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Study of frictional ratchet where asymmetry arises due to the phase difference between the periodic friction coefficient and the periodic potential considering the movement of Brownian particles

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***Abstract**:- Noise has been usually considered to be a hindrance for quite some time. However, in the last two decades of the last century, its beneficial aspects have been rigorously studied. In this paper I have presented a special form of noise – thermal noise. The dynamics of a Brownian particle suspended in a heat bath (which acts as a source of thermal noise) at some temperature is considered. Under equilibrium conditions, the observance of directed transport or the so called ratchet effect is excluded. However, in a non-equilibrium situation, the Brownian particle exhibits ratchet effect if the medium in which it undergoes motion offers asymmetry. As a result of this, various forms of ratchets have been proposed. Here, I write up one form of a ratchet (inhomogeneous frictional ratchet) where the asymmetry arises due to the phase difference between the periodic friction coefficient and the periodic potential where the Brownian particle moves. This simple model is found to approximately mimic many real systems like molecular motors moving along microtubules and also systems that can be designed artificially.*

***Keywords:** Brownian motion, noise, non-equilibrium process, ratchet effect, periodic potential, symmetry breaking, inhomogeneous frictional ratchets, and molecular motors.*

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I. Introduction

Nature manifests itself in sublime ways and one of its manifestations is in its beneficial application of noise. Usually noise is normally thought of as a nuisance - as a destructive interference in signal detection and a hindrance in transmission of information. For example, one-to-one conversation in a crowded room, difficulty in focusing when the mind is interrupted by various thoughts and inability of cell phone network detection due to unfavorable weather conditions, all just tending to eclipse and obscure the desired information. However, over the past thirty years or so, a wide range of studies in a variety of systems - climatic models [1, 2], electronic circuits [3], neurophysiologic systems [4, 5, 6, 7], perceptual systems [8, 9, 10] - to name a few, have shown that noise may indeed enhance signal detection and transmission. Its essential role in physical processes was pioneered by Smoluchowski [11] in 1912. In fact it was because of the random jittery motion of a suspended particle (Brownian particle) in a colloidal solution, first officially recorded by the Botanist Robert Brown [12], and then theorized by Einstein [13], that was led to the confirmation that matter has an underlying microscopic structure within it. In other words, the Brownian particle was undergoing motion in a noisy environment. Physically, noise is used to describe fluctuations about the mean deterministic stationary value of a physical quantity. From the mathematical viewpoint, noise is a random variable whose values fluctuate unpredictably in time. Nonetheless, it should have well-defined properties like the mean, correlation and the other moments. This means that although the random variable by itself can take different values for each of its "realizations", its statistical properties however must remain constant. There is a large literature on the different kinds of noise that can arise in physical and physiological systems [14, 15]. In this review we focus mainly on one type of noise -thermal noise (also known as Johnson-Nyquist noise) -fluctuations that are ever present in any system due to its non-zero absolute temperature.

The main motivation for studying noise induced processes is in the domain of cellular motion and transport, especially molecular motors which aid the motion of proteins along periodic structures called microtubules by converting ATP into mechanical work [16]. The idea of constructing machines on an atomic scale was first discussed by Feynman in his talk on December 1959 [17]. Although not in the atomic scale, his dreams now seems to be a reality with the fabrication of molecular machines first developed by a French group led by Sauvage [18], Stoddart [19] and Feringa [20] who were eventually awarded with the Nobel Prize in Chemistry in 2016 [21]. Thermal fluctuations which can be safely ignored for the motion of macroscopic objects cannot be swept aside for microscopic objects like molecular machines. Thermal forces and viscous forces due to Brownian motion are more conspicuous and these small machines must either exploit the effects of thermal noise or otherwise overcome them altogether. Nature has provided a host of small machines, both linear and rotary, in the background of thermal noise. These natural biological machines perform their functions with remarkable efficiency and accuracy [22] and can thus serve as a guide for scientists who aim to design such artificial machinery.

a spool that was attached to a load. The whole setup was enclosed by a gas at thermal equilibrium, say Under non-equilibrium conditions, the cooperation of noise with nonlinearity can be seemingly counterintuitive. Numerous theoretical, numerical and experimental works reveal that there are a plethora of non-equilibrium systems which use noise as a driving force. Some of these phenomena are noise-induced transitions [15, 23], coherence resonance [24], noise-induced transport in ratchets [25, 26], resonant activation [27], noise-induced pattern formation [28] and stochastic resonance [1, 29]. This classification is by no means exhaustive. In this review, we focus only on the phenomenon of ratchet effect. Below we give a simple description of this phenomenon.

The concept of ratchet effect has dated many centuries back with the works of Archimedes, Seebeck, Maxwell, Curie, and others [26]. But the thought experiment by Smoluchowski [11] happens to be the first major contribution in this field. This was then popularized and extended some fifty years later by Feynman and was put as a chapter in his celebrated lectures on Physics [30]. The main idea was this: under isothermal conditions, it is impossible to rectify unbiased random fluctuations to generate directed transport for a microscopic system despite the presence of any intrinsic or fabricated asymmetry. We describe this thought experiment below in the same spirit as that of Feynman.

As shown in Fig. 1, a ratchet and pawl device was considered to be connected to the vanes by an axle. Halfway along the length of the axle was considered at temperature 'T'. The gas molecules undergoes Brownian motion and on doing so will hit the vanes which will in turn rotate the ratchet (inherently asymmetric) either clockwise or anti-clockwise. The purpose of

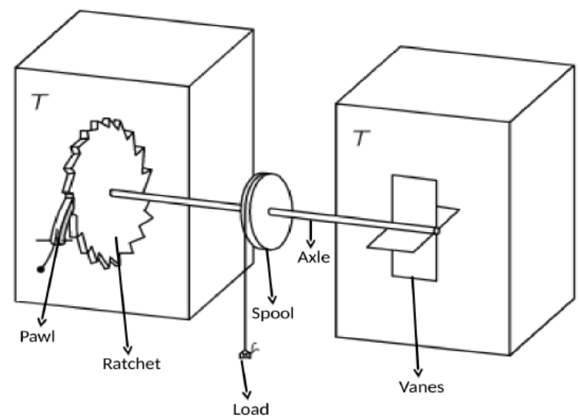

Fig.1: A ratchet and Pawl device

this setup is to allow the ratchet to rotate either clockwise only or either anti-clockwise only and if it were supposed to rotate in both directions then the pawl must forbid such a scenario. So if the ratchet system can rotate say clockwise only (according to the diagram), then it may be able to lift the load, which in turns increases the potential energy of the load thus enabling the ratchet to perform work even though the load were applied in a direction opposite to the direction of the rotation of the ratchet. Looking at such a scenario, it seems quite possible and convincing enough that a net transport has occurred by rectifying Brownian motion. But knowing how natural systems behave at equilibrium, we know that such a process does not occur otherwise the gadget would end up violating the Second Law of Thermodynamics. What needs to be examined properly here is the working of the pawl which closely resembles a sort of Maxwell's Demon [31]. It is important to stress that such a gadget was thought of to take place in the microscopic domain and the pawl itself must be extremely microscopic in order to allow a clockwise rotation only. But since the pawl is microscopic, it itself will also be subjected to undergo Brownian motion. So when if lifts itself up, it surely will not be in a position to prevent the ratchet from rotating anti-clockwise. Such kind of instances will on the average be anti-clockwise and hence overall the gadget will prefer no net rotation thus keeping the Second Law intact. Since this device was initially thought of by Smoluchowski and later popularized by Feynman, it was called after them as Smoluchowski-Feynman ratchet. Later this ratchet was experimentally realized on a molecular scale [32, 33, 34, 35]. Feynman however extended that the gadget may prefer a certain direction only if the gas molecules surrounding the vanes and those surrounding the ratchet and pawl are at different temperatures. Such a device is now under the domain of non-equilibrium thermodynamics since the isothermal condition is no longer kept. This device is named only after Feynman as the Feynman ratchet. So in perspective, directed transport in spatially periodic systems under isothermal conditions may occur only if the system were to be driven away from equilibrium by adding either a deterministic or a stochastic perturbation or in the presence of temperature gradients. In this review, we put a write up only of the effects of a co-sinusoid on a multi-stable periodic potential system at constant temperatures. But despite breaking thermodynamic equilibrium on periodic systems, the principle of detailed balance will still prevent a net transport to occur if the potential is symmetric. So an additional criterion to be realized is the breaking of this symmetry. This can be carried out either by introducing an asymmetric forcing of zero mean per period or by driving with a symmetrical force but introducing in homogeneities in the medium as in the form of friction. Following the above aforementioned criteria, many variations of ratchets have been proposed, developed and experimentally realized [26, 36, 37, 38]. All these proposed ratchets can be clubbed together into two basic classes: pulsating ratchets and tilting ratchets. In pulsating ratchets, the perturbation varies the shape of the potential without affecting its periodicity. In such ratchets there may be cases where the particles need not surmount any potential barriers, yet noise is indispensable to generate current in such models. Pulsating ratchets are further divided into two basic forms: fluctuating potential ratchets (on-off ratchets or flashing ratchets) and travelling potential ratchets. Tilting ratchets on the other hand are those for which the perturbation changes the average slope of the potential keeping in mind that the average of the perturbation over a period is zero. When the potential is of the form of a ratchet potential and the perturbation is symmetric, the tilting ratchet is called a rocking ratchet. However a tilting ratchet is called an asymmetric tilting ratchet when the potential is symmetric but the perturbation is asymmetric (zero mean over a period). Another variant of a tilting ratchet could be by varying the friction of the medium which could be done either spatially or temporally. Here, both the potential and the perturbation may be symmetric but symmetry breaking is realized by considering a similar symmetric friction coefficient which is out of phase with respect to the potential. Such variants of the tilting ratchet scheme are called inhomogeneous ratchets or frictional ratchets. In this short review, we only put a write up about frictional ratchets and the essential idea of how they work.

II. Frictional Ratchets: Modelling And Working

Frictional ratchets mentioned in this review are usually modeled by studying the motion of an ensemble of non-interacting Brownian particles, each of mass m, moving in a periodic potential $V(x) = -V_0 sin(kx)$ coupled to a heat bath at temperature *T*. This heat bath acts as a source of thermal noise represented by ξ. Moreover, the medium in which these particles move is inhomogeneous. An inhomogeneous medium is a medium where the friction coefficient $\gamma(x)$ offered by it is also periodic in nature as the potential $V(x)$. The friction coefficient is taken to be of the form of $\gamma(x) = \gamma_0\{1 - \lambda \sin(kx + \theta)\}$. Clearly, both the potential and the friction coefficient are sinusoidal in nature but have a phase difference θ between them. Fig. 2 shows a typical plot comparing the potential and the friction coefficient with a phase difference between them. This phase difference introduces the necessary asymmetry in the system. The quantity γ_0 is the average value of the friction coefficient for one period, λ is the in homogeneity parameter and it specifies the strength of the friction coefficient, V_0 is the amplitude of the potential and k is the wave number. The non-equilibrium condition in this problem is introduced by driving the system with a periodic forcing of the form $F(t) = F_0 \cos(\omega t)$, where F_0 is the amplitude of the forcing with driving frequency ω (or driving period $\tau = \frac{2\pi}{\omega}$). The one – dimensional equation of motion of one such Brownian particle is given by the so called Langevin equation as

$$m\frac{d^2x}{dt^2} = -\gamma(x)\frac{dx}{dt} + F(t) - \frac{dV(x)}{dx} + \sqrt{\gamma(x)T}\xi(t).$$

The first term in the RHS of the above equation refers to the damping force as given by Stoke's Law [39], the second term is the periodic force, the third term is the gradient of the potential and

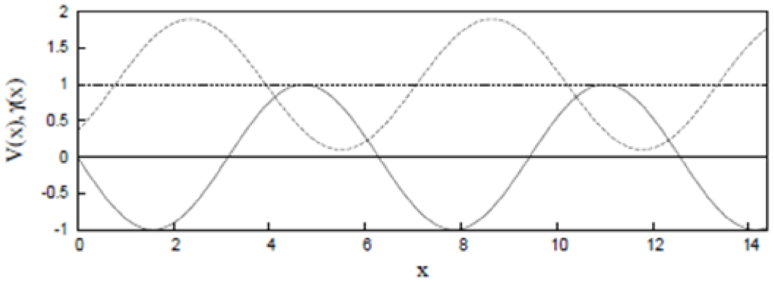

Fig. 2: The curve with unbroken lines represents the potential *V(x)* and the broken line represents $\gamma(x)$. The horizontal lines denote the corresponding average value per period. Here, $\theta = 0.75\pi$.

the last term represent the random forces (noise) at temperature *T*. Due to the presence of the periodic force, an effective potential $U(x) = V(x) - xF(t)$, is manifested into the system. The periodic force not only tilts the potential but also changes the barrier height. This is shown in Fig. 3 below.

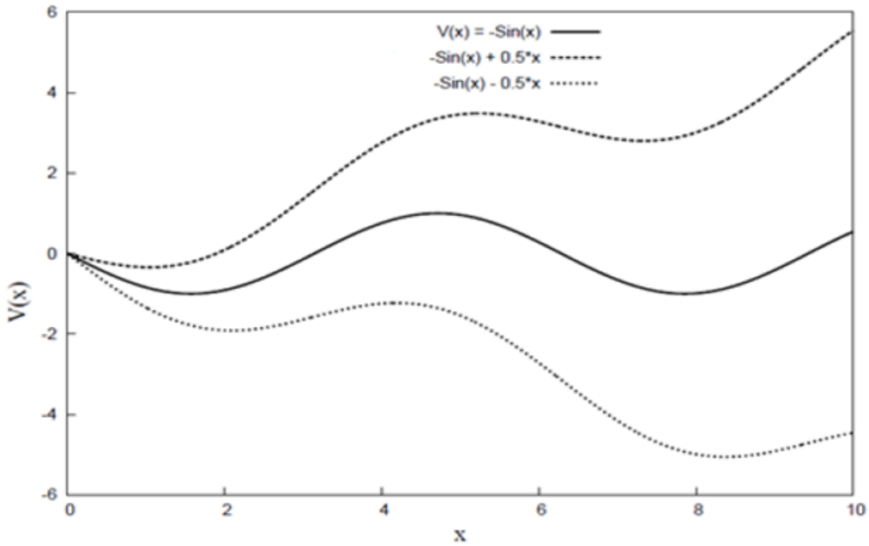

Fig. 3: The middle curve is the potential *V(x)* which gets tilted at different times of the forcing *F(t)*. For clarity we have chosen F_0= *0.5*.

Due to the form of the potential and the frictional coefficient, analytical methods of solving such an equation have not yet been developed. Thus numerical methods are resorted to solving. The dimensionless form [40] of the equation (by setting m, V_0 and $k = 1$) is usually solved numerically, say by Heun's method, for *n* Brownian particles and then ensemble averaging is done. In this review, we consider the case where the system is under damped ($\gamma_0 < \omega$). Though over damped approximations have been earlier studied extensively [22, 41, 42], yet the results presented when inertia is taken into consideration [43, 44] are also shown to be different and interesting in contrast to their over damped counterparts.

Fig. 4 shows a plot of ratchet current $< \bar{v} >$ as a function of noise strength *T* (temperature) for a typical under damped system (the symbol <...> represent ensemble averaging carried over *n* Brownian particles). The values of the parameters taken are described in the caption of the figure. In Fig. 4, it is seen that as temperature is increased, the system generates a current in the negative direction (along –X axis). For every value of temperature considered, the error bars over the velocity has also been shown.

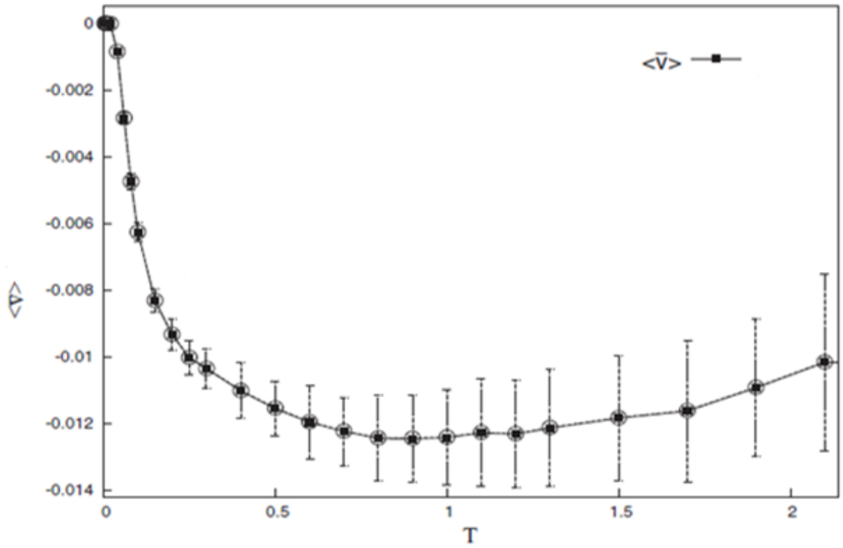

Fig. 4: Here the parameters taken are $\theta = 0.5\pi, \lambda = 0.9, \gamma_0 = 0.07, \tau = 7.7, F_0 = 0.2$.

In Fig. 5, we give a brief plausible explanation of how the system generates a current leftwards for the parameters considered. It may be mentioned here that since taking $\gamma_0 = 0.07$, $F_0 = 0.2$, the clarity of the graph may be reduced, so instead $\gamma_0 = 1.0$, $F_0 = 0.5$ is taken without compromising the working of the ratchet.

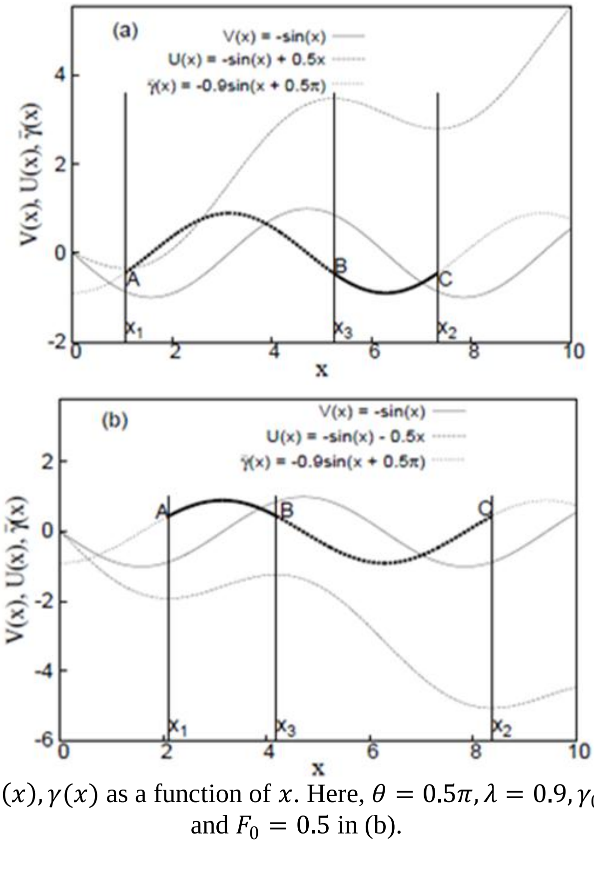

Fig. 5: Variation of $V(x), U(x), \gamma(x)$ as a function of x. Here, $\theta = 0.5\pi, \lambda = 0.9, \gamma_0 = 1$ with $F_0 = -0.5$ in (a) and $F_0 = 0.5$ in (b).

Let us consider Fig. 5a where the forcing tilts the potential with positive slope thus generating an effective potential *U(x)* as shown in the figure. On traversing *U(x)* from A to B (rightward) and from C to B (leftward), it is seen that the potential barrier rightward is much larger in comparison to that of leftward motion. Moreover, it is seen from the figure that the average friction (bold curve and bold- dotted curve) from A to B is also larger than that from C to B. Also, since *U(x)* tilts with positive slope, it is preferable for the particle to move leftwards.

Now let us consider Fig. 5b where the forcing tilts the potential with negative slope thus generating an effective potential *U(x)* as shown in the figure. On traversing *U(x)* from A to B (rightward) and from C to B (leftward), it is seen that the potential barrier rightward is much smaller in comparison to that of leftward motion. Moreover, it is seen from the figure that the average friction (bold curve and bold- dotted curve) from A to B is larger than that from C to B.

Summarizing the above two scenarios, we see that:

a) When potential is tilted with positive slope:

Rightwards: potential barrier is large and so is average friction

Leftwards: potential barrier is small and so is average friction

b) When potential is tilted with negative slope:

Rightwards: Potential barrier is small but average friction is large

Leftwards: Potential barrier is large but average friction is small

Thus, on an average the Brownian particles prefer small potential barrier to surmount with smaller average friction being offered by the medium so that mobility is enhanced. Thus, in such a case, it is understandable that the average displacement is leftwards.

III. Systems With Space-Dependent Friction

At a glance, it appears as though systems having space-dependent friction are artificial but they do actually occur in nature especially in biological systems, for example molecular motors moving along the periodic structure of microtubules experience space-dependent friction [45]. Also, in Josephson junctions the equation of motion have terms analogous to space-dependent friction [46] and the motion of ad atoms on the surface of a crystal of identical atoms have been justified by the mode-coupling theory that friction is periodically varying [47]. Systems with space dependent friction can also be designed artificially. For example, the case of Brownian motion in confined geometries [48] where the experimenters studied the diffusion of silica spheres (between 10^{-6} m and 3 x 10^{-6}m in diameter) diluted and subjected to ultrasonic vibrations in ultrapure water confined in a glass chamber. Fine wires with diameters ranging from 6 to 100 micrometer were used as spacers thus mimicking the periodic potential. On calculating the diffusion coefficient of the silica spheres, a deviation from the usual Stoke-Einstein law was observed.

IV. Conclusion

I presented above a short article on a class of ratchet – the frictional ratchet. In order to obtain a net current, it is important to drive the system out of equilibrium in conjunction with asymmetry. Here, the asymmetry is obtained by introducing a phase difference between the periodic potential and the friction coefficient whilst the non-equilibrium condition arises by driving the potential with a periodic force. In such ratchets noise plays an important role whereby it assists the particles to surmount the potential barriers. A plausible explanation of how this ratchet works has also been written in the text. A possible application of such a ratchet is in separation of micro sized particles having different diffusion constants. Much of the work on these ratchets is still left to be researched. For example, the quantitative behavior of the magnitude of average velocity with particle of different masses is still unexplored.

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