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Decelerating Bianchi Type VI_0 Universe Model with Time Dependent ∧ Term

Dr. Sapna Shrimali[1], Teena Trivedi[2]
[1](*Associate professor, Department of Mathematics and Statistics, Pacific College of Basic Science and Applied Sciences, Pacific University, Udaipur(Rajasthan), India)*
[2](*Research Scholar, Department of Mathematics and Statistics, Pacific College of Basic Science and Applied Sciences, Pacific University, Udaipur (Rajasthan), India)*

Abstract: *Decelerating Bianchi Type VI_0Universe Modelwithtimedependent∧Termisinvestigated.To obtain the determinant solution of Einstein's field equation, we assume* $A = B^n$ *and* $\theta \propto \sigma$. *Physical and geometrical properties are also discussed.*
Keywords:*Bianchi Type VI_0, decelerating, Cosmology*

I. Introduction

Cosmology has long been considered as a speculative field. It is the branch of Astronomy which deals with large structure of Universe.The theory of relativity is intimately connected with the theory of space and time Einstein equations are used for constructing model of universe; the universe was static initially, further astronomers pointed out homogeneity and isotropy of matter distribution.

Kibble and Valenkin[1, 2] initiated phase transitions in the early universe, which can give rise to microscopic topological defects such as vacuum domain walls, strings, walls bounded by strings, and monopoles connected by strings. Cosmic String has originally given by Letelier.He investigated the model formed by massive string [3, 4], which was used as Bianchi type I and "Kantowski-Sachs" type of cosmological models.The basic virtue of inflation in the deflationary picture has been discussed by Gasperini[5].

Bianchi type I-IX cosmological models are important in sense of strings, isotropic, homogeneous etc. In past five decades relativists has been interested in constructing string cosmological model. Borrow[6] initiated the model Bianchi type VI_o of universe and explained solution of cosmological problem. Some exact solutions of Bianchi type VI0 for perfect fluid distributions satisfying specific equation of state[7]. Ellis and McCollu[8, 9] investigated solution of Einstein field equation for Bianchi type VI0 space time in stiff fluid. Dunn and Tupper [10]obtained the solution of a class of Bianchi type VI0 perfect fluid cosmological model associated with electromagnetic field. Reddy and Rao[11]presented on some Bianchi type cosmological model in biometric theory of gravitation. Shri Ram[12] presented an algorithm for generating exact perfect fluid solution of Einstein field equation, not satisfying the equation of state, for spatially homogeneous cosmological model of Bianchi type VI0. Singh and Singh [13]has been obtained the solution of string cosmological models with magnetic field in General Relativity. Some exact solution of string cosmological model has investigated by several researchers [14, 15, 16, 17] Xing-Xiang[18,19,20] has obtained solution of Bianchi string cosmological model with bulk viscosity and magnetic field. Bianchi type III for cloud string cosmological model described by Tikekar& Patel[20]. Chakraborty*et al.*[21, 22, 23]investigated string cosmological model in general realtivity. In Bianchi Type VI0 string cosmological model Tikekar and Patel[24] obtained some exact solutions. Bianchi Type I and Bianchi Type III investigated by Bali *et al.*[25, 26, 27, 28]

Two parameter of Einstein's field equation is cosmological constant ∧ and gravitational constant G plays the role of coupling constant between geometry and matter in Einstein field equation. Shrimali and Joshi[29, 30, 31, 32] obtained the solution of Bianchi type III cosmological model in general Relativity. Pradhan and Bali[33] obtained the solution of magnetized Bianchi type VI0 Barotropic massive string universe with decaying vacuum energy density. Verma and Ram[34] investigated the solution of Bianchi-Type VI0 Bulk Viscous Fluid Models with Variable Gravitational and Cosmological Constants. Pradhan*et. al.*[35 36] obtained dark energy model in Bianchi Type VI0

Recently, Bali and Poonia[37] investigated Bianchi Type VI0 Inflationary Cosmological Model in General Relativity. Tyagi*et.al*[38, 39, 40]obtained Bianchi Type VI0 homogeneous cosmological model for anti-stiff perfect fluid for time dependent ∧ in general relativity Inhomogeneous cosmological model for stiff perfect fluid distribution in general relativity and Barotropic perfect fluid in creation field theory with time dependent cosmological model . Bali *et .al*[40, 41] and Bhoyar*et.al*[42] has investigated Bianchi Type VI0 in general relativity.

II. Field Equation

We consider Bianchi type VI_0 space time metric in the form of

$$ds^2 = -dt^2 + A^2 dx^2 + B^2 e^{-2mx} dy^2 + C^2 e^{2mx} dz^2 \tag{1}$$

Where A, B and C are function of time t and m is constant. The energy momentum tensor for a bulk viscous fluid distribution is given by

$$T_i^j = (\rho + \overline{p}) v_i v^j + \overline{p} g_i^j \tag{2}$$

$$\overline{p} = p - \xi v^i_{;i}$$

Here ρ, p, $\overline{\mathrm{p}}$, is energy densities, isotropic pressure, bulk viscous pressure respectively. The velocity vector of fluid satisfies

$$v_i v^j = -1 = -u_i u^j \tag{3}$$

$$u^i v_i = 0 \tag{4}$$

The vector $u_i u^j$ describes the direction of string or direction or anisotropy.

The Einstein field equation

$$R_{ij} - \frac{1}{2} R g_{ij} = -8\pi G T_{ij} + \wedge g_{ij} \tag{5}$$

R_{ij} is known as Ricci tensor and T_{ij} is the energy momentum tensor for matter.

For the line element (1) and the field equation (5) can be written as

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}\dot{C}}{BC} + \frac{m^2}{A^2} = -8\pi G \overline{p} + \wedge \tag{6}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{C}}{C} + \frac{\dot{A}\dot{C}}{AC} - \frac{m^2}{A^2} = -8\pi G \overline{p} + \wedge \tag{7}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} - \frac{m^2}{A^2} = -8\pi G \overline{p} + \wedge \tag{8}$$

$$\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{A}\dot{C}}{AC} + \frac{\dot{B}\dot{C}}{BC} - \frac{m^2}{A^2} = 8\pi G \rho + \wedge \tag{9}$$

$$\left(\frac{\dot{B}}{B}-\frac{\dot{C}}{C}\right)=0 \tag{10}$$

Dot on B and C denotes the ordinary differentiation with respect to t. An additional equation for time changes of G and $\wedge$ is obtained by the divergence of Einstein tensor

$$\left(R_i^j-\frac{1}{2}Rg_i^j\right)_{;j}$$

This leads to

$$\left(8\pi GT_i^j-\wedge g_i^j\right)_{;j}=0$$

$$8\pi\dot{G}\rho+\dot{\wedge}+8\pi G\left[\dot{\rho}+\left(\rho+\bar{p}\right)\left(\frac{\dot{A}}{A}+\frac{\dot{B}}{B}+\frac{\dot{C}}{C}\right)\right]=0 \tag{11}$$

Using equation (3), equation (11) split into (12) and (13)

$$\left[\dot{\rho}+\left(\rho+\bar{p}\right)\left(\frac{\dot{A}}{A}+\frac{\dot{B}}{B}+\frac{\dot{C}}{C}\right)\right]=0 \tag{12}$$

$$\dot{\wedge}+8\pi\dot{G}\rho=8\pi G\xi\left(\frac{\dot{A}}{A}+\frac{\dot{B}}{B}+\frac{\dot{C}}{C}\right)^2 \tag{13}$$

The average scale factor S for the metric (1) is defined by

$$S^3=ABC \tag{14}$$

The volume scalar factor V is given by

$$V=S^3=ABC \tag{15}$$

The generalize mean Hubble parameter H is given by

$$H=\frac{1}{3}\left(H_1+H_2+H_3\right) \tag{16}$$

Where $H_1=\frac{\dot{A}}{A}, H_2=\frac{\dot{B}}{B}, H_3=\frac{\dot{C}}{C}$

The expansion scalar θ and shear scalar σ are given by

$$\theta = v^i_{;i} = \left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C} \right) \tag{17}$$

And

$$\sigma^2 = \frac{1}{3}\left(\frac{\dot{A}^2}{A^2} + \frac{\dot{B}^2}{B^2} + \frac{\dot{C}^2}{C^2} - \frac{\dot{A}\dot{B}}{AB} - \frac{\dot{B}\dot{C}}{BC} - \frac{\dot{A}\dot{C}}{AC} \right) \tag{18}$$

The deceleration parameter q is given by

$$q = -1 + \frac{d}{dt}(H) \tag{19}$$

The sign of q indicates condition of model inflation

III. SOLUTION OF FIELD EQUATION

We first assume that the expansion scalar is proportional to shear scalar. This condition leads to

$$A = B^n \tag{20}$$

Where n is positive constant. We assume n=1

From equation (10), we have

$$B = \mu C \tag{21}$$

μ is constant of integration. From equation (21), we take μ=1 so that

$$B = C \tag{22}$$

Using equation (6) and (7) with equation (22), we have

$$\frac{\ddot{B}}{B} - \frac{\ddot{A}}{A} + \frac{\dot{B}^2}{B^2} - \frac{\dot{A}\dot{B}}{AB} + \frac{2m^2}{A^2} = 0 \tag{3}$$

$$(1-\mathrm{n})\frac{\ddot{B}}{B} + (1-n^2)\frac{\dot{B}}{B} + \frac{2m^2}{B^{2n}} = 0 \tag{17}$$

$$\ddot{B} + (1+\mathrm{n})\frac{\dot{B}^2}{B} + \frac{2m^2}{1-n}B^{1-2n} = 0 \tag{18}$$

To solve equation (18), we denote $\dot{B} = \eta$ then $\ddot{B} = \eta \frac{d\eta}{dB}$ and Equation (18) reduced into first order first degree differential equation in the following form

$$\frac{d}{dB}\eta^2+\frac{2(n+1)}{B}\eta^2=k_1^2B^{1-2n} \quad (19)$$

Where $k_1^2=\frac{4m^2}{n-1}$

$$\eta=\frac{\sqrt{k_1^2B^4+k_2^2}}{2B^{n+1}} \quad (20)$$

$$\frac{2B^{n+1}}{\sqrt{k_1^2B^4+k_2^2}}dB=dt \quad (21)$$

Model I: n=2

$$\frac{2B^3dB}{\sqrt{k_1^2B^4+k_2^2}}=dt \quad (22)$$

$$B^4=\frac{(t+k_3)^2+k_2^2}{k_1^2} \quad (23)$$

Where k_1, k_2 and k_3 are constant.

$$ds^2=-dt^2+\left[\frac{(t+k_3)^2+k_2^2}{k_1^2}\right]^{2n}dx^2+\left[\frac{(t+k_3)^2+k_2^2}{k_1^2}\right]^2e^{-2mx}dy^2+\left[\frac{(t+k_3)^2+k_2^2}{k_1^2}\right]^2e^{2mx}dz^2 \quad (24)$$

IV. PHYSICAL KINEMATICAL PARAMETER

We can find the physical and geometrical parameter by using equation (24)
The Spatial Volume is given by

$$V=\frac{(t+k_3)^2+k_2^2}{k_1^2} \quad (24)$$

The Hubble parameter is given by

$$H=\frac{2}{3}\left[\frac{t+k_3}{(t+k_3)^2+k_2^2}\right] \quad (25)$$

The expansion scalar is given by

$$\theta=\frac{2(t+k_3)}{(t+k_3)^2+k_2^2} \quad (26)$$

The Shear Scalar is given by

$$\sigma=\frac{1}{2\sqrt{3}}\left[\frac{t+k_3}{(t+k_3)^2+k_2^2}\right] \quad (27)$$

The Deceleration parameter is given by

$$q=-1+\frac{2}{3}\left[\frac{k_2^2-(t+k_3)^2}{\left[(t+k_3)^2+k_2^2\right]^2}\right] \quad (28)$$

For the model 27, we observe that the spatial volume V is increases with time t. For large value t it becomes infinite. θ, H and σ decreases as time t increases. It vanish for large value of t. Thus the model has a big bang singularity at finite time t. it is continuously expanding Shearing non rotating.Since $t \xrightarrow{\lim} \infty \frac{\sigma}{\theta} = \text{constant}$ therefore the model doesn't approach isotropy

V. Conclusion

In this paper, we have presented exact solution of Einstein fields equation for Bianchi type VI_0 Space time under the assumption that expansion scalar is proportional to shear scalar. The physical and the kinematical parameters are decreasing function of time, for large value of t it tends to zero. The universe model decelerating and doesn't approach isotropy.

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