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# Thermal properties of hot Infinite Nuclear matter

M. M. H. El-Gogary [1a], M. H. M. Soleiman [1]
[1](*Physics Department, Faculty of Science, Cairo University, Giza 12613, EGYPT*)

***Abstract:***
*In the present study a method is developed to get and explore thermal properties of hot infinite nuclear matter in the phase of quark gluon plasma-gas. Two models (modified from MIT model to fit experimental data in nuclei collisions) are used as case studies. The models were introduced in the form of parametric barotropic equations of state for quark gluon plasma gas. The volumetric specific heat capacity at constant volume $c_v$, the adiabatic speed of sound $c_s$, and isentropic compressibility $\kappa_s$ of the quark gluon plasma (QGP) have been calculated as continuous functions of total energy density. It is found that the speed of sound increases as the energy content of QGP increases, while $c_v$ and $\kappa_s$ shows physical behavior like dilute gases.*

***Key Word**: MIT model; Buprenorphine; volumetric specific heat capacity; isentropic compressibility curve; Barotropic Equation of state.*

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## I. Introduction

The derivation of thermodynamic properties of infinite nuclear matter from equations of state (EOS) is not a straightforward from the mathematical point of view, because of hidden assumptions included in the formulation of the equation of state. it is standard to assemble three-dimensional tables of its properties. A fundamental basis is that full (EOS) tables be thermodynamically reliable so as not to produce deceptive and unphysical entropy during hydrodynamical simulations [1]. A few equations of state (EOS) of neutron gas and QGP have been adjusted and concentrated as those states of matter are fundamental in depicting the layered construction of neutron stars, and as these states of matter are difficult to be produced in nuclear labs on the earth [2]. The neutron gas and QGP thermodynamic properties are derived directly from the energy per particle of the system [3]. These methods of derivations give solid numbers and hide the dependence of thermodynamic properties on the thermodynamic parameters like temperature (T), chemical potential (μ), and total energy density (ϵ). In heavy-ion collisions and in the inner core of dense neutron stars, Quark gluon plasma (QGP) occurs also in gas-state. Several EOS's are proposed in publications [4]. Equations of the gas-state QGP status are some variations of the original bag concept, but are generalized to incorporate data collected from the CERN Large Hadron Collider experiments and the Brookhaven Relativistic Strong Ion Collider – U.S.A. In the present work, a method is proposed to drive thermodynamic quantities from polytropic equation of state, requires only four partial derivatives and the assumption that, the chemical potential (μ) is independent of the temperature (T) at constant volume. The procedure is extended to one EOS of neutron gas and four EOS of QGP gas. By this method, it is possible to calculate the following thermodynamic quantities; volumetric specific heat capacities at constant volume ($c_v$), speed of sound ($c_s$), and the isentropic compressibility ($\kappa_s$). The calculated quantities are functions of total energy density (ϵ) and additionally in chemical potential. The first part will present the preferred equations of state for process verification. The second section provides a summary of the schema of the proposed system. Third section discusses the application of the method to the models and shows the resulting thermodynamic quantities as functions of total energy density for each model.

## II. EOS models

**MIT bag-model;**

The quarks and gluons in the MIT bag model move freely inside the bag, so the bag compresses its parts to form the deconfined phase. The plasma constituents are moving free through large spatial regions because of the large number and high energy densities of the quarks and emits hadrons as black body emits photons. The quark-antiquark pair creation is enhanced with high temperature. For zero values of all conserved charges and non-interacting massless quarks, the EoS of the bag model is given as [4, 5]:

$$p = \frac{37\pi^2}{90}T^4 - B \qquad \text{and} \qquad \varepsilon = \frac{37\pi^2}{30}T^4 + B, \tag{1}$$

where, B is the bag constant with positive value. From equation (1) we can get the following relation:

$$p[\varepsilon(t)] = \frac{1}{3}[\varepsilon(t) - 4B]. \quad (2)$$

**Model 1 (A-bag model):**

By modify the bag model to fit the thermodynamic functions of the quark gluon plasma to the lattice data, the equations of state of QGP should include the following features, linear or quadratic in temperature term in the pressure function. The form of energy density is the same as in the standard bag model but with negative value of the bag constant B. The pressure and energy density are given by [4]:

$$p_1 = \frac{\sigma_1}{3}T^4 - AT - B_1 \qquad \text{and} \qquad \varepsilon_1 = \sigma_1 T^4 + B_1, \quad (3)$$

The pressure as function of energy density is given by the following relation:

$$P_1 = \frac{1}{3}[\varepsilon_1(t) - 4B_1] - A[\frac{\varepsilon_1(t)-4B_1}{\sigma_1}]^{\frac{1}{4}}. \quad (4)$$

Where the parameters, $\sigma_1 = 4.73$, $A = 3.93T_c^3$ and $B_1 = -2.37T_c^4$ are taken from [5], and $T_c$ is the critical temperature of the quark gluon plasma.

Model 2 (C-bag model):

At high temperatures, equation of state which could be compared quantitatively with the Monte-Carlo quantum chromodynamic lattice results and shows the best fit of thermodynamic functions is known as the C-bag Model. The pressure and total energy density functions are found in [6]. Pisarski had suggested the introduction of the terms proportional to $T^2$. The negative value of the bag constant is also required in this model to fit it with the lattice data. The pressure and total energy density functions are given respectively as [4]:

$$P_2 = \frac{\sigma_2}{3}T^4 - CT^2 - B_2, \qquad \text{and} \qquad \varepsilon_2 = \sigma_2 T^4 - CT^2 + B_2, \quad (5)$$

and the Eos is given by the relation:

$$P_2[\varepsilon_2(t)] = \frac{1}{3\sigma_2}\{[\varepsilon_2(t) - 4B_2] - C\left[C + \sqrt{C^2 + 4\sigma_2[\varepsilon_2(t) - B_2]}\right]\}. \quad (6)$$

Where, $\sigma_2 = 13.01$, $C = 6.06T_c^2$ and $B_2 = -2.34T_c^4$, also from [5].

## III. Method of Four Partial Derivatives:

The proposed method introduces consistent and simple formulation of the thermodynamic quantities in terms of p, $\epsilon$ and their derivatives with respect to T and μ.

The method assumptions:

a – The equation of state is polytropic,

$$p = p(\epsilon), \quad (7)$$

b- p and $\epsilon$ are functions of chemical potential μ and (free parameter, say) temperature T,

$$\epsilon = f(V, \mu), \quad (8)$$

$$V = V(\mu, T), \quad (9)$$

That is,

$$p = p(\mu, T). \quad (10)$$

c- The chemical potential μ is independent of T at constant V or the dependence could be negligible,

$$\left(\frac{\partial\mu}{\partial T}\right)_V = 0, \quad (11)$$

d- The internal energy U is proportional to $\epsilon$,

$$U = V\,\epsilon, \quad (12)$$

e- Validity of thermodynamic laws.

f- Validity of Maxwell's relations.

The focus is on the derivations of the following thermodynamic quantities:

$$c_S = \sqrt{\left(\frac{\partial p}{\partial \epsilon}\right)_S}, \quad (13)$$

$$C_V = Vc_V = \left(\frac{\partial U}{\partial T}\right)_V = V\left(\frac{\partial \epsilon}{\partial T}\right)_V, \quad (14)$$

$$\kappa_S = -\frac{1}{V}\left(\frac{\partial V}{\partial P}\right)_S, \quad (15)$$

where $c_S$ is the speed of sound in matter, $c_V$ is the volumetric specific heat capacities at constant volume, U is the internal energy of the system, and $\kappa_S$ is the compressibility at constant entropy.

The Maxwell's relations used in this work are:

$$\left(\frac{\partial U}{\partial V}\right)_S = -p, \tag{16}$$
$$\left(\frac{\partial U}{\partial s}\right)_V = T, \tag{17}$$

The first and the second internal energy equations [7] are:

$$\left(\frac{\partial U}{\partial V}\right)_T = T\left(\frac{\partial S}{\partial V}\right)_T - p, \tag{18}$$
$$\left(\frac{\partial U}{\partial p}\right)_T = -T\left(\frac{\partial V}{\partial T}\right)_p - p\left(\frac{\partial V}{\partial p}\right)_T, \tag{19}$$

The derivatives appears in equations (13-15) will be given in the following paragraphs, in terms of six quantities; the first two are p and $\epsilon$, and the other four are $\left(\frac{\partial \epsilon}{\partial T}\right)_\mu, \left(\frac{\partial \epsilon}{\partial \mu}\right)_T, \left(\frac{\partial p}{\partial T}\right)_\mu, \left(\frac{\partial p}{\partial \mu}\right)_T,$

-The derivative $\left(\frac{\partial \epsilon}{\partial T}\right)_V$ and $\left(\frac{\partial U}{\partial T}\right)_V$:

$$d\epsilon = \left(\frac{\partial \epsilon}{\partial \mu}\right)_T d\mu + \left(\frac{\partial \epsilon}{\partial T}\right)_\mu dT, \tag{20}$$
$$\left(\frac{\partial \epsilon}{\partial T}\right)_V = \left(\frac{\partial \epsilon}{\partial \mu}\right)_T \left(\frac{\partial \mu}{\partial T}\right)_V + \left(\frac{\partial \epsilon}{\partial T}\right)_\mu = \left(\frac{\partial \epsilon}{\partial T}\right)_\mu, \tag{21}$$

Thus, using the identity (12);

$$\left(\frac{\partial U}{\partial T}\right)_V = V\left(\frac{\partial \epsilon}{\partial T}\right)_V = V\left(\frac{\partial \epsilon}{\partial T}\right)_\mu. \tag{22}$$

-The derivatives $\left(\frac{\partial \epsilon}{\partial p}\right)_S$,

$$\left(\frac{\partial \epsilon}{\partial p}\right)_S = \frac{\left(\frac{\partial \epsilon}{\partial \mu}\right)_T\left(\frac{\partial \mu}{\partial T}\right)_S + \left(\frac{\partial \epsilon}{\partial T}\right)_\mu}{\left(\frac{\partial p}{\partial \mu}\right)_T\left(\frac{d\mu}{\partial T}\right)_S + \left(\frac{\partial p}{\partial T}\right)_\mu} = \left(\frac{\partial \epsilon}{\partial T}\right)_\mu / \left(\frac{\partial p}{\partial T}\right)_\mu, \tag{23}$$

where $\left(\frac{\partial \epsilon}{\partial \mu}\right)_T \equiv \left(\frac{\partial p}{\partial \mu}\right)_T \equiv 0$ for the selected models. There is no explicit dependence on the chemical potential $\mu$.

-The derivative $\left(\frac{\partial V}{\partial P}\right)_S$,

From the mathematical relation [7];

$$\left(\frac{\partial V}{\partial P}\right)_S \left(\frac{\partial p}{\partial \epsilon}\right)_S \left(\frac{\partial \epsilon}{\partial V}\right)_S = 1,$$

It's true that,

$$\left(\frac{\partial V}{\partial P}\right)_S = \left(\frac{\partial \epsilon}{\partial p}\right)_S / \left(\frac{\partial \epsilon}{\partial V}\right)_S = -V\frac{1}{p+\epsilon}\frac{\left(\frac{\partial \epsilon}{\partial \mu}\right)_T\left(\frac{\partial \mu}{\partial T}\right)_S + \left(\frac{\partial \epsilon}{\partial T}\right)_\mu}{\left(\frac{\partial p}{\partial \mu}\right)_T\left(\frac{d\mu}{\partial T}\right)_S + \left(\frac{\partial p}{\partial T}\right)_\mu} = -V\frac{1}{p+\epsilon}\left(\left(\frac{\partial \epsilon}{\partial T}\right)_\mu / \left(\frac{\partial p}{\partial T}\right)_\mu\right). \tag{24}$$

The derivations of the matter properties in reference [8] had assumed the dependence of the fermion density n, the total energy density $\epsilon$, and the pressure p on the chemical potential ($\mu$) and the temperature (T). It was normal among authors working in theoretical and mathematical models to get the isothermal compressibility of the equation [9]:

$$\kappa_T = 9\left[\frac{\partial}{\partial \rho}\left(\rho^2 \frac{\partial(E/A)}{\partial \rho}\right)\right]_{\rho=\rho_0, T=0}. \tag{25}$$

Where, $\rho_0$ is the ground state density of the nucleons in the nucleus of atomic mass number A and nucleon density $\rho$, and $E/A$ is the total energy per particle (i.e. nucleon). This equation is limited to a system at zero absolute temperature and requires a mechanical function of $E/A$. The method in this work is general to find $\kappa_s$ directly from the equation of state and at any temperature allowed by the validity of the EoS. Reference [9] uses the derivatives $\left(\frac{\partial \epsilon}{\partial T}\right)_\mu, \left(\frac{\partial \epsilon}{\partial \mu}\right)_T, \left(\frac{\partial p}{\partial T}\right)_\mu, \left(\frac{\partial p}{\partial \mu}\right)_T, \left(\frac{\partial n}{\partial T}\right)_\mu$, and $\left(\frac{\partial n}{\partial \mu}\right)_T$ to drive the thermodynamic properties. The suggested scheme in this work is more economic and logic as well as based on realistic assumptions.

The isentropic compressibility $\kappa_s$ is calculated as function of energy density $\epsilon$ for models 1 and 2, and they are graphed in figure 3. The isentropic compressibility $\kappa_s$ is found to be $\sim 10^{-12}$ MeV$^{-4}$, which means the stiffness of the QGP-gas.

## IV. Results and discussion

The behaviors of the thermodynamic properties as functions of energy density $\epsilon$ are obtained numerically according to the scheme described in section 2. The comparison of the thermodynamic quantities between different models reveals peculiarities of different models for infinite nuclear matter. In this research, the behaviors of the thermodynamic quantities as functions of the total energy density are shown four neutron gas and two different versions of the QGP gas.

The behavior of the speed of sound squared $c_s^2$ with the energy density $\epsilon$ is obtained in figure 1. The speed of sound increases for QGP-matter and takes constants value $c_s^2 = 0.33$ for MIT-model. The curves are below the MIT model for models 1,2 and are saturated in regions with high energy densities.

Figure 2 gives the behavior of $c_v$ with energy density $\epsilon$ for QGP Models MIT, 1, and 2. The behavior of volumetric heat capacity $c_v$ is physical for all studied models. The studied EoS's show large values of heat volumetric capacity at constant volume $\sim 10^{10}\ \mathrm{MeV}^3$.

The isentropic compressibility $\kappa_s$ is calculated as function of energy density $\epsilon$ for models 1 and 2, and they are graphed in figure 3. The isentropic compressibility $\kappa_s$ is found to be $\sim 10^{-12}\ \mathrm{MeV}^{-4}$, which means the stiffness of the QGP-gas.

## V. Conclusion

1-In the present work a method to derive thermodynamic properties from parametric equations of state of infinite nuclear matter is discovered. Thermodynamic parameters are derived for two case studies, which describe QGP matter, and are different modifications of the MIT-bag model. Smooth functions of quantities; $c_s$, $c_V$, $\kappa_s$ are derived in terms of p and $\epsilon$, and four derivatives $\left(\frac{\partial \epsilon}{\partial T}\right)_\mu$, $\left(\frac{\partial \epsilon}{\partial \mu}\right)_T$, $\left(\frac{\partial p}{\partial T}\right)_\mu$, $\left(\frac{\partial p}{\partial \mu}\right)_T$ . It has been found that the speeds of sound of in QGP matter increases and saturate in models 1 and 2. It is less than $0.33$.

2-The compressibility $\kappa_s$ is obtained directly from the equations of state as smooth functions of $T$ and $\epsilon$.

3-The derivation scheme in this work is economic in that, it requires only six quantities and derivatives to obtain the thermodynamic properties. Other method requires at least eight or more derivatives to get some of the thermodynamic properties.

Future work will extend this schema to accommodate polytropic EOSs in parametric forms and with explicit dependence on chemical potential$\mu$, and the method will be applied to more case studies.

## References

[1] Constantinos Constantinou, Brian Muccioli, Madappa Prakash, and James M. Lattimer, "Thermal properties of supernova matter: The bulk homogeneous phase", Phys.Rev C 89, (2014), 065802.

[2] James M. Lattimer, "Neutron Stars", 33rd SLAC Summer Institute on Particle Physics (SSI 2005), 25 July - 5 August 2005.

[3] Yu. B. Rumer and M. Sh. Ryvkin, "Thermodynamics, Statistical Physics, and Kinetics", Published by MIR Publishers, 1980.

[4] S.M. Sanches Jr., F.S. Navarra, D.A. Fogaça "The quark gluon plasma equation of state and the expansion of the early Universe", Nuclear Physics A, 937, (2015), 1–16.

[5] V.V. Begun, M.I. Gorenstein, O.A. Mogilevsky, "MODIFIED BAG MODELS FOR THE QUARK GLUON PLASMA EQUATION OF STATE", Int. J. Mod. Phys. E 20, (2011), 1805.

[6] Pisarski, "Effective theory of Wilson lines and deconfinement", Phys. Rev. D 74, (2006), 121703.

[7] Mark W. Zemansky, Richard H. Dittman, " Heat and Thermodynamics", 7th ed, Copyright ©1997.

[8] Constantinos Constantinou, Brian Muccioli, Madappa Prakash, and James M. Lattimer, "Thermal properties of hot and dense matter with finite range interactions", Phys. Rev. C 92, (2015), 025801.

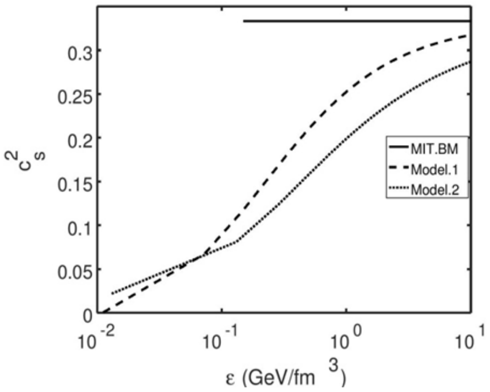
[9] Hua Zheng, Aldo Bonasera, "Non-Abelian behavior of α bosons in cold symmetric nuclear matter", Phys. Rev. C, 83, (2011), 057602.

## Figures Captions

Fig.1. The comparison of the speed of sound between model 1, model 2 and MIT bag model quark gluon plasma matter. The result of the original bad model calculated with the bag constant $B = 150\ \mathrm{MeV/fm^3}$.

Fig. 2. Volumetric specific heat capacity $c_V$ of quark gluon plasma matter of MIT - bag model, model 1, and model 2. Dotted line is the Volumetric heat capacity $c_v$ of model 2 scaled by factor 1.1.

Fig. 3. The comparison of the Isentropic compressibility $\kappa_s$ of quark gluon plasma matter, MIT bag model, model 1 and model 2. The solid line for the MIT-model is scaled by factor 100, and dashed line for model 1is scaled by facto

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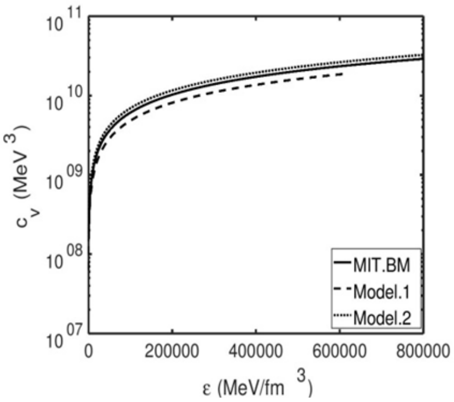


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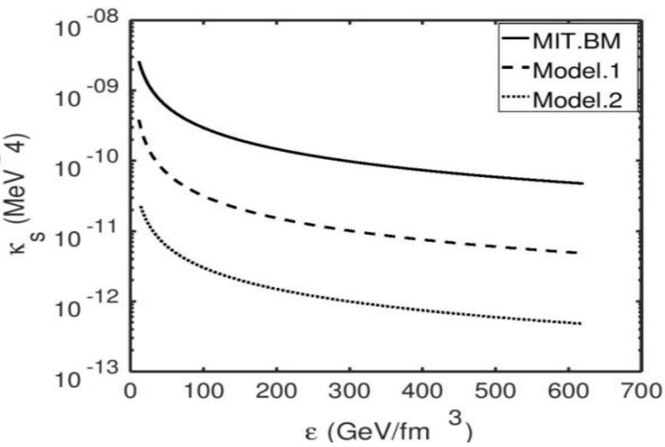


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