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On Soft Continuity And Soft Connectedness In Soft Generalized Topological Spaces

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Abstract:
In this paper, we introduced the notion of some new types of soft continuous functions such as soft minimal (μ, η) continuous, soft maximal (μ, η)-continuous and soft para (μ, η) continuous in soft generalized topological spaces. Some basic concepts and their properties of these types of soft continuous functions in soft generalized topological spaces are studied. Also, we introduced soft μ-connectedness and soft μ-hyperconnectedness in soft generalized topological spaces.

Key Word: *soft minimal (μ, η)-continuous; soft μ-connected; soft μ-hyperconnected.*

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I. Introduction

In 1999 D. Molodtsov [17] initiated the concept of soft set theory as a mathematical tool for modelling uncertainties. A soft set is a collection of approximate descriptions of an object. Maji et al. [16] have further improved the theory of soft sets. NaimCagman et al. [1] modified the definition of soft sets which is similar to that of Molodtsov. Muhammad Shabir et al. [25] introduced soft topological spaces. In 2002 A. Csaszar [9] introduced the concept of generalized topology and also studied some of its properties. Let X be a non-empty set and ξ be a collection of subsets of X. Then ξ is called a generalized topology (briefly GT) on X if and only if $\emptyset \in \xi$ and $G_i \in \xi$ for $i \in J$ implies $\bigcup_{i\in J} G_i \in \xi$. Sunil Jacob John et al. [13] introduced the concept of soft generalized topological spaces in 2014. In the year 2001 and 2003, F. Nakaoka and N.oda [18], [19] introduced and studied minimal/maximal open sets which are subclasses of open sets. Qays Shakir [21] introduced and studied the concept of minimal and maximal soft open sets in soft topological spaces. B. Roy and R. Sen [22] introduced the concept of maximal μ-open and minimal μ-closed sets via generalized topology. C. Chetana and K. Naganagouda [8] introduced the notion of soft minimal continuous and soft maximal continuous in soft topological spaces. Sunil Jacob John [14] also introduced some interesting properties of the soft mapping $\pi : S(U)_E \rightarrow S(U)_E$ which satisfy the condition $\pi F_B \subset \pi F_D$ whenever $F_B \subset F_D \subset F_{\tilde{E}}$ in soft π-open sets in soft generalized topological spaces in 2015. S.S. Benchalli et.al [6] introduced and studied the concept of weaker forms of soft nano open sets in soft nano topological spaces in 2017. I. Basdouri et.al [3] introduced connected and hyperconnected generalized topological spaces in 2016.

These concepts motivate us to enhance our study in soft minimal (μ, η)-continuous, soft maximal (μ, η)-continuous and soft para (μ, η) continuous in soft generalized topological spaces. Also, we introduced soft μ-connectedness and soft μ-hyperconnectedness in soft generalized topological spaces.

II. Preliminaries

Definition: 2.1 [14]
A soft set F_A on the universe U is defined by the set of ordered pairs $F_A = \{(e,\ f_A(e)) / e \in E,\ f_A(e) \in \mathcal{P}(U)\}$ where $f_A : E \longrightarrow \mathcal{P}(U)$ such that $f_A(e) = \emptyset$ if $e \notin A$. Here f_A is called an approximate function of the soft set F_A. The value of $f_A(e)$ may be arbitrary. Some of them may be empty,some may have nonempty intersection. The set of all soft sets over U with E as the parameter set will be denoted by $S(U)_E$ or simply $S(U)$.

Definition: 2.2 [14]
Let $F_A \in S(U)$. If $f_A(e) = \emptyset$, for all $e \in E$, then F_A is called an empty soft set, denoted by $F_\emptyset$. $f_A(e) = \emptyset$ means there is no element in U related to the parameter e in E. Therefore we do not display such elements in the soft sets as it is meaningless to consider such parameters.

Definition: 2.3 [14]
Let $F_A \in S(U)$. If $f_A(e) = U$, for all $e \in A$, then F_A is called an A-universal soft set, denoted by $F_{\tilde{A}}$. If A = E, then the A-universal soft set is called an universal soft set, denoted by $F_{\tilde{E}}$.

Definition: 2.4 [14]
Let $F_A \in S(U)$. Then, the soft complement of F_A, denoted by $(F_A)^c$, is defined by the approximate function $f_{A^c}(e) = (f_A(e))^c$, where $(f_A(e))^c$ is the complement of the set $f_A(e)$, that is, $(f_A(e))^c = U \setminus f_A(e)$ for all $e \in E$.

Definition: 2.5 [13]
Let $F_A \in S(U)$. A Soft Generalized Topology (SGT) on F_A, denoted by μ (or) μ_{F_A} is a collection of soft subsets of F_A having the following properties:

i. $F_\emptyset \in \mu$
ii. $\{F_{A_i} \subseteq F_A$ / i ∈ J ⊆ N$\} \subseteq \mu \implies \bigcup_{i\in J} F_{A_i} \in \mu$
The pair (F_A, μ) is called a Soft Generalized Topological Space (SGTS).
Observe that $F_A \in \mu$ must not hold.

Definition: 2.6 [13]
Let (F_A, μ) be a SGTS. Then every element of μ is called a soft μ-open set.

Definition: 2.7 [18]
Let (X, τ) be a topological space. A nonempty open set U of X is said to be a minimal open set if and only if any open set which is contained in U is $\emptyset$ or U.

Definition: 2.8 [19]
Let (X, τ) be a topological space. A proper nonempty open subset U of X is said to be a *maximal open set* if any open set which contains U is X or U.

Definition: 2.9 [20]
Let F be a subset of a topological space X. Then the following *duality principle* holds:
(1) F is a minimal closed set if and only if $X - F$ is a maximal open set.
(2) F is a maximal closed set if and only if $X - F$ is a minimal open set.

Definition: 2.10 [5]
Any open subset U of a topological space X is said to be a paraopen set if it is neither a minimal open nor a maximal open set.

Definition: 2.11 [5]
Any closed subset F of a topological space X is said to be a paraclosed set if and only if its complement $(X - F)$ is a paraopen set.

Definition: 2.12 [21]
A proper nonempty soft open subset F_K of a soft topological space $(F_A, \tilde{\tau})$ is said to be minimal soft open set if any soft open set which is contained in F_K is $F_\emptyset$ or F_K.

Definition 2.13 [21]
A proper nonempty soft open subset F_K of a soft topological space $(F_A, \tilde{\tau})$ is said to be maximal soft open set if any soft open set which contains F_K is F_A or F_K.

Definition: 2.14 [14]
Let (F_A, μ) be a SGTS and $F_B \subseteq F_A$. Then the soft μ-interior of F_B, denoted by $i_\mu(F_B)$ is defined as the soft union of all soft μ-open subsets of F_B. Note that $i_\mu(F_B)$ is the largest soft μ-open set that is contained in F_B.

Definition: 2.15 [14]
Let (F_A, μ) be a SGTS and $F_B \subseteq F_A$. Then the soft μ-closure of F_B, denoted by $c_\mu(F_B)$ is defined as the soft intersection of all soft μ-closed super sets of F_B. Note that $c_\mu(F_B)$ is the smallest soft μ-closed superset of F_B.

Definition: 2.16 [14]

Let $(F_{\tilde{E}}, \mu)$ be a SGTS. Then a soft set $F_G \subset F_{\tilde{E}}$ is said to be a soft μ-semi-open set iff $F_G \subset c_\mu i_\mu F_G$ (i.e, the case when $\pi = c_\mu i_\mu$). The class of all soft μ-semi-open sets is denoted by $\delta_{(\mu)}$ *or* δ_μ.

Definition: 2.17 [14]

Let $(F_{\tilde{E}}, \mu)$ be a SGTS. Then a soft set $F_G \subset F_{\tilde{E}}$ is said to be a soft μ-pre-open set iff $F_G \subset i_\mu c_\mu F_G$ (i.e, the case when $\pi = i_\mu c_\mu$). The class of all soft μ-pre-open sets is denoted by $\rho_{(\mu)}$ *or* ρ_μ.

Definition: 2.18 [14]

Let $(F_{\tilde{E}}, \mu)$ be a SGTS. Then a soft set $F_G \subset F_{\tilde{E}}$ is said to be a soft μ-α-open set iff $F_G \subset i_\mu c_\mu i_\mu F_G$ (i.e, the case when $\pi = i_\mu c_\mu i_\mu$). The class of all soft μ-α-open sets is denoted by $\alpha_{(\mu)}$ *or* α_μ.

Definition: 2.19 [14]

Let $(F_{\tilde{E}}, \mu)$ be a SGTS. Then a soft set $F_G \subset F_{\tilde{E}}$ is said to be a soft μ-β-open set iff $F_G \subset c_\mu i_\mu c_\mu F_G$ (i.e, the case when $\pi = c_\mu i_\mu c_\mu$). The class of all soft μ-β-open sets is denoted by $\beta_{(\mu)}$ *or* β_μ.

Throughout this paper, we symbolize soft generalized topological space, soft generalized topological subspace, soft minimal (μ,η)-continuous, soft maximal (μ,η)-continuous, soft para (μ,η)-continuous and the set of all soft μ-interior and soft μ-closure in a soft generalized topological space $(F_{\tilde{E}}, \mu)$ by SGTS, SGTSS, soft M*in* (μ,η)-continuous, soft M*ax* (μ,η)-continuous, soft P*ara* (μ,η)-continuous, $S\mu - Int(F_{\tilde{E}})$ and $S\mu - Cl(F_{\tilde{E}})$ respectively.

III. SOFT MINIMAL (μ, η)-CONTINUOUS

Definition: 3.1

Let (F_A, μ) and (F_B, η) be two SGTS's. A soft mapping $\psi_\chi : (F_A, \mu) \longrightarrow (F_B, \eta)$ is said to be soft minimal (μ, η)-continuous (soft M*in* (μ, η)-continuous) if for each soft minimal η-open set F_L in F_B, its inverse image ${\psi_\chi}^{-1}(F_L)$ is a soft μ-open set in F_A.

Example: 3.2

Let $\mathcal{V} = \{v_1, v_2, v_3, v_4, v_5, v_6\}$, $\mathcal{E} = \{\widehat{e_1}, \widehat{e_2}, \widehat{e_3}\}$, $\mathfrak{A} = \{\widehat{e_1}, \widehat{e_2}\} \cong \mathcal{E}$, then $(F_{\mathfrak{A}}, \mu) = \{F_\emptyset, F_{\mathfrak{A}_1}, F_{\mathfrak{A}_2}, F_{\mathfrak{A}_3}\}$ is a SGTS where

$F_\emptyset = \{(\widehat{e_1}, \emptyset), (\widehat{e_2}, \emptyset)\}$
$F_{\mathfrak{A}} = \{(\widehat{e_1}, \{v_1, v_2, v_3, v_4, v_5\}), (\widehat{e_2}, \{v_1, v_2, v_4, v_5\})\}$
$F_{\mathfrak{A}_1} = \{(\widehat{e_1}, \{v_1, v_3, v_4, v_5\}), (\widehat{e_2}, \{v_1, v_4, v_5\})\}$
$F_{\mathfrak{A}_2} = \{(\widehat{e_1}, \{v_1, v_3, v_5\}), (\widehat{e_2}, \{v_1, v_5\})\}$
$F_{\mathfrak{A}_3} = \{(\widehat{e_1}, \{v_4, v_5\}), (\widehat{e_2}, \{v_1, v_4\})\}$

Let $\mathcal{K} = \{k_1, k_2, k_3, k_4, k_5\}$, $\mathfrak{J} = \{j_1', j_2', j_3'\}$, $\mathfrak{B} = \{j_1', j_2'\} \cong \mathfrak{J}$, then $(F_{\mathfrak{B}}, \eta) = \{F_\emptyset, F_{\mathfrak{B}_1}, F_{\mathfrak{B}_2}, F_{\mathfrak{B}_3}\}$ is a SGTS where

$F_\emptyset = \{(j_1', \emptyset), (j_2', \emptyset)\}$
$F_{\mathfrak{B}} = \{(j_1', \{k_1, k_2, k_3, k_4\}), (j_2', \{k_1, k_2, k_3\})\}$
$F_{\mathfrak{B}_1} = \{(j_1', \{k_2, k_3, k_4\}), (j_2', \{k_2, k_3\})\}$
$F_{\mathfrak{B}_2} = \{(j_1', \{k_3, k_4\}), (j_2', \{k_3\})\}$
$F_{\mathfrak{B}_3} = \{(j_1', \{k_2, k_4\}), (j_2', \{k_2, k_3\})\}$

Define a map $\psi : \mathcal{V} \longrightarrow \mathcal{K}$ by $\psi(v_1) = k_3$, $\psi(v_2) = k_2$, $\psi(v_3) = k_4$, $\psi(v_4) = k_1$, $\psi(v_5) = k_3$, $\psi(v_6) = k_5$ and $\chi : \mathcal{E} \longrightarrow \mathfrak{J}$ by $\chi(\widehat{e_1}) = j_1'$, $\psi(\widehat{e_2}) = j_2'$, $\psi(\widehat{e_3}) = j_3'$. Then $\psi_\chi : (F_{\mathfrak{A}}, \mu) \longrightarrow (F_{\mathfrak{B}}, \eta)$ is a soft M*in* (μ, η)-continuous function.

Definition: 3.3

The SGTS (F_A, μ) is said to be soft μ-$\mathcal{T}_{\mathrm{M}in}$space if for every non-null proper soft μ-open subset of F_A is soft minimal μ-open.

Proposition: 3.4

Each soft (μ, η)-continuous function is soft M*in* (μ, η)-continuous.

Proof:

Let $\psi_\chi : (F_A,\mu) \longrightarrow (F_B,\eta)$ be a soft (μ,η)-continuous function. Let F_{B_i} be any soft minimal η-open set in F_B. Since each soft minimal η-open set is a soft η-open set, F_{B_i} is a soft η-open set in F_B. By hypothesis, ${\psi_\chi}^{-1}(F_{B_i})$ is a soft μ-open set in F_A.Hence $\psi_\chi : (F_A,\mu) \longrightarrow (F_B,\eta)$ is soft Min (μ,η)-continuous.

Remark: 3.5

The converse of the preceding proposition holds only if $\psi_\chi : (F_A,\mu) \longrightarrow (F_B,\eta)$ is an onto map and F_B is a soft η-$\mathcal{T}_{\mathrm{M}in}$space.

Proof:

Let $\psi_\chi : (F_A,\mu) \longrightarrow (F_B,\eta)$ be a soft Min (μ,η)-continuous, onto map. Let F_{B_k}be any nonempty proper soft η-open set in F_B. Since F_B is a soft η-$\mathcal{T}_{\mathrm{M}in}$ space, F_{B_k} is a soft minimal η-open set in F_B. Also, by the definition of soft Min (μ,η)-continuous, ${\psi_\chi}^{-1}(F_{B_k})$ is a soft μ-open set in F_A. Hence $\psi_\chi : (F_A,\mu) \longrightarrow (F_B,\eta)$ is soft (μ,η)-continuous.

Remark: 3.6

The composition of two soft Min (μ,η)-continuous function need not be soft Min (μ,η)-continuous.

Theorem: 3.7

The previous remark holds only if $\psi_\chi : (F_A,\mu) \longrightarrow (F_B,\eta)$ is a soft (μ,η)-continuous function and $\Phi_\zeta : (F_B,\eta) \longrightarrow (F_C,\xi)$ is a soft Min (η,ξ)-continuous map.

Proof:

Let F_{C_i} be any soft minimal ξ-open set in F_C. As Φ_ζ is a soft Min (η,ξ)-continuous function, ${\Phi_\zeta}^{-1}(F_{C_i})$ is a soft η-open set in F_B. Since ψ_χ is soft (μ,η)-continuous, ${\psi_\chi}^{-1}({\Phi_\zeta}^{-1}(F_{C_i}))$ is a soft μ-open set in F_A. Hence $\Phi_\zeta\ o\ \psi_\chi$ is a soft Min (μ,η)-continuous map.

Theorem: 3.8

Let (F_A,μ) and (F_B,η) be two SGTS's. If $\psi_\chi : (F_A,\mu) \longrightarrow (F_B,\eta)$ is a soft Min (μ,η)-continuous mapping and (F_H,ξ) is a SGTSS of F_A, then the restricted map $\psi_{\chi|F_H} : (F_H,\xi) \longrightarrow (F_B,\eta)$ is soft Min (ξ,η)-continuous.

Proof:

Let F_{B_l} be any soft minimal η-open set in F_B. Then, by definition 3.1, ${\psi_\chi}^{-1}(F_{B_l})$ is a soft μ-open set in F_A. Consider $(\psi_{\chi|F_H})^{-1}(F_{B_l}) = F_H \tilde{\cap} {\psi_\chi}^{-1}(F_{B_l})$, where $F_H \tilde{\cap} {\psi_\chi}^{-1}(F_{B_l})$ is a soft ξ-open set in F_H which implies $(\psi_{\chi|F_H})^{-1}(F_{B_l})$ is a soft ξ-open set in F_H. Hence $\psi_{\chi|F_H} : (F_H,\xi) \longrightarrow (F_B,\eta)$ is soft Min (ξ,η)-continuous.

IV. SOFT MAXIMAL (μ,η)-CONTINUOUS

Definition: 4.1

Let (F_A,μ) and (F_B,η) be two SGTS's. A soft mapping $\psi_\chi : (F_A,\mu) \longrightarrow (F_B,\eta)$ is said to be soft maximal (μ,η)-continuous (soft Max (μ,η)-continuous) if for each soft maximal η-open set F_G in F_B, its inverse image ${\psi_\chi}^{-1}(F_G)$ is a soft μ-open set in F_A.

Definition: 4.2

The SGTS (F_A,μ) is said to be soft μ-$\mathcal{T}_{\mathrm{M}ax}$ space if for every non-null proper soft μ-open subset of F_A is soft maximal μ-open.

Proposition: 4.3

Each soft (μ,η)-continuous function is soft Max (μ,η)-continuous.

Proof:

Let $\psi_\chi : (F_A,\mu) \longrightarrow (F_B,\eta)$ be a soft (μ,η)-continuous function. Let F_{B_i} be any soft maximal η-open set in F_B. Since each soft maximal η-open set is a soft η-open set, F_{B_i} is a soft η-open set in F_B. As ψ_χ is soft (μ,η)-continuous, its inverse image ${\psi_\chi}^{-1}(F_{B_i})$ is a soft μ-open set in F_A. Therefore $\psi_\chi : (F_A,\mu) \longrightarrow (F_B,\eta)$ is soft Max (μ,η)-continuous.

Remark: 4.4

The converse of the proposition 4.3 is true only if $\psi_\chi : (F_A, \mu) \longrightarrow (F_B, \eta)$ is an onto map and F_B is a soft η-$\mathcal{T}_{\mathrm{M}ax}$ space.

Proof:

Let $\psi_\chi : (F_A, \mu) \longrightarrow (F_B, \eta)$ be a soft M*ax* (μ, η)-continuous, onto map. Let F_{B_j} be any nonempty proper soft η-open set in F_B. Since F_B is a soft η-$\mathcal{T}_{\mathrm{M}ax}$ space, F_{B_j} is a soft maximal η-open set in F_B. Also, by the definition of soft M*ax* (μ, η)-continuous, ${\psi_\chi}^{-1}\left(F_{B_j}\right)$ is a soft μ-open set in F_A. Hence $\psi_\chi : (F_A, \mu) \longrightarrow (F_B, \eta)$ is soft (μ, η)-continuous.

Remark: 4.5

The composition of two soft M*ax* (μ, η)-continuous function need not be soft M*ax* (μ, η)-continuous.

Theorem: 4.6

The above remark is true only if $\psi_\chi : (F_A, \mu) \longrightarrow (F_B, \eta)$ is a soft (μ, η)-continuous function and $\Phi_\zeta : (F_B, \eta) \longrightarrow (F_C, \xi)$ is a soft M*ax* (η, ξ)-continuous map.

Proof:

Let F_{C_t} be any soft maximal ξ-open set in F_C. As Φ_ζ is a soft M*ax* (η, ξ)-continuous function, ${\Phi_\zeta}^{-1}\left(F_{C_t}\right)$ is a soft η-open set in F_B. Since ψ_χ is soft (μ, η)-continuous, ${\psi_\chi}^{-1}({\Phi_\zeta}^{-1}\left(F_{C_t}\right))$ is a soft μ-open set in F_A. Hence $\Phi_\zeta \; o \; \psi_\chi$ is a soft M*ax* (μ, η)-continuous map.

Theorem: 4.7

Let (F_A, μ) and (F_B, η)be two SGTS's. If $\psi_\chi : (F_A, \mu) \longrightarrow (F_B, \eta)$ is a soft M*ax* (μ, η)-continuous mapping and (F_H, ξ) is a SGTSS of F_A, then the restricted map $\psi_{\chi|F_H} : (F_H, \xi) \longrightarrow (F_B, \eta)$ is soft M*ax* (ξ, η)-continuous.

Proof:

Let F_{B_w} be any soft maximal η-open set in F_B. Then, by definition 4.1, ${\psi_\chi}^{-1}\left(F_{B_w}\right)$ is a soft μ-open set in F_A. Consider $(\psi_{\chi|F_H})^{-1}\left(F_{B_w}\right) = F_H \mathbin{\tilde{\cap}} {\psi_\chi}^{-1}\left(F_{B_w}\right)$, where $F_H \mathbin{\tilde{\cap}} {\psi_\chi}^{-1}\left(F_{B_w}\right)$ is a soft ξ-open set in F_H which implies $(\psi_{\chi|F_H})^{-1}\left(F_{B_w}\right)$ is a soft ξ-open set in F_H. Hence $\psi_{\chi|F_H} : (F_H, \xi) \longrightarrow (F_B, \eta)$ is soft M*ax* (ξ, η)-continuous.

V. SOFT PARA (μ, η)-CONTINUOUS

Definition: 5.1

Let (F_A, μ) and (F_B, η) be two SGTS's. A soft mapping $\psi_\chi : (F_A, \mu) \longrightarrow (F_B, \eta)$ is said to be soft para (μ, η)-continuous (soft P*ara* (μ, η)-continuous) if for each soft para η-open set F_s in F_B, its inverse image ${\psi_\chi}^{-1}(F_s)$ is a soft μ-open set in F_A.

Definition: 5.2

The SGTS (F_A, μ) is said to be soft μ-$\mathcal{T}_{\mathrm{Para}}$ space if for every non-null proper soft μ-open subset of F_A is soft para μ-open.

Proposition: 5.3

Each soft (μ, η)-continuous function is soft P*ara* (μ, η)-continuous.

Proof:

Let $\psi_\chi : (F_A, \mu) \longrightarrow (F_B, \eta)$ be a soft (μ, η)-continuous function. Let F_{B_i} be any soft para η-open set in F_B. Since each soft para η-open set is a soft η-open set, F_{B_i} is a soft η-open set in F_B. As ψ_χ is soft (μ, η)-continuous, its inverse image ${\psi_\chi}^{-1}(F_{B_i})$ is a soft μ-open set in F_A. Hence $\psi_\chi : (F_A, \mu) \longrightarrow (F_B, \eta)$ is soft P*ara* (μ, η)-continuous.

Remark: 5.4

The converse of the above proposition holds only if $\psi_\chi : (F_A, \mu) \longrightarrow (F_B, \eta)$ is an onto map and F_B is a soft η- $\mathcal{T}_{\mathrm{Para}}$ space.

Proof:

Let $\psi_\chi : (F_A, \mu) \longrightarrow (F_B, \eta)$ be a soft P*ara* (μ, η)-continuous, onto map. Let F_{B_l} be any nonempty proper soft η-open set in F_B. Since F_B is a soft η- $\mathcal{T}_{\text{Para}}$ space, F_{B_l} is a soft para η-open set in F_B. Also, by the definition of soft P*ara* (μ, η)-continuous, ${\psi_\chi}^{-1}(F_{B_l})$ is a soft μ-open set in F_A. Hence $\psi_\chi : (F_A, \mu) \longrightarrow (F_B, \eta)$ is soft (μ, η)-continuous.

Remark: 5.5

The composition of two soft P*ara* (μ, η)-continuous function need not be soft P*ara* (μ, η)-continuous.

Theorem: 5.6

The above remark holds only if $\psi_\chi : (F_A, \mu) \longrightarrow (F_B, \eta)$ is a soft (μ, η)-continuous function and $\Phi_\zeta : (F_B, \eta) \longrightarrow (F_C, \xi)$ is a soft P*ara* (η, ξ)-continuous map.

Proof:

Let F_{C_m} be any soft para ξ-open set in F_C. As Φ_ζ is a soft P*ara* (η, ξ)-continuous function, ${\Phi_\zeta}^{-1}(F_{C_m})$ is a soft η-open set in F_B. Since ψ_χ is soft (μ, η)-continuous, ${\psi_\chi}^{-1}({\Phi_\zeta}^{-1}(F_{C_m}))$ is a soft μ-open set in F_A. Hence $\Phi_\zeta \ o \ \psi_\chi$ is a soft P*ara* (μ, η)-continuous map.

Theorem: 5.7

Let (F_A, μ) and (F_B, η) be two SGTS's. If $\psi_\chi : (F_A, \mu) \longrightarrow (F_B, \eta)$ is a soft P*ara* (μ, η)-continuous mapping and (F_H, ξ) is a SGTSS of F_A, then the restricted map $\psi_{\chi|F_H} : (F_H, \xi) \longrightarrow (F_B, \eta)$ is soft P*ara* (ξ, η)- continuous.

Proof:

Let F_{B_z} be any soft para η-open set in F_B. Then, by definition 5.1, ${\psi_\chi}^{-1}(F_{B_z})$ is a soft μ-open set in F_A. Consider $(\psi_{\chi|F_H})^{-1}(F_{B_z}) = F_H \tilde{\cap} {\psi_\chi}^{-1}(F_{B_z})$, where $F_H \tilde{\cap} {\psi_\chi}^{-1}(F_{B_z})$ is a soft ξ-open set in F_H which implies $(\psi_{\chi|F_H})^{-1}(F_{B_z})$ is a soft ξ-open set in F_H. Hence $\psi_{\chi|F_H} : (F_H, \xi) \longrightarrow (F_B, \eta)$ is soft P*ara* (ξ, η)-continuous.

VI. SOFT μ-CONNECTEDNESS

Definition: 6.1

Let $(F_{\tilde{E}}, \mu)$ be a SGTS. Then a soft μ-separation of $F_{\tilde{E}}$ is a pair of soft disjoint non-null soft μ-open subsets of $F_{\tilde{E}}$ whose soft union is $F_{\tilde{E}}$.

Definition: 6.2

A space $F_{\tilde{E}}$ is said to be soft μ-connected if there does not exist a soft μ-separation of $F_{\tilde{E}}$.

Definition: 6.3

Let $(F_{\tilde{E}}, \mu)$ be a SGTS. Then $(F_{\tilde{E}}, \mu)$ is said to be soft μ-connected if there $\nexists$ two soft μ-open sets $F_J, F_K \tilde{\in} \mu - F_\emptyset$ such that $F_J \tilde{\cap} F_K = F_\emptyset$ and $F_J \tilde{\cup} F_K = F_{\tilde{E}}$ or else $(F_{\tilde{E}}, \mu)$ is soft μ-disconnected.

Example: 6.4

Let $\mathcal{W} = \{w_1, w_2, w_3\}$, $E = \{\widehat{\mathcal{g}_1}, \widehat{\mathcal{g}_2}, \widehat{\mathcal{g}_3}\}$ and $\mu = \{F_\emptyset, F_{A_1}, F_{A_2}, F_{A_3}, F_{A_4}, F_{\tilde{E}}\}$ is a SGTS where

$F_{A_1} = \{(\widehat{\mathcal{g}_1}, \{w_1, w_2\}), (\widehat{\mathcal{g}_2}, \{w_1, w_3\})\}$

$F_{A_2} = \{(\widehat{\mathcal{g}_1}, \{w_1, w_3\}), (\widehat{\mathcal{g}_2}, \{\mathcal{W}\})\}$

$F_{A_3} = \{(\widehat{\mathcal{g}_1}, \{w_2, w_3\}), (\widehat{\mathcal{g}_2}, \{w_1, w_2\})\}$

$F_{A_4} = \{(\widehat{\mathcal{g}_1}, \{w_2\}), (\widehat{\mathcal{g}_2}, \{w_1\})\}$

Then $(F_{\tilde{E}}, \mu)$ is a soft μ-connected space.

Remark: 6.5

Even though for each parameter g, $(F_{\tilde{E}}, \mu_g)$ is soft μ-connected, the space $(F_{\tilde{E}}, \mu)$ need not be soft μ-connected.

Definition: 6.6

Let $(F_{\tilde{E}}, \mu)$ be a SGTS. Any soft μ-open subset $F_Q \tilde{\subseteq} F_{\tilde{E}}$ is said to be soft μ-connected, if it is soft μ-connected as a SGTSS.

Theorem: 6.7

Let $(F_{\tilde{E}}, \mu)$ be a SGTS. Then the following are equivalent:

(i) $F_{\tilde{E}}$ is soft μ-connected.

(ii) $F_{\tilde{E}}$ cannot be written as the soft union of two soft disjoint non-null soft μ-closed sets.

(iii) $F_{\tilde{E}}$ cannot be written as the soft union of two non-null soft sets and such that $F_{\mathcal{K}} \tilde{\cap}\ S\mu - Cl(F_{\mathcal{L}}) = F_{\mathcal{L}} \tilde{\cap} S\mu - Cl(F_{\mathcal{K}}) = F_{\emptyset}$

(iv) $F_{\emptyset}$ and $F_{\tilde{E}}$ are the only soft sets which are both soft μ-open and soft μ-closed in $F_{\tilde{E}}$.

Proof:

(i) ⇒ (ii):

Suppose $F_{\tilde{E}} = F_{\mathcal{K}} \tilde{\cup} F_{\mathcal{L}}$, where $F_{\mathcal{K}}$ and $F_{\mathcal{L}}$ are two non-null soft μ-closed sets such that $F_{\mathcal{K}} \tilde{\cap} F_{\mathcal{L}} = F_{\emptyset}$. Since $F_{\mathcal{K}}$ and $F_{\mathcal{L}}$ are soft μ-closed sets, $F_{\mathcal{K}}{}^{\tilde{c}}$ and $F_{\mathcal{L}}{}^{\tilde{c}}$are soft μ-open. From hypothesis, $F_{\mathcal{K}} \tilde{\cap} F_{\mathcal{L}} = F_{\emptyset}$ which conclude that $F_{\mathcal{L}}$ and $F_{\mathcal{K}}$ are soft μ-open sets. Hence $F_{\tilde{E}}$ is the soft union of two soft disjoint non-null soft μ-open sets. Thus $F_{\tilde{E}}$ is soft μ-disconnected which is a contradiction.

(ii) ⇒ (iii):

Consider $F_{\tilde{E}} = F_{\mathcal{K}} \tilde{\cup} F_{\mathcal{L}}$, where $F_{\mathcal{K}}$ and $F_{\mathcal{L}}$ are two non-null soft sets such that $F_{\mathcal{K}} \tilde{\cap}\ S\mu - Cl(F_{\mathcal{L}}) = F_{\mathcal{L}} \tilde{\cap} S\mu - Cl(F_{\mathcal{K}}) = F_{\emptyset}$. Let $\alpha \tilde{\in}\ S\mu - Cl(F_{\mathcal{K}})$. Since $F_{\mathcal{L}} \tilde{\cap} S\mu - Cl(F_{\mathcal{K}}) = F_{\emptyset}, \alpha \tilde{\notin} F_{\mathcal{L}}$. Since $F_{\tilde{E}} = F_{\mathcal{K}} \tilde{\cup} F_{\mathcal{L}}$, $\alpha \tilde{\in}\ F_{\mathcal{K}}$ which implies $S\mu - Cl(F_{\mathcal{K}})\ \tilde{\subseteq}\ F_{\mathcal{K}}$. Hence $F_{\mathcal{K}}$ is soft μ-closed. Similarly, $F_{\mathcal{L}}$ is soft μ-closed. Also, $F_{\mathcal{K}} \tilde{\cap} F_{\mathcal{L}} = F_{\mathcal{K}} \tilde{\cap}\ S\mu - Cl(F_{\mathcal{L}}) = F_{\emptyset}$.Hence $F_{\tilde{E}} = F_{\mathcal{K}} \tilde{\cup} F_{\mathcal{L}}$, where $F_{\mathcal{K}} \neq F_{\emptyset}$ and $F_{\mathcal{L}} \neq F_{\emptyset}$, $F_{\mathcal{K}}$ and $F_{\mathcal{L}}$ are two soft μ-closed sets and $F_{\mathcal{K}} \tilde{\cap} F_{\mathcal{L}} = F_{\emptyset}$ which contradicts (ii).

(iii) ⇒ (iv):

Suppose (iv) is not true. Then there exists $F_{\mathcal{K}} \tilde{\subseteq}\ F_{\tilde{E}}$ such that $F_{\mathcal{K}} \neq F_{\tilde{E}}$ and $F_{\mathcal{K}} \neq F_{\emptyset}$ where $F_{\mathcal{K}}$ is both soft μ-open and soft μ-closed. Let $F_{\mathcal{K}}{}^{\tilde{c}} = F_{\mathcal{L}}$.Then $F_{\mathcal{L}} \neq F_{\emptyset}$ is both soft μ-open and soft μ-closed. Hence $F_{\tilde{E}} = F_{\mathcal{K}} \tilde{\cup} F_{\mathcal{L}}$, $S\mu - Cl(F_{\mathcal{K}})\ \tilde{\cap}\ F_{\mathcal{L}} = F_{\emptyset}$. Similarly $F_{\mathcal{K}} \tilde{\cap}\ S\mu - Cl(F_{\mathcal{L}}) = F_{\emptyset}$. Hence $F_{\tilde{E}} = F_{\mathcal{K}} \tilde{\cup} F_{\mathcal{L}}$, where $S\mu - Cl(F_{\mathcal{K}})\ \tilde{\cap}\ F_{\mathcal{L}} = F_{\emptyset}$ and $F_{\mathcal{K}} \tilde{\cap}\ S\mu - Cl(F_{\mathcal{L}}) = F_{\emptyset}$ which is a ⇒⇐ to (iii).

(iv)⇒ (i):

Suppose $F_{\tilde{E}}$ is soft μ-disconnected. Then $F_{\tilde{E}} = F_{\mathcal{K}} \tilde{\cup} F_{\mathcal{L}}$, $F_{\mathcal{K}}$ and $F_{\mathcal{L}}$ are two non-null soft μ-open sets and $F_{\mathcal{K}} \tilde{\cap} F_{\mathcal{L}} = F_{\emptyset}$. Then $F_{\mathcal{L}}{}^{\tilde{c}} = F_{\mathcal{K}}$. Since $F_{\mathcal{L}}$ is soft μ-open, $F_{\mathcal{K}}$ is soft μ-closed. Also, $F_{\mathcal{K}} \neq F_{\emptyset}$ and $F_{\mathcal{K}} \neq F_{\tilde{E}}$, since $F_{\mathcal{L}} \neq F_{\emptyset}$. Hence $F_{\mathcal{K}}$ is a proper non-null soft subset of $F_{\tilde{E}}$ which is both soft μ-open and soft μ-closed which is a contradiction to (iv).

Theorem 6.8:

Let $(F_{\tilde{E}}, \mu)$ be a SGTS. Then $F_{\tilde{E}}$ is soft μ-connected if and only if the only soft subsets of $F_{\tilde{E}}$ that are both soft μ-open and soft μ-closed in $F_{\tilde{E}}$ are $F_{\emptyset}$ and $F_{\tilde{E}}$.

Proof:

Suppose there exists a proper soft subset $F_{\mathcal{D}}$ which is both soft μ-open and soft μ-closed. Let $F_{\mathcal{C}} = F_{\mathcal{D}}{}^{\tilde{c}}$ Suppose $F_{\mathcal{D}}$ is soft μ-open, then $F_{\mathcal{C}}$ is soft μ-closed and $F_{\mathcal{D}}$ is soft μ-closed, then $F_{\mathcal{C}}$ is soft μ-open. Then $F_{\mathcal{D}} \neq F_{\emptyset}$ and $F_{\mathcal{C}} \neq F_{\emptyset}$, $F_{\mathcal{C}} \tilde{\cap}\ F_{\mathcal{D}} = F_{\emptyset}$ and $F_{\tilde{E}} = F_{\mathcal{C}} \tilde{\cup}\ F_{\mathcal{D}} \Rightarrow F_{\tilde{E}}$ is soft μ-disconnected which is a contradiction. Thus, $F_{\emptyset}$ and $F_{\tilde{E}}$ are the only soft μ-subsets of $F_{\tilde{E}}$ that are both soft μ-open and soft μ-closed in $F_{\tilde{E}}$.

Conversely, Suppose $F_{\tilde{E}}$ is soft μ-disconnected. Then $F_{\tilde{E}} = F_{\mathcal{U}} \tilde{\cup}\ F_{\mathcal{V}}$ where $F_{\mathcal{U}} \neq F_{\emptyset}$, $F_{\mathcal{V}} \neq F_{\emptyset}$, $F_{\mathcal{U}}$ and $F_{\mathcal{V}}$ are soft μ-open sets and $F_{\mathcal{U}} \tilde{\cap}\ F_{\mathcal{V}} = F_{\emptyset}$. Then $F_{\mathcal{V}}{}^{\tilde{c}} = F_{\mathcal{U}}$. Since $F_{\mathcal{V}}$ is soft μ-open, $F_{\mathcal{U}}$ is soft μ-closed. Also $F_{\mathcal{U}} \neq F_{\emptyset}$ and $F_{\mathcal{U}} \neq F_{\tilde{E}}$, since $F_{\mathcal{V}} \neq F_{\emptyset}$. Hence $F_{\mathcal{U}}$ is a proper non-null soft subset of $F_{\tilde{E}}$ which is both soft μ-open and soft μ-closed.

Theorem: 6.9

If $F_{\mathcal{A}}$ and $F_{\mathcal{B}}$ are two soft μ-open sets which forms a soft μ-disconnection in a SGTS $(F_{\tilde{E}}, \mu)$ and $F_{\mathcal{T}}$ is a soft μ-connected SGTSS of $(F_{\tilde{E}}, \mu)$. Then $F_{\mathcal{T}}$ is contained in $F_{\mathcal{A}}$ or $F_{\mathcal{B}}$.

Proof:

Let $F_{\mathcal{A}}$and $F_{\mathcal{B}}$ be two soft μ-open sets in a SGTS $(F_{\tilde{E}}, \mu)$.Then $F_{\mathcal{A}} \tilde{\cap}\ F_{\mathcal{T}} \neq F_{\emptyset}$ and $F_{\mathcal{B}} \tilde{\cap}\ F_{\mathcal{T}} \neq F_{\emptyset}$ are two non-null soft μ-open subsets of $F_{\mathcal{T}}$ such that $(F_{\mathcal{A}} \tilde{\cap}\ F_{\mathcal{T}}) \tilde{\cap} (F_{\mathcal{B}} \tilde{\cap}\ F_{\mathcal{T}}) = F_{\emptyset}$ and $(F_{\mathcal{A}} \tilde{\cap}\ F_{\mathcal{T}}) \tilde{\cup}\ (F_{\mathcal{B}} \tilde{\cap}\ F_{\mathcal{T}}) = F_{\mathcal{T}}$. Hence $F_{\mathcal{A}} \tilde{\cap}\ F_{\mathcal{T}} = F_{\emptyset}$ or $F_{\mathcal{B}} \tilde{\cap}\ F_{\mathcal{T}} = F_{\emptyset}$. If $F_{\mathcal{A}} \tilde{\cap}\ F_{\mathcal{T}} = F_{\emptyset}$, then $F_{\mathcal{T}}$ is contained in $F_{\mathcal{B}}$ and $F_{\mathcal{B}} \tilde{\cap}\ F_{\mathcal{T}} = F_{\emptyset}$, then $F_{\mathcal{T}}$ is contained in $F_{\mathcal{A}}$. Therefore $F_{\mathcal{T}}$ is contained in $F_{\mathcal{A}}$ or $F_{\mathcal{B}}$.

Theorem: 6.10

The soft union of a collection of soft μ-connected sets of a SGTS $(F_{\tilde{E}}, \mu)$ that have a common point is soft μ-connected.

Proof:

Let F_{A_i} be a collection of soft μ-connected subsets of a SGTS $(F_{\tilde{E}}, \mu)$. Suppose F_Y is not soft μ-connected. Then there exists two non-null soft disjoint soft μ-open sets $F_{\mathcal{P}}$ and $F_{\mathcal{Q}}$ such that $F_Y = F_{\mathcal{P}} \tilde{\cup} F_{\mathcal{Q}}$. Let α be a common point in that collection. Then $\alpha \tilde{\in} F_Y$ which implies either $\alpha \tilde{\in} F_{\mathcal{P}}$ or $\alpha \tilde{\in} F_{\mathcal{Q}}$. Since each F_{A_i} is soft μ-connected, F_{A_i} is contained in $F_{\mathcal{P}}$ or $F_{\mathcal{Q}}$. If $\alpha \tilde{\in} F_{\mathcal{P}}$, then F_{A_i} is not contained in $F_{\mathcal{Q}}$ which implies $F_{\mathcal{Q}} = F_{\emptyset}$. Similarly, if $\alpha \tilde{\in} F_{\mathcal{Q}}$, then F_{A_i} is not contained in $F_{\mathcal{P}}$ which implies $F_{\mathcal{P}} = F_{\emptyset}$ which is a contradiction. Hence F_Y is soft μ-connected.

Theorem: 6.11

Let $F_{\mathcal{R}}$ be a SGTSS of a SGTS $(F_{\tilde{E}}, \mu)$. Then $F_{\mathcal{R}}$ is soft μ-separated if neither of which contains a soft μ-limit point of the other and $F_{\mathcal{R}}$ is soft μ-connected if there does not exist a soft μ-separation of $F_{\mathcal{R}}$.

Proof:

Suppose $F_{\mathcal{R}}$ is soft μ-separated. Then $F_{\mathcal{G}} \neq F_{\emptyset}$, $F_{\mathcal{H}} \neq F_{\emptyset}$, $F_{\mathcal{G}} \tilde{\cap} F_{\mathcal{H}} = F_{\emptyset}$, $F_{\mathcal{G}} \tilde{\cup} F_{\mathcal{H}} = F_{\tilde{E}}$, where $F_{\mathcal{G}}$ and $F_{\mathcal{H}}$ are soft μ-open sets. We have $F_{\mathcal{G}}$ and $F_{\mathcal{H}}$ are both soft μ-open and soft μ-closed which imply that $F_{\mathcal{G}} \tilde{\cap} S\mu - Cl(F_{\mathcal{H}}) = F_{\mathcal{G}} \tilde{\cap} (F_{\mathcal{H}} \tilde{\cup} (F_{\mathcal{H}})') = F_{\emptyset}$. Suppose $F_{\mathcal{G}}$ is soft μ-open and $F_{\mathcal{H}}$ is soft μ-closed. Then $F_{\mathcal{G}} \tilde{\cap} F_{\mathcal{H}} = F_{\emptyset} \Rightarrow F_{\mathcal{G}} \tilde{\cap} (F_{\mathcal{H}})' = F_{\emptyset}$. Similarly, if $F_{\mathcal{G}}$ is soft μ-closed and $F_{\mathcal{H}}$ is soft μ-open, then $(F_{\mathcal{G}})' \tilde{\cap} F_{\mathcal{H}} = F_{\emptyset}$. Let $F_{\mathcal{C}} \neq F_{\emptyset}$, $F_{\mathcal{D}} \neq F_{\emptyset}$, $F_{\mathcal{C}} \tilde{\cap} S\mu - Cl(F_{\mathcal{D}}) = F_{\emptyset}$, $S\mu - Cl(F_{\mathcal{C}}) \tilde{\cap} F_{\mathcal{D}} = F_{\emptyset}$, $F_{\mathcal{R}} = F_{\mathcal{C}} \tilde{\cup} F_{\mathcal{D}}$. Now, $S\mu - Cl(F_{\mathcal{C}}) \tilde{\cap} F_{\mathcal{R}} = (S\mu - Cl(F_{\mathcal{C}}) \tilde{\cap} F_{\mathcal{C}}) \tilde{\cup} (S\mu - Cl(F_{\mathcal{C}}) \tilde{\cap} F_{\mathcal{D}}) = F_{\mathcal{C}}$ which implies $F_{\mathcal{C}}$ is soft μ-closed in $F_{\mathcal{R}}$. Similarly, $F_{\mathcal{D}}$ is soft μ-closed in $F_{\mathcal{R}}$ which implies $F_{\mathcal{C}}$ and $F_{\mathcal{D}}$ are both soft μ-open and soft μ-closed in $F_{\mathcal{R}}$. Hence $F_{\mathcal{R}}$ is soft μ-separated.

Theorem: 6.12

Let $F_{\mathcal{P}}$ be a soft μ-connected subset of a SGTS $(F_{\tilde{E}}, \mu)$ and F_I be a soft subset of $F_{\tilde{E}}$ such that $F_{\mathcal{P}} \tilde{\subseteq} F_I \tilde{\subseteq} S\mu - Cl(F_{\mathcal{P}})$, then F_I is soft μ-connected.

Proof:

Suppose F_I is not soft μ-connected. Then there exists two non-null soft μ-open sets F_K and F_V such that $F_K \tilde{\cap} F_V = F_{\emptyset}$ and $F_K \tilde{\cup} F_V = F_I$. Then $F_{\mathcal{P}} \tilde{\subseteq} F_K \tilde{\cup} F_V$ which implies either $F_{\mathcal{P}} \tilde{\subseteq} F_K$ or $F_{\mathcal{P}} \tilde{\subseteq} F_V$. By theorem: 6.11, $S\mu - Cl(F_K) \tilde{\cap} F_V = F_{\emptyset}$ and $S\mu - Cl(F_{\mathcal{P}}) \tilde{\subseteq} S\mu - Cl(F_K)$, $F_I \tilde{\cap} F_V = F_{\emptyset}$ which implies $F_V = F_{\emptyset}$ which is a contradiction. Also, since $F_K \tilde{\cap} S\mu - Cl(F_V) = F_{\emptyset}$ and $S\mu - Cl(F_{\mathcal{P}}) \tilde{\subseteq} S\mu - Cl(F_V)$, $F_I \tilde{\cap} F_K = F_{\emptyset}$ which implies $F_K = F_{\emptyset}$ which is a contradiction. Thus F_I is soft μ-connected.

Corollary: 6.13

Let F_A be a soft μ-connected SGTSS of $(F_{\tilde{E}}, \mu)$. Then $S\mu - Cl(F_A)$ is soft μ-connected.

VII. SOFT μ-CONNECTEDNESS IN WEAKER FORMS OF SOFT μ-OPEN SETS

Definition: 7.1

Let $(F_{\tilde{E}}, \mu)$ be a SGTS. Then $F_{\tilde{E}}$ is called

(i) soft μ-ρ-connected if $\nexists$ two soft μ-pre-open sets F_B, $F_Z \tilde{\in} \mu - F_{\emptyset}$ such that $F_B \tilde{\cap} F_Z = F_{\emptyset}$ and $F_B \tilde{\cup} F_Z = F_{\tilde{E}}$

(ii) soft μ-δ-connected if $\nexists$ two soft μ-semi-open sets F_B, $F_Z \tilde{\in} \mu - F_{\emptyset}$ such that $F_B \tilde{\cap} F_Z = F_{\emptyset}$ and $F_B \tilde{\cup} F_Z = F_{\tilde{E}}$

(iii) soft μ-α-connected if $\nexists$ two soft μ-α-open sets F_B, $F_Z \tilde{\in} \mu - F_{\emptyset}$ such that $F_B \tilde{\cap} F_Z = F_{\emptyset}$ and $F_B \tilde{\cup} F_Z = F_{\tilde{E}}$

(iv) soft μ-β-connected if $\nexists$ two soft μ-β open sets F_B, $F_Z \tilde{\in} \mu - F_{\emptyset}$ such that $F_B \tilde{\cap} F_Z = F_{\emptyset}$ and $F_B \tilde{\cup} F_Z = F_{\tilde{E}}$

Corollary: 7.2

Let $(F_{\tilde{E}}, \mu)$ be a SGTS. Then the following are equivalent:

(i) $F_{\tilde{E}}$ is soft μ-β-connected

(ii) $F_{\tilde{E}}$ is soft μ-δ-connected

(iii) $F_{\tilde{E}}$ is soft μ-α-connected

(iv) $F_{\tilde{E}}$ is soft μ-connected

Proof:
(i) ⇒ (ii):

Let $F_{\tilde{E}}$ be soft μ-β-connected. Contrarily, $F_{\tilde{E}}$ is soft μ-δ-connected. Then there exists two non-null soft μ-semi-open sets $F_{\mathcal{R}}, F_{\mathcal{S}} \tilde{\in} \mu - F_{\emptyset}$ such that $F_{\mathcal{R}} \tilde{\cap} F_{\mathcal{S}} = F_{\emptyset}$ and $F_{\mathcal{R}} \tilde{\cup} F_{\mathcal{S}} = F_{\tilde{E}}$. Since every soft μ-δ-open is soft μ-β-open, there exists two non-null soft μ-β-open sets $F_{\mathcal{R}}, F_{\mathcal{S}} \tilde{\in} \mu - F_{\emptyset}$ such that $F_{\mathcal{R}} \tilde{\cap} F_{\mathcal{S}} = F_{\emptyset}$ and $F_{\mathcal{R}} \tilde{\cup} F_{\mathcal{S}} = F_{\tilde{E}}$ which implies $F_{\tilde{E}}$ is not soft μ-β-connected. Hence $F_{\tilde{E}}$ is soft μ-δ-connected.

(ii) ⇒ (iii):

Let $F_{\tilde{E}}$ be soft μ-δ-connected. Suppose not, $F_{\tilde{E}}$ is soft μ-α-connected. Then there exists two non-null soft μ-α-open sets $F_{\mathcal{R}}, F_{\mathcal{S}} \tilde{\in} \mu - F_{\emptyset}$ such that $F_{\mathcal{R}} \tilde{\cap} F_{\mathcal{S}} = F_{\emptyset}$ and $F_{\mathcal{R}} \tilde{\cup} F_{\mathcal{S}} = F_{\tilde{E}}$. Since every soft μ-α-open set is soft μ-semi-open, then there exists two non-null soft μ-semi-open sets $F_{\mathcal{R}}, F_{\mathcal{S}} \tilde{\in} \mu - F_{\emptyset}$ such that $F_{\mathcal{R}} \tilde{\cap} F_{\mathcal{S}} = F_{\emptyset}$ and $F_{\mathcal{R}} \tilde{\cup} F_{\mathcal{S}} = F_{\tilde{E}}$ which implies $F_{\tilde{E}}$ is not soft μ-δ-connected. Hence $F_{\tilde{E}}$ is soft μ-α-connected.

(iii) ⇒ (iv):

Let $F_{\tilde{E}}$ be soft μ-α-connected. If not, $F_{\tilde{E}}$ is soft μ-connected. Then there exists two non-null soft μ-open sets $F_{\mathcal{R}}, F_{\mathcal{S}} \tilde{\in} \mu - F_{\emptyset}$ such that $F_{\mathcal{R}} \tilde{\cap} F_{\mathcal{S}} = F_{\emptyset}$ and $F_{\mathcal{R}} \tilde{\cup} F_{\mathcal{S}} = F_{\tilde{E}}$. We know that every soft μ-open set is soft μ-α-open. Therefore there exists two non-null soft μ-α-open sets $F_{\mathcal{R}}, F_{\mathcal{S}} \tilde{\in} \mu - F_{\emptyset}$ such that $F_{\mathcal{R}} \tilde{\cap} F_{\mathcal{S}} = F_{\emptyset}$ and $F_{\mathcal{R}} \tilde{\cup} F_{\mathcal{S}} = F_{\tilde{E}}$ which implies $F_{\tilde{E}}$ is not soft μ-α-connected. Hence $F_{\tilde{E}}$ is soft μ-connected.

Corollary: 7.3
Let $(F_{\tilde{E}}, \mu)$ be a SGTS. Then the following are equivalent:
(i) $F_{\tilde{E}}$ is soft μ-β-connected
(ii) $F_{\tilde{E}}$ is soft μ-ρ-connected
(iii) $F_{\tilde{E}}$ is soft μ-α-connected

Proof:
(i) ⇒ (ii):

Let $F_{\tilde{E}}$ be soft μ-β-connected. On the other hand, $F_{\tilde{E}}$ is not soft μ-ρ-connected. Then there exists two non-null soft μ-pre-open sets $F_{\mathcal{R}}, F_{\mathcal{S}} \tilde{\in} \mu - F_{\emptyset}$ such that $F_{\mathcal{R}} \tilde{\cap} F_{\mathcal{S}} = F_{\emptyset}$ and $F_{\mathcal{R}} \tilde{\cup} F_{\mathcal{S}} = F_{\tilde{E}}$. Since every soft μ-pre-open set is a soft μ-β-open set, then there exists two non-null soft μ-β-open sets $F_{\mathcal{R}}, F_{\mathcal{S}} \tilde{\in} \mu - F_{\emptyset}$ such that $F_{\mathcal{R}} \tilde{\cap} F_{\mathcal{S}} = F_{\emptyset}$ and $F_{\mathcal{R}} \tilde{\cup} F_{\mathcal{S}} = F_{\tilde{E}}$ which implies $F_{\tilde{E}}$ is not soft μ-β-connected. Hence $F_{\tilde{E}}$ is soft μ-ρ-connected.

(ii) ⇒ (iii):

Let $F_{\tilde{E}}$ be soft μ-ρ-connected. On the contrary, $F_{\tilde{E}}$ is soft μ-α-connected. Then there exists two non-null soft μ-α-open sets $F_{\mathcal{R}}, F_{\mathcal{S}} \tilde{\in} \mu - F_{\emptyset}$ such that $F_{\mathcal{R}} \tilde{\cap} F_{\mathcal{S}} = F_{\emptyset}$ and $F_{\mathcal{R}} \tilde{\cup} F_{\mathcal{S}} = F_{\tilde{E}}$. Also, since every soft μ-α-open set is soft μ-pre-open, then there exists two non-null soft μ-pre-open sets $F_{\mathcal{R}}, F_{\mathcal{S}} \tilde{\in} \mu - F_{\emptyset}$ such that $F_{\mathcal{R}} \tilde{\cap} F_{\mathcal{S}} = F_{\emptyset}$ and $F_{\mathcal{R}} \tilde{\cup} F_{\mathcal{S}} = F_{\tilde{E}}$ which implies $F_{\tilde{E}}$ is not soft μ-ρ-connected. Hence $F_{\tilde{E}}$ is soft μ-α-connected.

VIII. SOFT μ-HYPERCONNECTEDNESS IN WEAKER FORMS OF SOFT μ-OPEN SETS

Definition: 8.1
Let $(F_{\tilde{E}}, \mu)$ be a SGTS. Then $F_{\tilde{E}}$ is called
(i) soft μ-hyperconnected if for each non-null soft μ-open subset F_A of $F_{\tilde{E}}$ is soft μ-dense.
[(i.e.) $S\mu - Cl(F_A) = F_{\tilde{E}}$]
(ii) soft μ-ρ-hyperconnected if for each non-null soft μ-pre-open subset F_A of $F_{\tilde{E}}$ is soft μ-dense.
(iii) soft μ-δ-hyperconnected if for each non-null soft μ-semi-open subset F_A of $F_{\tilde{E}}$ is soft μ-dense.
(iv) soft μ-α-hyperconnected if for each non-null soft μ-α-open subset F_A of $F_{\tilde{E}}$ is soft μ-dense.
(v) soft μ-β-hyperconnected if for each non-null soft μ-β-open subset F_A of $F_{\tilde{E}}$ is soft μ-dense.

Corollary: 8.2
Let $(F_{\tilde{E}}, \mu)$ be a SGTS. Then the following are equivalent:
(i) $F_{\tilde{E}}$ is soft μ-β-hyperconnected
(ii) $F_{\tilde{E}}$ is soft μ-δ-hyperconnected
(iii) $F_{\tilde{E}}$ is soft μ-α-hyperconnected
(iv) $F_{\tilde{E}}$ is soft μ-hyperconnected

Proof:
(i) ⇒ (ii):

Let $F_{\tilde{E}}$ be soft μ-β-hyperconnected. Let $F_{\mathcal{W}}$ be any non-null soft μ-semi-open subset of $F_{\tilde{E}}$. As every soft μ-semi-open set is soft μ-β-open which imply that $F_{\mathcal{W}}$ is a non-null soft μ-β-open subset of $F_{\tilde{E}}$. By hypothesis, we have $S\mu - Cl(F_{\mathcal{W}}) = F_{\tilde{E}}$. Hence $F_{\tilde{E}}$ is soft μ-δ-hyperconnected.

(ii) ⇒ (iii):

Let $F_{\tilde{E}}$ be soft μ-δ-hyperconnected. Let $F_{\mathcal{W}}$ be any non-null soft μ-α-open subset of $F_{\tilde{E}}$. Since each soft μ-α-open is soft μ-semi-open, $F_{\mathcal{W}}$ is a non-null soft μ-semi-open subset of $F_{\tilde{E}}$. Also, $F_{\tilde{E}}$ being soft μ-δ-hyperconnected, we have $S\mu - Cl(F_{\mathcal{W}}) = F_{\tilde{E}}$. Hence $F_{\tilde{E}}$ is soft μ-α-hyperconnected.

(iii) ⇒ (iv):

Let $F_{\tilde{E}}$ be soft μ-α-hyperconnected. Let $F_{\mathcal{W}}$ be any non-null soft μ-open subset of $F_{\tilde{E}}$. By known result, every soft μ-open set is soft μ-α-open ⇒ $F_{\mathcal{W}}$ is a non-null soft μ-α-open subset of $F_{\tilde{E}}$. Under the hypothesis that $F_{\tilde{E}}$ is soft μ-α-hyperconnected, we have $S\mu - Cl(F_{\mathcal{W}}) = F_{\tilde{E}}$. Hence $F_{\tilde{E}}$ is soft μ-hyperconnected.

Corollary: 8.3
Let $(F_{\tilde{E}}, \mu)$ be a SGTS. Then the following are equivalent:
(i) $F_{\tilde{E}}$ is soft μ-β-hyperconnected
(ii) $F_{\tilde{E}}$ is soft μ-ρ-hyperconnected
(iii) $F_{\tilde{E}}$ is soft μ-α-hyperconnected

Proof:
(i) ⇒ (ii):

Let $F_{\tilde{E}}$ be soft μ-β-hyperconnected. Let $F_{\mathcal{W}}$ be any non-null soft μ-pre-open subset of $F_{\tilde{E}}$. As every soft μ-pre-open set is soft μ-β-open which implies $F_{\mathcal{W}}$ is a non-null soft μ-β-open subset of $F_{\tilde{E}}$. By hypothesis, we have $S\mu - Cl(F_{\mathcal{W}}) = F_{\tilde{E}}$. Hence $F_{\tilde{E}}$ is soft μ-ρ-hyperconnected.

(ii) ⇒ (iii):

Let $F_{\tilde{E}}$ be soft μ-ρ-hyperconnected. Let $F_{\mathcal{W}}$ be any non-null soft μ-α-open subset of $F_{\tilde{E}}$. Since each soft μ-α-open is soft μ-pre-open, $F_{\mathcal{W}}$ is a non-null soft μ-pre-open subset of $F_{\tilde{E}}$. Since $F_{\tilde{E}}$ is soft μ-ρ-hyperconnected, $S\mu - Cl(F_{\mathcal{W}}) = F_{\tilde{E}}$. Hence $F_{\tilde{E}}$ is soft μ-α-hyperconnected.

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