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Algebraic Structure Of (α, β)-Cut Of Fermatean Fuzzy Set Over Near Ring

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Abstract
Fermatean fuzzy sets (FFSs), an extension of intuitionistic fuzzy sets and Pythagorean fuzzy sets, provide a more generalized framework for handling uncertainty. In this paper, we introduce the concept of (α, β)-cut in the context of FFSs over near ring and investigate some algebraic structure of (α, β)-cut sets in FFSs over near rings. The notions cut sets of Fermatean fuzzy ideals (FFIs) are discussed. Also, we establish some conditions for which cut set of FFI over near ring form FFI over near rings.

Keywords: *Intuitionistic fuzzy set, pythagorean fuzzy set, fermatean fuzzy set, fermatean fuzzy ideals, cut set of fermatean fuzzy set, fermatean fuzzy homomorphism near-rings.*

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I. Introduction

Zadeh [16] introduced the idea of fuzzy set which has a membership function, μ that assigns to each element of the universe of discourse, a number from the unit interval [0, 1] to indicate the degree of belongingness to the set under consideration. The notion of fuzzy sets generalizes classical sets theory by allowing intermediate situations between the whole and nothing. In a fuzzy set, a membership function is defined to describe the degree of membership of an element to a class. The membership value ranges from 0 to 1, where 0 shows that the element does not belong to a class, 1 means belongs, and other values indicate the degree of membership to a class. For fuzzy sets, the membership function replaced the characteristic function in crisp sets. Since the pioneering work of Zadeh [16], the fuzzy set theory has been used in different disciplines such as management sciences, engineering, mathematics, social sciences, statistics, signal processing, artificial intelligence, automata theory, medical and life sciences.

The concept of fuzzy sets theory seems to be inconclusive because of the exclusion of nonmembership function and the disregard for the possibility of hesitation margin. Atanassov [9] critically studied these short comings and proposed a concept called intuitionistic fuzzy sets (IFSs) . The construct (that is, IFSs) incorporates both membership function, μ and nonmembership function, ν with hesitation margin, π (that is, neither membership nor nonmembership functions), such that $\mu + \nu \leq 1$ and $\mu + \nu + \pi = 1$. The notion of IFSs provides a flexible framework to elaborate uncertainty and vagueness. There are lot of research work done in area of IFSs in [2, 3, 4, 5].

There are situations where $\mu + \nu \geq 1$ unlike the cases capture in IFSs. This limitation in IFS naturally led to a construct, called Pythagorean fuzzy sets (PFSs). Pythagorean fuzzy set (PFS) proposed in [15], is a new tool to deal with vagueness considering the membership grade, μ and nonmembership grade, ν satisfying the conditions $0 \leq \mu \leq 1$ or $0 \leq \nu \leq 1$, and also, it follows that $\mu^2 + \nu^2 + \pi^2 \leq 1$, where π is the pythagorean fuzzy set index.

The concept of fuzzy subnear-ring and ideal were first introduced by Abou-Zaid in [1]. The notion of fuzzy ideals of near rings with interval valued membership functions introduced by B. Davvaz [10] in 2001. In [13], Kuncham et. al., introduced fuzzy prime ideal of near-rings. Here in this paper we study the properties of Intuitionistic fuzzy ideals of near ring with the help of their (α, β)-cut sets.

The rest of the paper organized as follows. In Section 2, the preliminaries and some definitions are given and present some algebraic structures of fermatean fuzzy sets. In Section 3, we introduced the notion of cut set of Fermatean fuzzy sets and Fermatean fuzzy ideals as well as investigated some interesting results on these sets. Finally, a conclusion is made in Section 4.

II. Preliminaries And Definition

In this section, we recall the related concepts to the fuzzy sets, the intuitionistic fuzzy sets and the pythagorean fuzzy sets as the definition of intuitionistic fuzzy set, pythagorean fuzzy set. The definition of upper and lower cut on a fuzzy set are represented. We also give an analysis of this concept when applied to the pythagorean fuzzy set.

Definition 2.1 *Let $\mathcal{N}$ be a non-empty set with two binary operations $+$ and $\cdot$, then it is called near ring if*
(i) $(\mathcal{N},+)$ is a group
(ii) $(\mathcal{N},.)$ is a semigroup
(iii) $(x+y).z = x.z + y.z$ for all $x,y,z \in \mathcal{N}$.
It is a right near ring since it satisfies the right distributive law.
If the condition (iii) is replace by $z.(x+y) = z.x + z.y$ for all $x,y,z \in \mathcal{N}$, then it is left near ring.
In this paper we use the word “near ring" instead of “right near ring".
We denote xy instead of $x.y$.
A near ring $\mathcal{N}$ is called zero symmetric if $x.0 = 0$ for all $x \in \mathcal{N}$.

Definition 2.2 *A fuzzy set A in X is*

$$A = \{\langle x, \varpi_A(x)\rangle | x \in X\},$$

where $\varpi_A: X \to [0,1]$ is a mapping called the membership function of the fuzzy set A.
The complement of μ is $\bar{\mu}(x) = 1 - \mu(x)$ for all $x \in X$.
Definition 2.3 *An intuitionistic fuzzy set (IFS) I in X is*

$$I = \{\langle x, \varpi_I(x), \varrho_I(x)\rangle : x \in X\},$$

where the functions $\varpi_I(x)$ and $\varrho_I(x)$ are the degree of membership and the degree of non-membership of the element $x \in X$ respectively. Also $\varpi_I: X \to [0,1]$, $\varrho_I: X \to [0,1]$ and $0 \le \varpi_I(x) + \varrho_I(x) \le 1$, for all $x \in X$.
The degree of indeterminacy $\pi_I(x) = 1 - \varpi_I(x) - \varrho_I(x)$.
Definition 2.4 *A pythagorean fuzzy set P is given by*

$$P = \{\langle x, \varpi_P(x), \varrho_P(x)\rangle | x \in X\},$$

where $\varpi_P(x): X \to [0,1]$ denotes the degree of membership and $\varrho_P(x): X \to [0,1]$ denotes the degree of non-membership of the element $x \in X$ to the set A respectively with the condition that $0 \le (\varpi_P(x))^2 + (\varrho_P(x))^2 \le 1$.
The degree of indeterminacy $\pi_P(x) = \sqrt{1 - (\varpi_P(x))^2 - (\varrho_P(x))^2}$.

Definition 2.5 *A Pythagorean fuzzy set P in universe of discourse X is represented as*

$$P = \{\langle x, \varpi_P(x), \varrho_P(x)\rangle | x \in X\},$$

where $\varpi_P(x): X \to [0,1]$ denotes the worth of membership and $\varrho_P(x): X \to [0,1]$ represents the worth to which the element $x \in X$ is not a member of the set P, with the condition that

$$0 \le (\varpi_P(x))^2 + (\varrho_P(x))^2 \le 1,$$

for all $x \in X$.
The worth of indeterminacy $h_P(x) = \sqrt{1 - (\varpi_P(x))^2 - (\varrho_P(x))^2}$.

Definition 2.6 *A Fermatean fuzzy set A in a finite universe of discourse X is furnished as*

$$A = \{\langle x, \varpi_A(x), \varrho_A(x)\rangle | x \in X\},$$

where $\varpi_A(x): X \to [0,1]$ denotes the worth of membership and $\varrho_A(x): X \to [0,1]$ represents the worth to which the element $x \in X$ is not a member of the set A, with the predicament that

$$0 \le (\varpi_A(x))^3 + (\varrho_A(x))^3 \le 1,$$

for all $x \in X$.
The worth of indeterminacy $h_A(x) = \sqrt[3]{1 - (\varpi_A(x))^3 - (\varrho_A(x))^3}$.

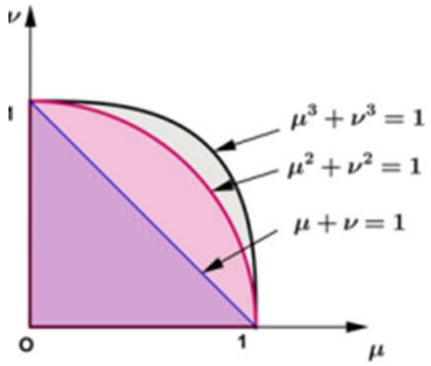

Figure 1: Spaces for IFSs, PFSs and FFSs

Some operations on Fermatean Fuzzy Numbers

Given three FFNs $\alpha = \langle \mu, \nu \rangle$, $\alpha_1 = \langle \varpi_1, \varrho_1 \rangle$ and $\alpha_2 = \langle \varpi_2, \varrho_2 \rangle$. Then

$$(1)\bar{\alpha} = \langle \nu, \mu \rangle$$

$$(2)\alpha_1 \vee \alpha_2 = \langle \max\{\varpi_1, \varpi_2\}, \min\varrho_1, \varrho_2 \rangle$$

$$(3)\ \alpha_1 \wedge \alpha_2 = \langle \min\{\varpi_1, \varpi_2\}, \max\varrho_1, \varrho_2 \rangle$$

$$(4)\alpha_1 \oplus \alpha_2 = \langle \sqrt{\varpi_1^2 + \varpi_2^2 - \varpi_1^2\varpi_2^2}, \varrho_1\varrho_2 \rangle$$

$$(5)\alpha_1 \otimes \alpha_2 = \langle \varpi_1\varpi_2, \sqrt{\varrho_1^2 + \varrho_2^2 - \varrho_1^2\varrho_2^2} \rangle$$

$$(6)\lambda \cdot \alpha = \langle \sqrt{1 - (1 - \mu^2)^\lambda}, \nu^\lambda \rangle, \lambda > 0.$$

$$(7)\alpha^\lambda = \langle \mu^\lambda, \sqrt{1 - (1 - \nu^2)^\lambda} \rangle, \lambda > 0.$$

Definition 2.7 *A FFS* $A = (\varpi_A, \varrho_A)$ *of a near ring* N *is called an Fermatean fuzzy subnear-ring of* N *if for all* $\varsigma_1, y \in \mathcal{N}$

(i) $\varpi_A(\varsigma_1 - \varsigma_2) \geq \min\{\varpi_A(\varsigma_1), \varpi_A(\varsigma_2)\}$ and $\varrho_A(\varsigma_1 - \varsigma_2) \leq \max\{\varrho_A(\varsigma_1), \varrho_A(\varsigma_2)\}$

(ii) $\varpi_A(\varsigma_1\varsigma_2) \geq \min\{\varpi_A(\varsigma_1), \varpi_A(\varsigma_2)\}$ and $\varrho_A(\varsigma_1\varsigma_2) \leq \max\{\varrho_A(\varsigma_1), \varrho_A(\varsigma_2)\}$.

Definition 2.8 *A FFS* $A = (\varpi_A, \varrho_A)$ *of a near ring* $\mathcal{N}$ *is called a Fermatean fuzzy Ideal of* $\mathcal{N}$ *if for all* $\varsigma_1, \varsigma_2, n \in \mathcal{N}$

(i) $\varpi_A(\varsigma_1 - \varsigma_2) \geq \min\{\varpi_A(\varsigma_1), \varpi_A(\varsigma_2)\}$ and $\varrho_A(\varsigma_1 - \varsigma_2) \leq \max\{\varrho_A(\varsigma_1), \varrho_A(\varsigma_2)\}$

(ii) $\varpi_A(\varsigma_1 n) \geq \varpi_A(\varsigma_1)$ and $\varrho_A(\varsigma_1 n) \leq \varrho_A(\varsigma_1)$

(iii) $\varpi_A(\varsigma_2 + \varsigma_1 - \varsigma_2) \geq \varpi_A(\varsigma_1)$ and $\varrho_A(\varsigma_2 + \varsigma_1 - \varsigma_2) \leq \varrho_A(\varsigma_1)$

(iv) $\varpi_A(n(\varsigma_1 + \varsigma_2) - n\varsigma_1) \geq \varpi_A(\varsigma_2)$ and $\varrho_A(n(\varsigma_1 + \varsigma_2) - n\varsigma_1) \leq \varrho_A(\varsigma_2)$

Theorem 2.1 *If* $A = (\varpi_A, \varrho_A)$ *be Fermatean fuzzy ideal of a near ring* $\mathcal{N}$*, then*

(i) $\varpi_A(\varsigma_1) \leq \varpi_A(0)$ and $\varpi_A(\varsigma_1) \geq \varpi_A(0)$ for all $\varsigma_1 \in \mathcal{N}$

(ii) $\varpi_A(-\varsigma_1) = \varpi_A(\varsigma_1)$ and $\varpi_A(-\varsigma_1) = \varpi_A(\varsigma_1)$ for all $\varsigma_1 \in \mathcal{N}$

(iii) $\varpi_A(\varsigma_1 + \varsigma_2) = \varpi_A(\varsigma_2 + \varsigma_1)$ and $\varpi_A(\varsigma_1 + \varsigma_2) = \varpi_A(\varsigma_2 + \varsigma_1)$ for all $\varsigma_1, \varsigma_2 \in \mathcal{N}$.

Proof. Trivial Proof

Proposition 1 *If* $A = (\varpi_A, \varrho_A)$ *be Fermatean fuzzy ideal of a near ring* $\mathcal{N}$*, then*

(i) $\varpi_A(\varsigma_1 - \varsigma_2) \geq \varpi_A(0) \Rightarrow \varpi_A(\varsigma_1) = \varpi_A(\varsigma_2)$

(ii) $\varrho_A(\varsigma_1 - \varsigma_2) \leq \varrho_A(0) \Rightarrow \varrho_A(\varsigma_1) = \varrho_A(\varsigma_2)$.

Proof. Trivial Proof.

Definition 2.9 *Let* $\mathcal{N}$ *and* $\mathcal{N}'$ *be two near rings. Then the mapping* $f: \mathcal{N} \to \mathcal{N}'$ *is called (near ring) homomorphism if for all* $\varsigma_1, \varsigma_2 \in \mathcal{N}$*, the following holds*

(i) $f(\varsigma_1 + \varsigma_2) = f(\varsigma_1) + f(\varsigma_2)$

(ii) $f(\varsigma_1\varsigma_2) = f(\varsigma_1)f(\varsigma_2)$

Theorem 2.2 *Let* $\mathcal{N}$ *and* $\mathcal{N}'$ *be two near rings and let* $f: \mathcal{N} \in \mathcal{N}'$ *a near ring epimorphism. Let* 0 *and* $0'$ *be additive identity element in* $\mathcal{N}$ *and* $\mathcal{N}'$ *respectively such that* $f(0) = 0'$*. If* $A = (\varpi_A, \varrho_A)$ *and* $B = (\varpi_B, \varrho_B)$ *are FFIs in* $\mathcal{N}$ *and* $\mathcal{N}'$ *respectively. Then*

(i) $\varpi_{f(A)}(0') = \varpi_A(0)$ and $\varrho_{f(A)}(0') = \varrho_A(0)$

(ii) $\varpi_{f^{-1}(B)}(0) = \varpi_B(0')$ and $\varrho_{f^{-1}(B)}(0) = \varrho_B(0')$.

Proof. Trivial Proof.

III. Main Results

Definition 3.1 *Let* A *be Fermatean fuzzy set of a universe set* X*. Then* (α, β)*-cut of* A *is a crisp subset* $\Upsilon_{\alpha,\beta}(A)$ *of the FFS* A *is given by* $\Upsilon_{\alpha,\beta}(A) = \{\varsigma_1 : \varsigma_1 \in X\ s.t.\ \varpi_A(\varsigma_1) \geq \alpha, \varrho_A(\varsigma_1) \leq \beta\}$*, where* $\alpha, \beta \in [0,1]$ *with* $\alpha, \beta \leq 1$.

Proposition 2 *If* A *and* B *be two FFSs of a universe set* X*, then*

(i) $\Upsilon_{\alpha,\beta}(A) \subseteq \Upsilon_{\delta,\theta}(A)$ if $\alpha \geq \delta$ and $\beta \leq \theta$

(ii) $\Upsilon_{1-\beta,\beta}(A) \subseteq \Upsilon_{\alpha,\beta}(A) \subseteq \Upsilon_{\alpha,1-\alpha}(A)$

(iii) $A \subseteq B$ implies $\Upsilon_{\alpha,\beta}(A) \subseteq \Upsilon_{\alpha,\beta}(B)$

(iv) $\Upsilon_{\alpha,\beta}(A \cap B) = \Upsilon_{\alpha,\beta}(A) \cap \Upsilon_{\alpha,\beta}(B)$

(v) $\Upsilon_{\alpha,\beta}(A \cup B) \supseteq \Upsilon_{\alpha,\beta}(A) \cup \Upsilon_{\alpha,\beta}(B)$ equality hold if $\alpha + \beta = 1$

(vi) $\Upsilon_{\alpha,\beta}(\cap A_i) = \cap \Upsilon_{\alpha,\beta}(A_i)$

(vii) $\Upsilon_{0,1}(A) = X$.

Proposition 3 *Let* $f: X \to Y$ *be a mapping. Then the following holds*

(i) $f(\Upsilon_{\alpha,\beta}(A)) \subseteq \Upsilon_{\alpha,\beta}(f(A)), \forall\ A \in FFS(X)$
(ii) $f^{-1}(\Upsilon_{\alpha,\beta}(B)) = \Upsilon_{\alpha,\beta}(f^{-1}(B)), \forall\ B \in FFS(Y)$.

Theorem 3.1 *If* $A = (\varpi_A, \varrho_A)$ *be FFI in near ring* $\mathcal{N}$, *then* $\Upsilon_{\alpha,\beta}(A)$ *is ideal of* $\mathcal{N}$ *if* $\varpi_A(0) \geq \alpha,\ \varrho_A(0) \leq \beta$.

Proof. Let $\varpi_A(0) \geq \alpha, \varrho_A(0) \leq \beta$. Clearly $\Upsilon_{\alpha,\beta}(A) \neq \phi$.
Let $\varsigma_1, \varsigma_2 \in \Upsilon_{\alpha,\beta}(A)$.
Then $\varpi_A(\varsigma_1) \geq \alpha, \varrho_A(\varsigma_1) \leq \beta$ and $\varpi_A(\varsigma_2) \geq \alpha, \varrho_A(\varsigma_2) \leq \beta$
$\Rightarrow \min\{\varpi_A(\varsigma_1), \varpi_A(\varsigma_2)\} \geq \alpha$ and $\max\{\varrho_A(\varsigma_1), \varrho_A(\varsigma_2)\} \geq \beta$.
Now, $\varpi_A(\varsigma_1 - \varsigma_2) \geq \min\{\varpi_A(\varsigma_1), \varpi_A(\varsigma_2)\} \geq \alpha$ and $\varrho_A(\varsigma_1 - \varsigma_2) \leq \max\{\varrho_A(\varsigma_1), \varrho_A(\varsigma_2)\} \leq \beta$
$\Rightarrow \varpi_A(\varsigma_1 - \varsigma_2) \geq \alpha$ and $\varrho_A(\varsigma_1 - \varsigma_2) \leq \beta$ and so $\varsigma_1 - \varsigma_2 \in \Upsilon_{\alpha,\beta(A)}$.
Thus $(\Upsilon_{\alpha,\beta}(A), +)$ is a subgroup of $(\mathcal{N}, +)$.
To show that $(\Upsilon_{\alpha,\beta}(A), +)$ is normal subgroup of $(\mathcal{N}, +)$
Let $\varsigma_1 \in \Upsilon_{\alpha,\beta}(A)$ be any element and $\varsigma_2 \in \mathcal{N}$.
$\therefore$ we have $\varpi_A(\varsigma_1) \geq \alpha, \varrho_A(\varsigma_1) \leq \beta$.
Since A is FFI of near ring $\mathcal{N}$.
$\therefore \varpi_A(\varsigma_2 + \varsigma_1 - \varsigma_2) \geq \varpi_A(\varsigma_1) \geq \alpha$ and $\varrho_A(\varsigma_2 + \varsigma_1 - \varsigma_2) \leq \varpi_A(\varsigma_1) \leq \beta$ implies that $\varsigma_2 + \varsigma_1 - \varsigma_2 \in \Upsilon_{\alpha,\beta}(A)$.
Thus $(\Upsilon_{\alpha,\beta}(A), +)$ is normal subgroup of $(\mathcal{N}, +)$.
Next to show that $\Upsilon_{\alpha,\beta}(A)\ \mathcal{N} \subseteq \Upsilon_{\alpha,\beta}(A)$. Let $\varsigma_1 \in \Upsilon_{\alpha,\beta}(A)$ and $n \in \mathcal{N}$.
Therefore, we have $\varpi_A(\varsigma_1) \geq \alpha, \varrho_A(\varsigma_1) \leq \beta$. As A is FFI of near ring $\mathcal{N}$.
$\therefore \varpi_A(\varsigma_1 n) \geq \varpi_A(\varsigma_1) \geq \alpha$ and $\varrho_A(\varsigma_1 n) \leq \varpi_A(\varsigma_1) \leq \beta$
$\Rightarrow \varpi_A(\varsigma_1 n) \geq \alpha$ and $\varrho_A(\varsigma_1 n) \leq \beta$.
And so $\varsigma_1 n \in \Upsilon_{\alpha,\beta}(A)$.
Thus $\Upsilon_{\alpha,\beta}(A)\mathcal{N} \subseteq \Upsilon_{\alpha,\beta}(A)$.
Next, to show that $n(\varsigma_1 + i) - n\varsigma_1 \in \Upsilon_{\alpha,\beta}(A)$; for all $\varsigma_1, n \in \mathcal{N}$ and $i \in \Upsilon_{\alpha,\beta}(A)$.
Let $i \in \Upsilon_{\alpha,\beta}(A)$ be any element.
Therefore, we have $\varpi_A(i) \geq \alpha, \varrho_A(i) \leq \beta$
As A is FFI of near ring $\mathcal{N}$, therefore we have
$\varpi_A(n(\varsigma_1 + i) - n\varsigma_1) \geq \varpi_A(i) \geq \alpha$ and $\varrho_A(n(\varsigma_1 + i) - n\varsigma_1) \leq \varrho_A(i) \leq \beta$
i.e. $\varpi_A(n(\varsigma_1 + i) - n\varsigma_1) \geq \alpha$ and $\varrho_A(n(\varsigma_1 + i) - n\varsigma_1) \leq \beta$ and so $n(\varsigma_1 + i) - n\varsigma_1 \in \Upsilon_{\alpha,\beta}(A)$.
Hence $\Upsilon_{\alpha,\beta}(A)$ is Ideal in near ring $\mathcal{N}$.

Theorem 3.2 *If* $A = (\varpi_A, \varpi_A)$ *is a FFS of a near ring* $\mathcal{N}$, *then* A *is FFI if and only if* $\Upsilon_{\alpha,\beta}(A)$ *is an ideal of* $\mathcal{N}$, *for all* $\alpha, \beta \in [0,1]$ *with* $\alpha + \beta \leq 1$ *and* $\varpi_A(0) \geq \alpha, \varrho_A(0) \leq \beta$.

Proof. If A be FFI of a near ring $\mathcal{N}$, then $\Upsilon_{\alpha,\beta}(A)$ is an ideal of $\mathcal{N}$, for all $\alpha, \beta \in [0,1]$ with $\alpha + \beta \leq 1$ and $\varpi_A(0) \geq \alpha, \varrho_A(0) \leq \beta$ follows from above Theorem.
Conversely, let $A = (\varpi_A, \varrho_A)$ be FFS of a near ring $\mathcal{N}$ such that $\Upsilon_{\alpha,\beta}(A)$ is an ideal of $\mathcal{N}$, for all $\alpha, \beta \in [0,1]$ with $\alpha + \beta \leq 1$ and $\varpi_A(0) \geq \alpha, \varrho_A(0) \leq \beta$.
To show that A is FFI of near ring $\mathcal{N}$.
Let $\varsigma_1, \varsigma_2 \in \mathcal{N}$ and $\alpha = \min\{\varpi_A(\varsigma_1), \varpi_A(\varsigma_2)\}$ and $\beta = \max\{\varrho_A(\varsigma_1), \varrho_A(\varsigma_2)\}$
$\Rightarrow \varpi_A(\varsigma_1) \geq \alpha, \varpi_A(\varsigma_2) \geq \alpha$ and $\varrho_A(\varsigma_1) \leq \beta, \varrho_A(\varsigma_2) \leq \beta$
$\Rightarrow \varpi_A(\varsigma_1) \geq \alpha, \varrho_A(\varsigma_1) \leq \beta$ and $\varpi_A(\varsigma_2) \geq \alpha, \varrho_A(\varsigma_2) \leq \beta$
Therefore, $\varsigma_1, \varsigma_2 \in \Upsilon_{\alpha,\beta}(A)$. As $\Upsilon_{\alpha,\beta}(A)$ is ideal in near ring $\mathcal{N} \Rightarrow \varsigma_1 - \varsigma_2 \in \Upsilon_{\alpha,\beta}(A)$
$\therefore \varpi_A(\varsigma_1 - \varsigma_2) \geq \alpha = \min\{\varpi_A(\varsigma_1), \varpi_A(\varsigma_2)\}$ and $\varrho_A(\varsigma_1 - \varsigma_2) \leq \beta = \max\{\varrho_A(\varsigma_1), \varrho_A(\varsigma_2)\}$.
Thus,

$$\varpi_A(\varsigma_1 - \varsigma_2) \geq \min\{\varpi_A(\varsigma_1), \varpi_A(\varsigma_2)\} \quad and \quad \varrho_A(\varsigma_1 - \varsigma_2) \leq \max\{\varrho_A(\varsigma_1), \varrho_A(\varsigma_2)\} \qquad (1)$$

As $\Upsilon_{\alpha,\beta}(A)\mathcal{N} \subseteq \Upsilon_{\alpha,\beta}(A)$ holds for all $\alpha, \beta \in [0,1]$ with $\alpha, \beta \leq 1$ and $\varpi_A(0) \geq \alpha, \varrho_A(0) \leq \beta$
Let $\varsigma_1 \in \Upsilon_{\alpha,\beta}(A)$ be such that $\varpi_A(\varsigma_1) = \alpha$ and $\varrho_A(\varsigma_1) = \beta$ and $n \in \mathcal{N}$ be any element.
Then $\varsigma_1 n \in \Upsilon_{\alpha,\beta}(A)$ and so $\varpi_A(\varsigma_1 n) \geq \alpha = \varpi_A(\varsigma_1)$ and $\varrho_A(\varsigma_1 n) \leq \beta = \varrho_A(\varsigma_1)$
i.e., $\varpi_A(\varsigma_1 n) \geq \varpi_A(\varsigma_1)$ and $\varrho_A(\varsigma_1 n) \leq \varrho_A(\varsigma_1)$.
And if $\varsigma_1 \in \Upsilon_{\alpha,\beta}(A)$ be such that $\varpi_A(\varsigma_1) = \alpha_1$ and $\varrho_A(\varsigma_1) = \beta_1 \leq 1 - \alpha_1$ where $\alpha_1 \geq \alpha$.
Then, $\varsigma_1 \in \Upsilon_{\alpha_1,\beta_1}(A)$. As $\Upsilon_{\alpha_1,\beta_1}(A)$ is Ideal in near ring N.
$\therefore \varsigma_1 n \in \Upsilon_{\alpha_1,\beta_1}(A)$
$\varpi_A(\varsigma_1 n) \geq \alpha_1 = \varpi_A(\varsigma_1)$ and $\varrho_A(\varsigma_1 n) \leq \beta_1 = \varrho_A(\varsigma_1)$ i.e. $\varpi_A(\varsigma_1 n) \geq \varpi_A(\varsigma_1)$ and $\varrho_A(\varsigma_1 n) \leq \varrho_A(\varsigma_1)$
Thus,

$$\varpi_A(\varsigma_1 n) \geq \varpi_A(\varsigma_1) \quad and \quad \varrho_A(\varsigma_1 n) \leq \varrho_A(\varsigma_1) \quad holds\ for\ all \quad \varsigma_1, n \in \mathcal{N}. \qquad (2)$$

Next to show that $\varpi_A(\varsigma_2 + \varsigma_1 - \varsigma_2) \geq \varpi_A(\varsigma_1)$ and $\varrho_A(\varsigma_2 + \varsigma_1 - \varsigma_2) \leq \varrho_A(\varsigma_1)$ holds for all $\varsigma_1, \varsigma_2 \in \mathcal{N}$.
As $(\Upsilon_{\alpha,\beta}(A), +)$ be normal subgroup of $(\mathcal{N}, +)$

Let $\varsigma_1 \in \Upsilon_{\alpha,\beta}(A)$ be such that $\varpi_A(\varsigma_1) = \alpha$ and $\varrho_A(\varsigma_1) = \beta$ and $\varsigma_2 \in \mathcal{N}$ be any element.
Then $(\varsigma_2 + \varsigma_1 - \varsigma_2) \in \Upsilon_{\alpha,\beta}(A) \Rightarrow \varpi_A(\varsigma_2 + \varsigma_1 - \varsigma_2) \geq \alpha = \varpi_A(\varsigma_1)$ and $\varrho_A(\varsigma_2 + \varsigma_1 - \varsigma_2) \leq \beta = \varrho_A(\varsigma_1)$
Now, if $\varsigma_1 \in \Upsilon_{\alpha,\beta}(A)$ be such that $\varpi_A(\varsigma_1) = \alpha_1$ and $\varrho_A(\varsigma_1) = \beta_1 \leq 1 - \alpha_1$, where $\alpha_1 \geq \alpha$
Then $\varsigma_1 \in \Upsilon_{\alpha_1,\beta_1}(A)$ As $\Upsilon_{\alpha_1,\beta_1}(A)$ is normal subgroup of $\mathcal{N}$.
So, $(\varsigma_2 + \varsigma_1 - \varsigma_2) \in \Upsilon_{\alpha_1,\beta_1}(A) \Rightarrow \varpi_A(\varsigma_2 + \varsigma_1 - \varsigma_2) \geq \alpha_1 = \varpi_A(\varsigma_1)$ and $\varrho_A(\varsigma_2 + \varsigma_1 - \varsigma_2) \leq \beta_1 = \varrho_A(\varsigma_1)$
i.e.,
$\varpi_A(\varsigma_2 + \varsigma_1 - \varsigma_2) \geq \varpi_A(\varsigma_1)\ \ and\ \ \varrho_A(\varsigma_2 + \varsigma_1 - \varsigma_2) \leq \varrho_A(\varsigma_1)\ \ holds\ for\ all\ \ \varsigma_1, \varsigma_2 \in \mathcal{N}.$ (3)
Next to show that $\varpi_A(n(\varsigma_1 + i) - n\varsigma_1) \geq \varpi_A(i)$ and $\varrho_A(n(\varsigma_1 + i) - n\varsigma_1) \leq \varrho_A(i)$ holds for all $\varsigma_1, n \in \mathcal{N}$, $i \in A$.
Take $i \in \Upsilon_{\alpha,\beta}(A)$ be an element such that $\varpi_A(i) = \alpha$ and $\varrho_A(i) = \beta$.
Since $\Upsilon_{\alpha,\beta}(A)$ is ideal of the near ring $\mathcal{N}$.
Therefore for $\varsigma_1, n \in \mathcal{N}$, we have $n(\varsigma_1 + i) - n\varsigma_1 \in \Upsilon_{\alpha,\beta}(A) \Rightarrow \varpi_A(n(\varsigma_1 + i) - n\varsigma_1) \geq \alpha = \varpi_A(i)$
and $\varrho_A(n(\varsigma_1 + i) - n\varsigma_1) \leq \beta = \varrho_A(i)$ and if $i \in \Upsilon_{\alpha,\beta}(A)$ be such that $\varpi_A(i) = \alpha_1$
and $\varrho_A(i) = \beta_1 \leq 1 - \alpha_1$, where $\alpha_1 \geq \alpha$
Then $i \in \Upsilon_{\alpha_1,\beta_1}(A)$ As $\Upsilon_{\alpha_1,\beta_1}(A)$ is ideal of $\mathcal{N}$.

$$\therefore n(\varsigma_1 + i) - n\varsigma_1 \in \Upsilon_{\alpha_1,\beta_1}(A)$$

$\Rightarrow \varpi_A(n(\varsigma_1 + i) - n\varsigma_1 \geq \alpha_1 = \varpi_A(i)$ and $\varrho_A(n(\varsigma_1 + i) - n\varsigma_1 \leq \beta_1 = \varrho_A(i)$
i.e.,
$\varpi_A(n(\varsigma_1 + i) - n\varsigma_1 \geq \varpi_A(i)\ \ and\ \ \varrho_A(n(\varsigma_1 + i) - n\varsigma_1 \leq \varrho_A(i)$ (4)
From (1), (2). (3) and (4) we find that A is FFI of near ring $\mathcal{N}$.

Theorem 3.3 *Let $\mathcal{N}$ be near ring. Then the FFS $A = \{\langle\varsigma_1, \varpi_A(\varsigma_1), \varrho_A(\varsigma_1)\rangle : \varsigma_1 \in \mathcal{N}, \varpi_A(\varsigma_1) = \varrho_A(0)$ and $\varrho_A(\varsigma_1) = \varrho_A(0)\}$ of $\mathcal{N}$ is FFI of the near ring $\mathcal{N}$.*

Proof. Taking $\alpha = \varpi_A(0)$ and $\beta = \varrho_A(0)$, then $\Upsilon_{\alpha,\beta}(A) = M$
Therefore by Theorem (2), A is FFI of near ring $\mathcal{N}$.

Theorem 3.4 *If A and B be two FFIs of a near ring $\mathcal{N}$, then $A \cap B$ is also FFI of $\mathcal{N}$.*

Proof. Since A and B be two FFIs of near ring $\mathcal{N}$. Then, $\Upsilon_{\alpha,\beta}(A)$ and $\Upsilon_{\alpha,\beta}(B)$ are ideals in near ring $\mathcal{N}$. Since intersection of two ideals in near ring is ideal in $\mathcal{N}$.
Therefore $\Upsilon_{\alpha,\beta}(A) \cap \Upsilon_{\alpha,\beta}(B)$ is ideal in $\mathcal{N}$
$\Rightarrow \Upsilon_{\alpha,\beta}(A \cap B)$ is ideal in $\mathcal{N}$.
$\Rightarrow A \cap B$ is FFI in near ring $\mathcal{N}$.

Theorem 3.5 *Let $\mathcal{N}$ and $\mathcal{N}'$ be two near rings and let $f: \mathcal{N} \to \mathcal{N}'$ be near ring homomorphism. If $B = (\varpi_B, \varrho_B)$ is a FFI in $\mathcal{N}'$, then the pre-image $f^{-1}(B)$ of B under f is a FFI of $\mathcal{N}$.*

Proof. Since B is FFI in near ring $\mathcal{N}' \Rightarrow \Upsilon_{\alpha,\beta}(B)$ is ideal in $\mathcal{N}'$, for all $\alpha, \beta \in [0,1]$ with $\alpha + \beta \leq 1$ and $\varpi_B(0) \geq \alpha, \varrho_B(0) \leq \beta$.
$\therefore f^{-1}(\Upsilon_{\alpha,\beta}(B))$ is ideal in $\mathcal{N}$.
But $f^{-1}(\Upsilon_{\alpha,\beta}(B)) = \Upsilon_{\alpha,\beta}(f^{-1}(B))$.
$\Rightarrow \Upsilon_{\alpha,\beta}(f^{-1}(B))$ is ideal in $\mathcal{N}$ and by using Theorem (5), we get $f^{-1}(B)$ is FFI in near ring $\mathcal{N}$.

Theorem 3.6 *Let $\mathcal{N}$ and $\mathcal{N}'$ be two near rings and let $f: \mathcal{N} \to \mathcal{N}'$ be epimorphism If $A = (\varpi_A, \varrho_A)$ is FFI of $\mathcal{N}$, then $f(A)$ is FFI of $\mathcal{N}'$.*

Proof. In view of Theorem (5), it is enough to show that $\Upsilon_{\alpha,\beta}(f(A))$ is ideal in $\mathcal{N}'$, for all $\alpha, \beta \in [0,1]$ with $\alpha + \beta \leq 1$ and $\varpi_{f(A)}(0') \geq \alpha, \varrho_{f(A)}(0') \leq \beta$.
Let $y_1, y_2 \in \Upsilon_{\alpha,\beta}(f(A))$ be any two elements, then $\varpi_{f(A)}(y_1) \leq \alpha, \varrho_{f(A)}(y_1) \geq \beta$ and $\varpi_{f(A)}(y_2) \leq \alpha, \varrho_{f(A)}(y_2) \geq \beta$.
Therefore, we have

$$f(\Upsilon_{\alpha,\beta}(A)) \subseteq \Upsilon_{\alpha,\beta}(f(A)), \forall A \in FFS(\mathcal{N})$$

Therefore $\exists$'s x_1 and x_2 in $\mathcal{N}$ such that $f(x_1) = y_1, f(x_2) = y_2$ and $\varpi_A(x_1) \geq \varpi_{f(A)}(y_1) \geq \alpha, \varrho_A(x_1) \leq \varrho_{f(A)}(y_1) \leq \beta$ and $\varpi_A(x_2) \geq \varpi_{f(A)}(y_2) \geq \alpha, \varrho_A(x_2) \leq \varrho_{f(A)}(y_2) \leq \beta$
$\Rightarrow \varpi_A(x_1) \geq \alpha, \varrho_A(x_1) \leq \beta$ and $\varpi_A(x_2) \geq \alpha, \varrho_A(x_2) \leq \beta$
$\Rightarrow \min\{\varpi_A(x_1), \varpi_A(x_2)\} \geq \alpha$ and $\max\{\varrho_A(x_1), \varrho_A(x_2)\} \leq \beta$
As A is FFI of near ring $\mathcal{N}$. Therefore
$\varpi_A(x_1 - x_2) \geq \min\{\varpi_A(x_1), \varpi_A(x_2)\} \geq \alpha$ and $\varrho_A(x_1 - x_2) \leq \max\{\varrho_A(x_1), \varrho_A(x_2)\} \leq \beta$
$\Rightarrow \varpi_A(x_1 - x_2) \geq \alpha$ and $\varrho_A(x_1 - x_2) \leq \beta$

$$\Rightarrow x_1 - x_2 \in \Upsilon_{\alpha,\beta}(A) \Rightarrow f(x_1 - x_2) \in f(\Upsilon_{\alpha,\beta}(A)) \subseteq \Upsilon_{\alpha,\beta}(f(A))$$

$\Rightarrow f(x_1) - f(x_2) \in \Upsilon_{\alpha,\beta}(f(A)) \Rightarrow y_1 - y_2 \in \Upsilon_{\alpha,\beta}(f(A))$.
Hence $(\Upsilon_{\alpha,\beta}(f(A)), +)$ is a subgroup of $(\mathcal{N}', +)$

Next to show that $(\Upsilon_{\alpha,\beta}(f(A)),+)$ is normal subgroup of $(\mathcal{N}',+)$
Let $y_1 \in \Upsilon_{\alpha,\beta}(f(A))$ and n' be any elements, then as above $\exists$'s x_1 and n in N such that $f(x_1) = y_1, f(n) = n'$ and $\varpi_A(x_1) \geq \varpi_{f(A)}(y_1) \geq \alpha, \varrho_A(x_1) \leq \varrho_{f(A)}(y_1) \leq \beta \Rightarrow \varpi_A(x_1) \geq \alpha, \varrho_A(x_1) \leq \beta$
$\Rightarrow x_1 \in \Upsilon_{\alpha,\beta}(A)$.
As A is FFI of $\mathcal{N}$, therefore $\Upsilon_{\alpha,\beta}(A)$ is ideal in $\mathcal{N}$

$$\Rightarrow (n + x_1 - n) \in \Upsilon_{\alpha,\beta(A)} \Rightarrow f(n + x_1 - n) \in f(\Upsilon_{\alpha,\beta}(A)) \subseteq \Upsilon_{\alpha,\beta}(f(A))$$

$\Rightarrow f(n) + (x_1) - (n) \in \Upsilon_{\alpha,\beta}(f(A))$ i.e. $(n' + y_1 - n') \in \Upsilon_{\alpha,\beta}(f(A))$.
Thus $(\Upsilon_{\alpha,\beta}(f(A)),+)$ is normal subgroup of $(\mathcal{N}',+)$.
Now to show that $\Upsilon_{\alpha,\beta}(f(A))\mathcal{N}' \subseteq \mathcal{N}'$. As above, we get $x_1\ n \in \mathcal{N}$
$\Rightarrow f(x_1 n) = f(x_1)f(n) = y_1 n' \in f(\mathcal{N}) = \mathcal{N}'$.
Further, to show that $n'(y_1 + y_2) - n'y_1 \in \Upsilon_{\alpha,\beta}(f(A))$ for all $y_1, y_2, n' \in \mathcal{N}'$.
As above $\exists' s\ x_1$ and x_2 in $\mathcal{N}$ such that $f(x_1) = y_1, f(x_2) = y_2$ and $f(n) = n'$ such that $x_1, x_2 \in \Upsilon_{\alpha,\beta}(A)$.
As $\Upsilon_{\alpha,\beta}(A)$ is ideal in $\mathcal{N}$. Therefore $(x_1 + x_2) - nx_1 \in \Upsilon_{\alpha,\beta}(A)$

$$\Rightarrow f(n(x_1 + x_2) - nx_1) \in f(\Upsilon_{\alpha,\beta}(A)) \subseteq \Upsilon_{\alpha,\beta}(f(A))$$

$$\Rightarrow f(n)(f(x_1) + f(x_2)) - f(n)f(x_1) = n'(y_1 + y_2) - n'y_1 \in \Upsilon_{\alpha,\beta}(f(A))$$

Hence f(A) is FFI in near ring $\mathcal{N}'$.

IV. Conclusion

Aim of this paper is to introduce the notions of fermatean fuzzy ideal of a near-ring and investigate some of its important results. In particular, we define concept cut set of fermatean fuzzy set and prove that cut set form a ferrmatean fuzzy ideal of a near ring. Also, represent the homomorphism of fermatean fuzzy ideal of near rings and prove some important properties. We hope that the research along this direction can be continued, and in fact, some results in this paper have already constituted a platform for further discussion concerning the future development of near-rings. In our future study of fermatean fuzzy structure of near-rings, may be describe fermatean fuzzy soft near-rings and its applications.

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References

[1] A. Abou-Zaid, On Fuzzy Subnear-Ring, Fuzzy Sets And Systems, 81 (1989) 383-393.
[2] A. K. Adak And M. Bhowmik, Interval Cut-Set Of Interval-Valued Intuitionistic Fuzzy Sets, African Journal Of Mathematics And Computer Sciences, 4(4) (2011) 192-200.
[3] A. K. Adak, M. Bhowmik And M. Pal, Semiring Of Interval-Valued Intuitionistic Fuzzy Matrices, Global Journal Of Computer Application And Technology, 1(3) (2011) 340-347.
[4] A. K. Adak, M. Bhowmik And M. Pal, Interval Cut-Set Of Generalized Interval-Valued Intuitionistic Fuzzy Sets, International Journal Of Fuzzy System Applications, 2 (3) (2012) 35-50.
[5] A. K. Adak, M. Bhowmik And M. Pal, Intuitionistic Fuzzy Block Matrix And Its Some Properties, Annals Of Pure And Applied Mathematics, 1(1) (2012) 13-31.
[6] A. K. Adak, M. Bhowmik And M. Pal, Some Properties Of Generalized Intuitionistic Fuzzy Nilpotent Matrices Over Distributive Lattice, International Journal Of Fuzzy Information And Engineering, 4(4) (2012) 371-387.
[7] A. K. Adak, M. Bhowmik And M. Pal, Semiring Of Generalized Interval-Valued Intuitionistic Fuzzy Matrices, World Applied Sciences Journal (Special Issue Of Applied Math), 16 (2012) 07-16.
[8] A. K. Adak, M. Bhowmik And M. Pal, Distributive Lattice Over Intuitionistic Fuzzy Matrices, The Journal Of Fuzzy Mathematics 21(2) (2013) 401-416.
[9] K. Atanassov, Intuitionistic Fuzzy Sets, Fuzzy Sets And Systems, 20 (1986) 87-96.
[10] B. Davvaz, Fuzzy R-Subgroups With Thresholds Of Near-Rings And Implication Operators, Soft Computing, 12 (2008) 875-879.
[11] Y. B. Jun, K. H. Kim, Y. H. Yon, Intuitionistic Fuzzy Ideals Of Near-Rings, J. Inst. Math. Comp. Sci. 12 (3) (1999) 221-228.
[12] K. H .Kim, Y. B. Jun, On Fuzzy R-Subgroups Of Near-Rings, J. Fuzzy Math., 8 (2000), 549-558.
[13] S. P. Kuncham, S. Bhavanari, Fuzzy Prime Ideal Of A Gamma Near Ring. Soochow Journal Of Mathematics 31 (1) (2005) 121-129.
[14] R. R. Yager, Abbasov Am. Pythagorean Membeship Grades, Complex Numbers And Decision Making. Int J Intell Syst 28 (2013) 436-452.
[15] R. R. Yager, Pythagorean Fuzzy Subsets. In: Proc Joint Ifsa World Congress And Nafips Annual Meeting, Edmonton, Canada, (2013) 57-61.
[16] L. A. Zadeh, Fuzzy Sets, Information And Control, 8(1965) 338-353.