IOSR Journal of Applied Physics (IOSR-JAP)
e-ISSN: 2278-4861.Volume 10, Issue 4 Ver. III (Jul. – Aug. 2018), 54-59
www.iosrjournals.org

Gravitational Waves and Predictions of K-Theory for Excited Gravitons in The Bulk

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Abstract: *The discovery of gravitational waves and its theoretical connection with quantum gravity leads to the idea of M-theory realization of topological solitons in the framework of Type I string theory. Using K-theory for SO(n) and Sp(n) Type I string theory it was possible to calculate the Ramon-Ramon charges of D-branesolitonic objects for extra dimensional space, that signalize about the presence of gauge fields of excited graviton type in the bulk.*

Date of Submission: 27-08-2018 Date of acceptance: 09-09-2018

I. Introduction

The discovery of gravitational waves (GW) on 14 September 2015 by LIGO and Virgo Collaborations and the confirmation of this discovery by later measurements during 2016-2017 was not only a vivid confirmation of Einstein's general theory of relativity, but also emphasized the generality of the four types of interactions in high-energy physics.

General theory of relativity (GTR) of Albert Einstein was completed in 1915 and the article "The Foundation of the General Theory of Relativity" was published in 1916 year[1], where he regarded the gravitational force not as the exterior one but as the "space-time" property. His geometric theoryof gravitation was confirmed by numerous experiments, including the explanation of the anomalous perihelion advance of the planet Mercury and by the deflection of starlight by the Sun in May 29, 1919[2]. GTR predicted the existence of GW and the appearance of irremovable physical divergences when considering black holes and space-time singularities[3]. The existence of the particle-wave dualism predicted by Louis de Broglie for three types of interactions (strong, weak and electromagnetic) and the wave-particle opening for gravitational interaction is an intuitive confirmation of the possibility of combining of all four kinds of interactions at high energies.

In this article we will consider the base of GTR, derive from it the formula for GW and consider the perspectives of the metric variations for the discovery of Kaluza-Klein particles at the LHC. In the framework of type I string theory we will calculate Ramon-Ramon charges, for orthogonal and symplectic groups, SO(n); Sp(n), and show the existence of the solitonic objects of Kaluza-Klein type in the space of extra dimensions.

II. General Theory Of Relativity

The feature of discovery of GW from the merging of black holes pair is thesimultaneous confirmation of their existence: GW and black holes. Einstein's equivalence principle, or the universal equality of inertial and passivegravitational masses leads to the need for a transition from the Mikowskimetric to the space with pseudo-Riemannian one. In this theory objects movealong the geodesics and intervals in four-dimensional space-time

$$ds^2 = g_{\mu\nu} dx^{\mu} dx^{\nu}$$

are given by the components of the metric tensor

$$g_{\mu\nu} = \begin{pmatrix} g_{00} & g_{01} & g_{02} & g_{03} \\ g_{10} & g_{11} & g_{12} & g_{13} \\ g_{20} & g_{21} & g_{22} & g_{23} \\ g_{30} & g_{31} & g_{32} & g_{33} \end{pmatrix}$$

Metric components determine the curvature of the space

$$R = g^{\mu\nu} R_{\mu\nu}$$

where $R_{\mu\nu}$ is Ricci tensor related to Riemann curvature tensor $R^{\alpha}_{\mu\alpha\nu}$ as

$$R_{\varepsilon\beta} = R^{\rho}_{\alpha\rho\beta} = \partial_{\rho}\Gamma^{\rho}_{\beta\alpha} - \partial_{\beta}\Gamma^{\rho}_{\rho\alpha} + \Gamma^{\rho}_{\rho\lambda}\Gamma^{\lambda}_{\beta\alpha} - \Gamma^{\rho}_{\beta\lambda}\Gamma^{\lambda}_{\rho\alpha}$$

and $\Gamma_{\alpha\beta\gamma}$ are Christoffel symbols derived from the metric

$$\Gamma_{\alpha\beta\gamma} = \frac{1}{2}\left(\frac{\partial g_{\alpha\beta}}{\partial x^{\gamma}} + \frac{\partial g_{\alpha\gamma}}{\partial x^{\beta}} - \frac{\partial g_{\beta\gamma}}{\partial x^{\alpha}}\right).$$

Thus, according to GTR a massive objects bend the space-time and movealong this curved ways. Einstein wrote: "matter tells space how to bend; space tells matter how to move"[4]. Einstein's field equations are the following

$$R_{\mu\nu} - \frac{R}{2}g_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4}T_{\mu\nu}\,.$$

The left side of it is the geometric properties of space-time (Λ is the cosmological constant) and the right part is expressed through the energy-momentumtensor, $T\mu\nu$, of the matter which includes the density and flux of energy and momentum in space-time[5].

III. Gravitational Waves

GW was predicted by Einstein in 1916[6] from the GTR as gravitationalradiation similar to electromagnetic radion. The essence of his work consistedin the addition to Kronecker delta the small metric tensor γμν :

$$g_{\mu\nu} = -\delta_{\mu\nu} + \gamma_{\mu\nu}$$

and in using an ansatz

$$\gamma'_{\mu\nu} = \alpha_{\mu\nu} f(x_1 + ix_4)$$

with $\alpha_{\mu\nu}$ - constants and f - real functions of $(x_1 + ix_4)$. Let us considerhow this addition to the metric leads to GW. As is known, gμν = δμν + hμν, where δμν is the Lorentz metric and $h_{\mu\nu}$ is a first order quantity, $h = h^{\alpha}_{\alpha} = \delta^{\sigma\lambda}h_{\sigma\lambda}$. We'll rewrite the Einstein equation of GTR with this addition to themetric. As

$$R_{\mu\nu} = -\frac{1}{2}\delta^{\sigma\lambda}h_{\mu\nu,\sigma\lambda} - \frac{1}{2}\left(h_{,\mu\nu} - h^{\beta}_{\mu,\nu\beta} - h^{\beta}_{\nu,\mu\beta}\right) \qquad (1)$$

then the equation

$$R_{\mu\nu} - \frac{R}{2}g_{\mu\nu} = \frac{8\pi G}{c^4}T_{\mu\nu}$$

transforms into the following expression

$$-\frac{1}{2}\delta^{\sigma\lambda}h_{\mu\nu,\sigma\lambda} + g_{\mu\nu}\,\frac{1}{4}\delta^{\sigma\lambda}h_{,\sigma\lambda} = \frac{8\pi G}{c^4}T_{\mu\nu} \qquad (2)$$

with the additional condition that the second term in (1) equal to zero,

$$\left(h^{\nu}_{\mu} - \frac{1}{2}\delta^{\nu}_{\mu}h\right)_{,\nu} = 0\,. \qquad (3)$$

Equations (2) and (3) can be rewritten into the equations

$$\Box\varphi^{\nu}_{\mu} = -\frac{16\pi G}{c^4}T^{\nu}_{\mu} \qquad (4)$$

and $\varphi^{\nu}_{\mu,\nu} = 0$ with $\varphi^{\nu}_{\mu} = h^{\nu}_{\mu} - \frac{1}{2}\delta^{\nu}_{\mu}h$ accordingly.

The question of the equation for particles with m = 0 and spin = 2 wasinvestigated by W. Pauli and M. Fierz[7].

The relativistic description ofsuch particles is connected with the equation $\Box\psi^{\nu}_{\mu} = 0$ with the additionalconditions $\psi^{\nu}_{\mu,\nu} = 0$. This equation is the (4) one with the energy-momentumtensor equal to zero. The solution of this equation is

$$\varphi \sim \frac{1}{r} = P_0(\cos\gamma)\frac{1}{r_1} + P_1(\cos\gamma)\frac{r}{r_1^2} + P_2(\cos\gamma)\frac{r^2}{r_1^3} + \ldots \quad .$$

As is known, the expression for quadrupole interaction with multipole moments q, p, Q_{ik}($n = \frac{\vec{R}}{R}$) is the following

$$\varphi = \frac{q}{R} + p\frac{n}{R^2} + 3Q_{ik}\frac{n_i n_k}{R^3} .$$

Therefore, we have the solution for the potential of the external field, which iscreated by some remote system (in our case these are two merging black holes)and in which the quadrupole-quadrupole interaction is the most important onefor uncharged systems. At the end of this section, it would be interesting tocompare gravitational and electromagnetic interactions, presented in Table 1, from[8].

Table no 1 : Comparision of electromagnetic and gravitational fields.

	Electromagnetism	Gravity
field	A_μ	φ_μ^ν
source	J^μ	T_μ^ν
gauge transformation	$A^\mu + \partial_\mu \lambda$	$h_\mu^\nu - \frac{1}{2}\delta_\mu^\nu h$
Lorenz gauge	$\partial^\mu A_\mu = 0$	$\varphi_{\mu,\nu}^\nu = 0$
field equation	$\Box A_\mu = 4\pi J_\mu$	$\Box \varphi_\mu^\nu = -\frac{16\pi G}{c^4} T_\mu^\nu$

I. Quantum gravity and topological invariants of D-branes

1. M-theory and AdS/CFT correspondence

For description of gravity with strong influence on (quantum) matter, are formulated quantum field theories in curved space-time[9]. The appearance ofsingularities at points with large curvature in quantum-mechanical descriptionof space-time led to the need for theory of quantum gravity. Despite the factthat there is no suitable theory that unifies all types of interactions, there exist a number of candidates[10]. One of such attempts which is characterizedby an additional property as six extra dimensions in addition to the fourdimensional space-time is string theory[11]. Later, supergravity theory[12], became a part of eleven-dimensional M-theory - consistent theory of quantumgravity. In paper[13] was considered the connection of M-theory and stringtheory with one dimension compactified on orbifold. There was stressed, that$E_8 \times E_8$heterotic string is related to an eleven-dimensional theory on theorbifold $R^{10} \times S^1/Z_2$, the 11D M-theory, when E_8 gauge fields are at each10D fixed point of this orbifold. The action of bosonic part is the following[14]

$S = S_{SG} + S_{YM}$,

where SSG is the familiar 11-dimensional supergravity and S_{YM} - the E_8gauge fields of Yang-Mills theories on the orbifold planes[13]. The metric ofsuch theory can be represented by formula

$$ds_{11}^2 = (1+b)\eta_{\mu\nu}dx^\mu dx^\nu + R^2(1+\hat{\gamma})(dx^{11})^2 + V^{1/3}(\Omega_{AB} + h_{AB})dx^A dx^B ,$$

where$b$, $\hat{\gamma}$ and h_{AB} depend on x^{11} and on coordinates of the space of extra dimensions - Calabi-Yau manifold. V and R are moduli for Calabi-Yau volumeand orbifold radius. The compactification of this theory on the Calabi-Yaumanifold leads to 5D N = 1 supersymmetric model with one dimension compactified on S^1/Z_2 ,from[15]. According to AdS/CFT correspondence[16], thespace-time can have more than three noncompact spatial dimensions if welive on a four-dimensional domain wall which is embedded in the higher dimensions. The Standard Model (SM) physics isdefined in four-dimensionalsubspace - D3-brane and gravity can propagate in bulk space-time. In AdSgeometry the extra dimension is strongly curved, so AdS/CFT correspondence is the "holographic" projection of the AdS to the gauge theory physics.Thus, we'll compare the properties of the space of extra dimensions with supersymmetric Yang-Mills theory in usual space-time. For example, in theAdS3 geometry, the Randall-Sundrum (RS) partition function is obtained byintegrating over the bulk metric with two patches R_1 and R_2 of AdS:

$$Z_{RS}[\gamma]=e^{-2S_1}\left(\int_{R_1\square R_2} Dg e^{-S_{EH}[g]-S_{GH}[g]}\right)$$

(where $S_{EH}=\frac{L}{16\pi}\int_{\Omega} d^3x\sqrt{g}\left(R+\frac{2}{l^2}\right)$ - the usual Einstein-Hilbert (EH) action with a negative cosmological constant $\left(\Lambda=-\frac{1}{l^2}\right)$, $S_{GH}=\frac{L_p}{8\pi}\int_{\partial\Omega} d^2x\sqrt{\gamma}K-$ Gibbons-Hawking (GH) boundary term, that depends on the bulk metric (*K*is the trace of the extrinsic curvature of the boundary, γ is the induced metricon the brane), S_1 is the brane action without matter, $S_1=(T/2)\int d^2x\sqrt{\gamma}$ .According to[16], this partition function is identified with the functional ofGreens functions of the RS CFT:

$$Z_{RS}[\gamma]=e^{-2W_{CFT}[\gamma]+2S_2[\gamma]}.$$

So, the RS-like model in AdS_3 is equivalent to a CFT coupled to gravity with action of matter on the brane, $S_2\propto\int d^dx\sqrt{\gamma}R$.

2. K-group calculations

According to the D-brane approach in M-theory, the ends of open strings arerestricted to D4-branes in the 11-dimensional space-time. The SM fields -harmonics of open string are localized on the D4-brane. The excited gravitonstates are closed string modes and propagate in the bulk. In Fig. 1 is presentedthe space-time in the Type I' string theory[17].

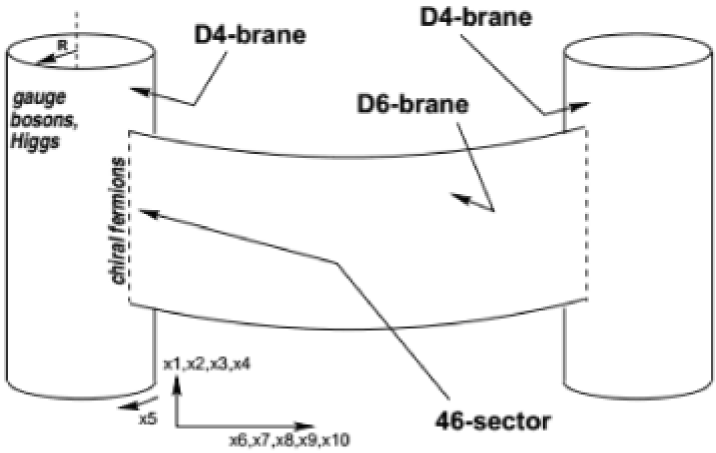

Figure no1: Schema of the space-time in the Type I' string theory.

The D4-brane corresponds to three space-time dimensions (usual space)plus one dimension compactified on the circle S^1. D6-brane corresponds to therest of the dimensions, five of which are the spatial dimensions.

D-branes of type I areunoriented and their supersymmetric vacuum configuration has a richer spectrum of particles for calibration group. In paper[17] was shown that the unification scale can became lower to the TeV scalethrough the extra dimensions, which lead to gauge coupling unification atscales of TeV, presented in the theory of grand unification physics – stringtheory, Fig 2.

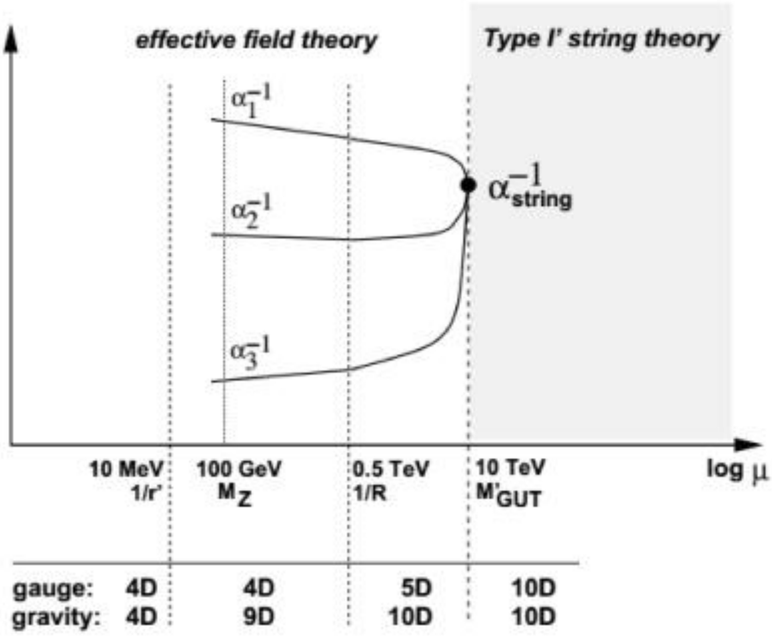

Figure no2: The evolution of the gauge couplings within a Type I' string theory.

The new unification scale is $M_{GUT} \approx 10$ TeV, the gravitational coupling feels five new dimensions at $(r')^{-1} \approx 10$ MeV and the gauge couplings feel a new dimension at $R^{-1} \approx 0.5$ TeV, from[17].

As is known, D-branes could be represented as fiber bundles[18]. Wecan compare the calculation of topological invariants of fiber bundles usingthe theory of categories[19], with the properties of solitonic objects whichare represented by such bundles[20]. As K-theory predicts the relations ofthe superstring theories and their D-brane spectra, for D-branes, consideredas topological solitons, is used formalism of K-theory, which reproduces thespectra of BPS and non-BPS D-branes.

We will consider the construction of Grothendieck group for a classification of vector bundles over compact manifolds[20]. Let $X=S^n$ bea compact manifold and let *Vect(X)* be the additive category of complexvector bundles over *X*. As I^k is the trivial bundle of rank *k* over *X*, i.e.$I^k \sim X \times C^k$, we can consider bundle *E* over *X* which is stably equivalentto bundle *F* , denoted by $E \sim F$, if $E \oplus I^j \sim F \oplus I^k$. The K-group of acompact manifold *X* can be defined as the Grothendieck group of the category $K(X) \equiv K(Vect(X))$; $K(X) = Vect(X) \times Vect(X)/\sim$, where *Vect(X)*- the set of isomorphism classes of complex vector bundles $E \rightarrow X$ with somestructure group. The equivalence relation in $Vect(X) \times Vect(X)$ is accordingto $(E; F) \sim (E'; F')$ such that $E \oplus F' \oplus G \sim E' \oplus F \oplus G$ with vector bundle$G \in Vect(X)$. $[E] = [F]$ in *K(X)* if *E* and *F* are stably equivalent. The elements of *K(X)* are virtual bundles, which represent functor from the categoryof compact topological spaces to the category of abelian groups, $X \rightarrow K(X)$.

In accordance with this known construction, we can consider the classification of Type I string theory for SO(n) and Sp(n) structure groups of fiberbundles with base *X*. As is shown in Fig.1, the schema of the space-time inthe Type I' string theory is presented by two types of D-branes: D4-branesand D6-branes. The excited graviton or Kaluza-Klein states propagates inthe bulk, as was highlighted above. So, the bulk or D6-branes are topologicalsolitons with brane spectra, presented by Ramon-Ramon charges, related toType I string theory spectra which can be calculated through K-group. ThisD6-branes are fiber bundles over X - noncompact space, which can be replacedby its one-point compactification, S^n; $X = S^n \times Z$, through the addition ofmanifold Z at infinity. In the paper[20], several types of string theories andtheir K-group classifications for the transverse space, S^nwere considered. ForType I string theory K-groups were calculated for extra dimensional spaceassociated with transverse space, S^n. Since we are working with five spatialdimensions, the transverse space for it is S^4 and:

$\widetilde{K}(S^4) = Z$ for SO(n) bundles for Type I spectrum of D5-branes; $\widetilde{K}(S^4) = Z$ forSp(n) bundles for Type I spectrum of D5-branes. The nonzero value of K-group together with the relations of D-brane spectra and string spectra[21], and according to AdS/CFT correspondence, weconclude about the existance of the excited states of gauge particles in thebulk.

IV. Conclusion

We have considered the string - Dp-branes -AdS correspondence, when Dp-branes are considered as p+1-dimensional hypersurfaces in space-time withthe endpoints of open strings or as solitonic topological defects in space-time.According to[22], D-branes as solitons are connected with fundamental stringstates and have provided the insights into the nature of space-time at veryshort distance scales. D-branes are also characterized by the determinationof their mass by their charge and D-brane charges take values in K-group.As D-branes are extra dimensional objects, which are of different geometry,there are the AdS/CFT correspondence, that correlates the geometry of spacetime with a certain content of the fields of the Yang-Mills theory. Therefore,the nonzero elements of K-group signals about the presence of excited gaugefields, including the graviton in the bulk, for orthogonal, SO(n) and simplectic, Sp(n), gauge bundles, that represents D-branesolitonic objects.

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