = \mu = \mu_2 = 4$, $r = 1 = x_2 = \vartheta$ and x for x_1, we set

$$H_n(x) = \frac{(2x)^n}{n!\;(-\lambda^1)_n} \sum_{i=0}^{n} \frac{(-p^1)^{nc^1-i}\, G_i^{\lambda^1}(x_1^r, s^1, p^1)}{(nc^1-i)!\; i!} \times F\left[\begin{matrix} -\frac{n}{2}, \frac{-n}{2}+\frac{1}{2} - nc^1 + i, 1+\lambda^1-n; \\ \qquad\qquad \frac{1}{x^2}\left(\frac{-1}{p^1}\right)^{c^1} \\ \text{——}; \end{matrix}\right]$$

   where $H_n(x)$ are the Hermite Polynomials.

2. **Legendre Polynomials :**

   On taking $p = 0 = q = g = h = u$; $\nu = 1 = r = x_2 = \vartheta = \mu = \mu_2$; $\beta_1 = 1$; $r_2 = 2$ and $\frac{x}{\sqrt{x^2-1}}$ for x, we get

$$P_n(x) = \frac{(x^2-1)^{\frac{-n}{2}}\, x^n}{(-\lambda^1)_n} \sum_{i=0}^{n} \frac{(-p^1)^{nc^1-i}\, G_i^{\lambda^1}(x_1^r, s^1, p^1)}{(nc^1-i)!\; i!} \times F\left[\begin{matrix} \frac{-n}{2}, \frac{-n}{2}+\frac{1}{2}, -nc^1 + i, 1+\lambda^1-n; \\ \qquad\qquad \frac{x^2-1}{x^2}\left(\frac{-1}{p^1}\right)^{c^1} \\ 1; \end{matrix}\right]$$

   where $P_n(x)$ are the Legendre Polynomials.

3. **Legendre Polynomials :**
   On Putting $p = q = 0 = h = u$; $r = 1 = \vartheta = x_2 = \nu$; $g = 1$ or 2, $E_1 = 1$; $E_2 = \upsilon = \beta_1$; $r_2 = 2 = \mu = \mu_2 = 4$, and z for x_1, we get

$$R_{n,\nu}\left(\frac{1}{z}\right) = \frac{(\nu)_n (2z)^n}{n!\;(-\lambda^1)_n} \sum_{i=0}^{n} \frac{(-p^1)^{nc^1-i}\, G_i^{\lambda^1}(x_1^r, s^1, p^1)}{(nc^1-i)!\; i!} \times F\left[\begin{matrix} -\frac{n}{2}, -\frac{n}{2}+\frac{1}{2}, -nc^1 + i, 1+\lambda^1-n; \\ \qquad\qquad \frac{-1}{z^2}\left(\frac{-1}{p^1}\right)^{c^1} \\ 1-\nu, -n, -n,\ \nu; \end{matrix}\right]$$

4. **Jackson Polynomials :**
   Putting $p = 0 = q = g = h = u = \nu$; $r_2 = 2 = r = 1 = x_2 = \upsilon$; $\mu = 4$, $= \mu_2 = -16$ and x for x_1, we set

$$\phi_n(x) = \frac{2^{2n} x^n}{n!\;(-\lambda^1)_n} \sum_{i=0}^{n} \frac{(-p^1)^{nc^1-i}\, G_i^{\lambda^1}(x_1^r, s^1, p^1)}{(nc^1-i)!\; i!} \times F\left[\begin{matrix} -\frac{n}{2}, \frac{-n}{2}+\frac{1}{2}, -nc^1 + i, 1+\lambda^1-n; \\ \qquad\qquad -\frac{1}{x^2}\frac{(-1)^{c^1+1}}{(-p^1)^{c^1}} \\ \text{——}; \end{matrix}\right]$$

   where $\phi_n(x)$ are the Jakson Polynomials.

5. **Humbert Polynomials [4]:**
   For $p = 0 = q = g = h = \nu = u$; $r_2 = 3 = \mu$; $r = 1 = \vartheta = x_2$; $\mu_2 = -27$ and λ for x_1, we get

$$h_n^*(y) = \frac{(3y)^n}{n!\ (-\lambda^1)_n} \sum_{i=0}^{n} \frac{(-p^1)^{nc^1-i}}{(nc^1-i)!} \frac{G_i^{\lambda^1}(x_1^r, s^1, p^1)}{i!} \times F\left[\begin{matrix} -\frac{n}{3}, \frac{-n+1}{3}, \frac{-n+2}{3}, -nc^1+i, 1+\lambda^1-n; \\ \qquad \frac{(-1)^{c^1+1}}{y^3(-p^1)^{c^1}} \\ \text{———}; \end{matrix}\right]$$

where $h_n^*(y)$ are the Humbert Polynomials.

6. **Lagrange Polynomials [5]:**
On taking $p = 0 = q = h = v$; $g = 1 = v = r = \vartheta = r_2 = \mu = \mu_2$; $\alpha_1 = a$; $E_1 = b$; $x_2 = y$ and x for x_1, we get

$$I_n^{(a,b)}(x, y) = \frac{(b)_n\ x^n}{n!\ (-\lambda^1)_n} \sum_{i=0}^{n} \frac{(-p^1)^{nc^1-i}}{(nc^1-i)!} \frac{G_i^{\lambda^1}(x_1^r, s^1, p^1)}{i!} \times F\left[\begin{matrix} -n, a; -nc^1+i, 1+\lambda^1-n; \\ \qquad \frac{y}{x}\left(\frac{-1}{p^1}\right)^{c^1} \\ 1-b-n; \end{matrix}\right] = g_n^{(a,b)}(x, y)$$

where $g_n^{(a,b)}(x, y)$ are the Lagrange Polynomials.

7. **Bedient Polynomials :**
Putting $q = 0 = u = v$; $p = 1 = h = g = r = x_2 = \upsilon = \mu_2$; $r_2 = 2 = \mu$; $c_1 = \alpha$, $E_1 = \beta$, $F_1 = \alpha + \beta$; and x for we get

$$G_n(\alpha, \beta, x) = \frac{(\alpha)_n (\beta)_n (2x)^n}{(\alpha+\beta)_n (-\lambda^1)^n\ n!} \sum_{i=0}^{n} \frac{(-p^1)^{nc^1-i}}{(nc^1-i)!} \frac{G_i^{\lambda^1}(x_1^r, s^1, p^1)}{i!} \times F\left[\begin{matrix} -\frac{n}{2}, -\frac{n}{2}+\frac{1}{2}, 1-\alpha-\beta-n; \\ \qquad \frac{-1}{x^2} \frac{(-1)^{c^1}}{(-p^1)^{c^1}} \\ 1-\alpha-n, 1-\beta-n; \end{matrix}\right]$$

where $G_n(\alpha, \beta, x)$ are the Bediend Polynomials.

8. **Gegenbouer Polynomials :**
On taking $p = 0 = q = h = v = u$; $g = 1 = r = x_2 = \upsilon$; $r_2 = 2 = \mu = \mu_2 = 4$; $E_1 = \upsilon$ and writting x for x_1, we get

$$C_n^v(x) = \frac{(\upsilon)_n (2x)^n}{n!(-\lambda^1)^n} \sum_{i=0}^{n} \frac{(-p^1)^{nc^1-i}}{(nc^1-i)!} \frac{G_i^{\lambda^1}(x_1^r, s^1, p^1)}{i!} \times F\left[\begin{matrix} \frac{-n}{2}, \frac{-n}{2}+\frac{1}{2}, -nc^1+i, 1+\lambda^1-n; \\ \qquad \frac{(-1)^{c^1+1}}{x^2(-p^1)^{c^1}} \\ 1-v-n; \end{matrix}\right]$$

where $C_n^{(\upsilon)}(x)$ are the Gegenbauer Polynomials.

V. CONCLUSION AND FUTURE SCOPE

In this paper we have obtained many interesting new results for the generalized hypergeometric polynomial set $B_n(x_1,x_2,x_3)$ followed by important and interesting particular cases. Out of these particular results some of them stand for well known and some of them are believed to be new. These are at most important for mathematicians, scientists and engineers.

VI. ACKNOWLEDGMENT

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