

Interpretation of the Coulomb's law for a Special Scenario

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Abstract: To date, there exists no evidence both theoretically and experimentally to confirm the breakdown of the Coulomb's law in any scenario. Here, we present a brief idea theoretically for a special case as to when and why the breakdown might occur in the Coulomb's law. We are not proposing that this is exactly the breakdown but more like an interpretation of the condition from a mathematical perspective and hence considering it as a special case of breakdown of the law. We also provide a possible physical scenario for the case with respect to singularity in the Coulomb's law term by utilizing George Cantor's concept of infinities. The entire work is entirely a viewpoint by utilizing various branches of physics.

Keywords: Coulomb, fusion, infinities, singularity.

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I. Introduction

In 1785 Coulomb discovered a law, called as the Coulomb's law that describes the force between two charged particles. The law is strictly applicable to point charges and acts along the line joining the centre of the two charges.

$$F = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^2} \quad (1)$$

It is also applicable for any arbitrary large distance between the charges. Also with cryogenic tests extending its validity to very low temperatures, it, therefore, has applications[8] in pedagogical physics.

In the last few years, there have been ideas to use spectroscopy to test deviations in the Coulomb's law with high precision on atomic length scales, by using the idea of particles[2] such as hidden photons. There also have been ways to obtain a modified version of Coulomb's law in two-and three-dimensional closed[7] spaces.

However during the late 19th century, despite Maxwell's apparent confidence, analysis of contemporary textbooks shows that until very recently, the inverse square law had been open to question, and its experimental base was far from robust. Following the death of Snow Harris, a major proponent of alternative electrical theories, in 1867, opposition[8] to Coulomb's law decreased significantly in Britain. In the 1870s, Maxwell, Thomson, and Everett moved rapidly to close the door on alternative theories by constructing an experimental tradition of null tests of the law that would be less open to critical scrutiny than Coulomb's experiments.

In this paper, we have considered a scenario where r (distance between the charges) = 0 in the Coulomb's law expression. We aim to provide a physical perspective of the proposed scenario. A lot of time questions have been asked as to what such mathematical condition might mean. This is an aim to shed light on the matter by using already existing concepts and then clubbing them together to provide an explanation. This is mostly a non-technical understanding of the phenomena by using classical, nuclear and quantum physics. We begin by providing a brief description of the topics that we intend to use in our final interpretation.

Singularity

In mathematics, singularity[12] refers to a point at which a given mathematical object is not defined or points where the mathematical objects are not well-behaved. Therefore it is a point at which a function, equation, surface, etc, become degenerate or just diverges towards infinity.

Consider the function

$$f(x) = \frac{1}{x} \quad (2)$$

The function is well behaved at all points except at $x = 0$. When you put $x = 0$, the function is not defined[13] because it explodes to infinity. Hence we say that the function has a singularity at $x = 0$.

George Cantor's idea of infinities

In one of his earliest papers, Cantor proved that the set of real numbers is "more numerous" than the set of natural numbers; this showed, for the first time, that there exist infinite sets of different sizes. Cantor's first proof that infinite[16] sets can have different cardinalities was published in 1874. The cardinality of a set is a measure of the "number of elements of the set"[15]. His proof demonstrates that the set of natural numbers and the set of real numbers have different cardinalities[12]. Cantor provided a stunning and instantly controversial proof that not only defined the nature of infinity, but it also revealed that multiple infinities existed, and some were larger than others[16]. Cantor called his various sets of different quantities of infinity 'transfinite' numbers — they are also known as cardinal numbers — and designated aleph-naught(or null) as the smallest transfinite number in existence[17]. Summarizing, we find that there are different types of infinities and in fact, some infinities might be larger than others.

Gravitational singularity

In the center of a black hole is a gravitational[18] singularity, a one-dimensional point which contains a huge mass in an infinitely small space, where density and gravity become infinite and space-time curves infinitely, and where the laws of physics as we know them to cease to operate. As the eminent American physicist Kip Thorne describes it, it is "the point where all laws of physics break down".

The existence of a singularity is often taken as proof that the theory of general relativity has broken down, which is perhaps not unexpected as it occurs in conditions where quantum effects should become important. It is conceivable that some future combined theory of quantum gravity (such as current research into superstrings) may be able to describe black holes without the need for singularities, but such a theory is still many years away. According to the "cosmic censorship" hypothesis[17], a black hole's singularity remains hidden behind its event horizon, in that it is always surrounded by an area which does not allow light to escape, and therefore cannot be directly observed. The only exception the hypothesis allows (known as a "naked" singularity)[18] is the initial Big Bang itself.

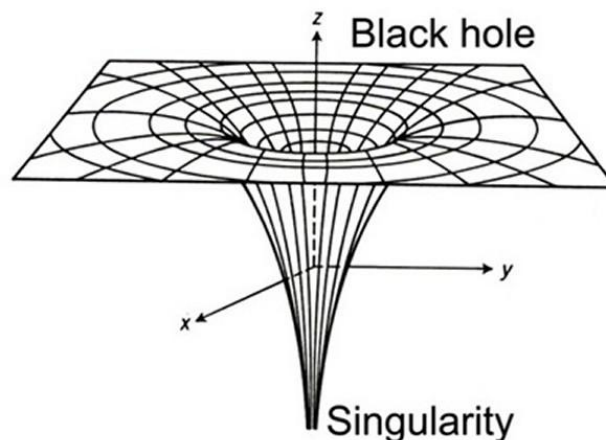


Figure – Illustration of a Black Hole Singularity[18].

Nuclear Reaction

In nuclear physics, nuclear fusion[21] is a reaction in which two or more atomic nuclei are combined to form one or more different atomic nuclei and subatomic particles (neutrons or protons). The difference in mass between the reactants and products is manifested as either the release or absorption of energy. This difference in mass arises due to the difference in atomic "binding energy" between the atomic nuclei before and after the reaction.

The release of energy with the fusion of light elements is due to the interplay of two opposing forces: the nuclear force, which combines together protons and neutrons, and the Coulomb force, which causes protons to repel each other. Protons are positively charged and repel each other by the Coulomb force, but they can nonetheless stick together, demonstrating the existence of another, short-range, force referred to as nuclear attraction. Light nuclei (or nuclei smaller than iron and nickel) are sufficiently small and proton-poor allowing the nuclear force[21] to overcome repulsion. This is because the nucleus is sufficiently small that all nucleons feel the short-range attractive force at least as strongly as they feel the infinite-range.

II. The Idea

The Coulomb's law expression from equation 1 can also be written as a function of the form

$$f(x) = \frac{1}{r^2} \quad (3)$$

Considering $r = 0$, we find

$$F = \infty \quad (4)$$

Force becomes infinity. This corresponds to zero distance between the charges. We, therefore assume that the centre of the two charges coincides after fusing as one. Therefore they exist together as one single entity and hence we can say they exist as a point charge.

III. Results

First and foremost after the two charges fuse into one, they are expected to have some energy. In electrostatics, there is a term called self-energy[8] of a single charge(spherical in shape with radius 'a'), that is calculated by considering that the sphere is produced by accumulating a succession of thin spherical shells.

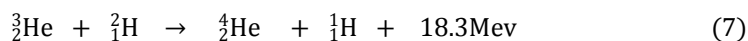
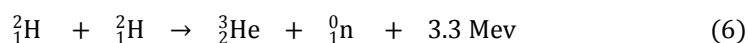
Where the total work required to build up the sphere = self-energy of the sphere. This is assumed to be the energy of the final charge after fusion, as according to the rules of electrostatics.

$$U = -\frac{q^2}{8\pi\epsilon_0} \left[\frac{1}{a} \right]_0^\infty \quad (5)$$

From the perspective of nuclear physics, the fusion of two charges leads to production of energy along with the final products.

For example,

When two deuterium[10] nuclei (${}^2_1\text{H}$) fuse, 3.3 MeV energy is released and the nucleus of helium isotope (${}^3_2\text{He}$) is formed. This helium isotope again gets fused with one deuterium nucleus to form a helium nucleus and 18.3 MeV energy is released in this process. The nuclear reactions are



Thus in all, three deuterium nuclei fuse to form a helium nucleus (${}^4_2\text{He}$) with a release of 21.6 MeV energy. A part of this energy is obtained in form of kinetic energy of neutron (${}^1_0\text{n}$) and proton (${}^1_1\text{H}$).

IV. Discussion

Nuclear fusion[10] is not possible at ordinary temperature and ordinary pressure. The reason is that when two nuclei approach each other, due to their positive charge, the electrostatic force of repulsion between them becomes too strong that they do not fuse. Hence to make the fusion possible, a high temperature(10^7K) and high pressure are required. At such a high temperature, both nuclei[21] due to thermal agitations acquire sufficient kinetic energy so as to overcome the force of repulsion between them when they approach each other and so they get fused.

Just as soon as the centre of two charges coincide at $r = 0$, the law becomes degenerate[14]. When we say something is degenerate, it means that it is the limiting case in which a class of objects changes its nature. Once it changes its nature, it usually belongs to another simpler class. For example, the point is a degenerate case of the circle as the radius approaches 0. The circle, in turn, is a degenerate form of an ellipse as the eccentricity approaches 0.

Singularity and infinity are related. According to Cantor that there are various types of infinities or hierarchy of infinities. Therefore, we can say there are various kinds of singularities(in this case also) for eg in general relativity we have a gravitational singularity.

Before fusion, the charges are accelerating towards each other, their fields can transport energy[9] irreversibly out to infinity, which we call radiation.

When a charged particle (such as an electron, a proton, or an ion) accelerates, it radiates[9] away energy in the form of electromagnetic waves, when a non-relativistic point charge accelerates or decelerates, the power is calculated by using Larmor's formula

$$P = \frac{\mu_0 q^2 a^2}{6\pi C} \quad (8)$$

The laws of Relativistic Electrodynamics[5] holds here, but we have not discussed here due to our purpose being entirely different.

V. Conclusion

The current scenario is proposed to be a case of electrostatic singularity, where we expect the charge is isolated and might be in equilibrium. What if singularity in the field of physics is a case where a particular entity(charge/particle) undergoes no interaction? like the above case of electrostatic singularity, where there is lone charge. We also see that the law is degenerate[12] at $r = 0$, this is where we believe the breakdown occurs as according to our intuition.

What if there exists a singularity in every field of physics? Like we already have in the context of general relativity inside a black hole. If the situation out above holds true, where we have a then we might have singularity from the perspective of classical physics

The electrostatic force between the positively charged nuclei is repulsive, but when the separation is small enough, the quantum effect[21] will tunnel through the wall. Therefore, the prerequisite for fusion is that the two nuclei be brought close enough together for a long enough time for quantum tunneling[22] to act. Therefore there is a possible need for quantum mechanics to come into the picture to explain the microscopic realm. However, we are still not sure if we surely need this.

There can be four different combinations of the charge here

1. + and -
2. + and +
3. - and -
4. N(no. of charges)+/- and N(no. of charges)+/-

In 2nd, 3rd and the 4th case as it looks there will be more energy required in the fusion of the charges. We don't know yet what will be the exact energy. But we are very much sure that the energy requirement for fusion in the 1st case will be very much less from the rest of the cases. So, therefore there is more work needed to be done. However, this needs to be done from a practical viewpoint.

The only way we can get an idea of the above-proposed scenario from a practical perspective is by utilizing the Large Hadron Collider (LHC), the world's largest and most powerful particle accelerator, where we can collide the two particles(charged) and look for the possibility of a resultant lone charge. We might have to tweak certain experimental steps in figuring out the desired results or it could be that we might benefit from a newer result. The area where we believe might get sufficient evidence would be the self-energy of the charge and whether the energy after the collision remains the same and if they differ, to what order of magnitude do they vary?

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