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Maxwell-Fredholm Equations for the Beam Deflection in Anisotropic Left-handed Propagating Media with Toroidal Symmetry

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Abstract:
We apply the recently created Maxwell-Fredholm equations to the description of the highly deflection of an electromagnetic beam immersed in a anisotropic media with left-handed properties with a preponderant toroidal symmetry, showing how we can model with an algebraic tool a very complicated electromagnetic interaction between the media and an electromagnetic beam relating the initial fields with the final ones through a simple matrix arrangement and very simple set of differential equations.
***Key Word**: Maxwell-Fredholm equations; left-handed media; toroidal symmetry; electromagnetic resonances.*

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I. Introduction

Nowadays, are several branches on applied physics and electromagnetics in which the toroidal symmetry becomes a trend, even, there is some kind of mixed way to attack flux problems by defining flux coordinates over a basic toroidal frame. On this work, we take the point of view of Maxwell-Fredholm equations (MFE) over toroidal coordinates to describe the deflection of a thin beam due to a strongly anisotropic, left-handed propagating media. We started by writing the hybrid Maxwell-Fredholm equations, that is:

$$rot\mathbf{E}_e(\omega) = -i\omega\boldsymbol{\mu} e^{-ih(\omega_e)}\mathbf{K}^{(\circ)}(\omega)\mathbf{H}_e(\omega)$$

(1)

$$rot\mathbf{H}_e(\omega) = i\omega\boldsymbol{\varepsilon} e^{-ih(\omega_e)}\mathbf{K}^{(\circ)}(\omega)\mathbf{E}_e(\omega)$$

(2)

$$\eta_e(\omega) = e^{ih(\omega_e)}$$

(3)

Next, we remember a recent paper[1] where we have demonstrated that it is possible to model the very complicated punctual interaction between an electromagnetic beam and a highly distorting media in some circumstances with the aid of a the permittivity tensor $\boldsymbol{\varepsilon}$, that is we can represent the electromagnetic fields at the final points of the beam trajectory in terms of their values at the initial points.

Then, we follow similar steps to pursue our goal in the specific problem of an electromagnetic thin beam travelling over the surface of a toroid from an initial point to a final location in a curl trajectory. We can see that it is possible to avoid more complicated computational calculations by employing an appropriate total permittivity tensor.

II. Justification for Using the MFE

As we can note the MFE formalism refers to a class of integral equations that in principle has not a source term, which is it appropriate for describe resonant phenomena[4]. But we have proved in a recent paper that there is an equation we named the Generalized Fredholm Equation (GFE) for describing both the resonant and non-resonant phenomena simultaneously:

$$\Phi(\mathbf{r},\omega) = \Phi^{(\circ)}(\mathbf{r},\omega) + \eta(\omega)\int_0^\infty \mathbf{K}_n^{m(\circ)}(\omega;\mathbf{r},\mathbf{s})\Phi(\mathbf{s};\omega)ds$$

(4)

Where

$$\Phi^{(\circ)}(\mathbf{r},\omega) = \Delta(\eta,\omega)[\eta(\omega) - g^{m(\circ)}(\mathbf{r},\omega)] + \Delta(\eta,\omega)[\eta(\omega) - \nu(\omega)]\int_0^\infty \mathbf{K}_n^{m(\circ)}(\omega,\mathbf{r},\mathbf{r}')g^n(\mathbf{r}',\omega)dr'$$

(5)

Is called the generalized source.

The GFE has a generalized source term (5) that can be treated as a tuning term that allow to choose the type of phenomena we want to describe by simple changing the value of the electromagnetic parameters, so it must be compatible with the particular mathematical structure of the MFE. So we should not worry about the validity of the last formalism for describing the problem of the bending of an electromagnetic beam, but we must take care in the sense that the solutions of the MFE are for a specific resonance. This poses a problem, the transmission of information on a beam. If we want to use the MFE for the broadcasting of information we must use precisely the information theory through the information packs we have defined previously in other place and combine the MFE and the information theory. Then we remember this procedure: first, there is a very important property of the resonant solutions for the generalized Fredholm equations that is, the orthogonally between different resonances. Indeed we are giving an alternative point of view as the established by Xiang-kun Kong et al.[2], concerning the physical interpretation of a resonance. Our own interpretation consider that resonances represent the condition for the breaking of the confinement of the evanescent waves If the resonances would constitute a band of resonant states, we could use these properties directly as a mathematical base to represent any kind of desired broadcasting signal, but then we have a problem because the set of resonant solutions is made up of a discrete set of frequencies that only permit a very limited information transmission, may be like a telegraph mode in which each individual frequency can be used as a signal succession, non-signal intervals. This last process is very far of our expectations for an efficient broadcasting. But, if we use some results we have obtained previously like the definition of information packs, we can reach our desired results. If we want to send a signal that is represented by a function $J(t)$ and assuming we know that the propagation medium bring us a set of resonances one of which we can call e so that the associated resonant frequency will be denoted by ω_e. Then we can project the original signal over a sub-space generated with the help of Communication theory[3] by the rule

$$J_e(t) = \sum_{-\infty}^{\infty} P_{m,e} \frac{\sin[\pi(\omega_e t - m)]}{\pi(\omega_e t - m)}$$

(6)

In expression (6) the development coefficients are:

$$P_{m,e} = J(\tfrac{m}{2\omega_e})$$

(7)

This gives us a set of projections of the original $J(t)$. Then, we can emit simultaneously the different projections $J_e(t)$ and when they arrive to their destination, we can unite all of them and rebuild the original $J(t)$. There is a limiting condition coming also from Communication theory about the frequencies appeared in every pack, that is, these frequencies cannot be major than the respective resonant frequency ω_e. During the broadcasting, the orthogonally properties of the resonances and the structure of the information packs guarantee that there is not interference between the different projections.

In this paper we do not show how we can apply equation (6) explicitly, but we suppose that the signal we enter through the initial electric and magnetic fields comes from the building of information packs. In this manner, we are using resonances in two different ways[4,5], first by using the Maxwell-Fredholm equations created for an explicitly homogeneous situation and second, by the projection of the original signal over the sub-spaces generated with the rules (6) and (7).

III. The Operator $\nabla \times \mathbf{F}$ in Toroidal Coordinates

In order to write explicitly the MFE we need the curl operator $\nabla \times \mathbf{F}$ in toroidal coordinates, that is:

$$\nabla \times \mathbf{F} = \hat{e}_\sigma \frac{(\cosh\tau - \cos\sigma)^2}{a^2 \sinh\tau} \left(\frac{\partial\left(F_\varphi \frac{a\sinh\tau}{\cosh\tau - \cos\sigma}\right)}{\partial\tau} - \frac{\partial\left(F_\tau \frac{a}{\cosh\tau - \cos\sigma}\right)}{\partial\varphi} \right)$$

$$+ \hat{e}_\tau \frac{(\cosh\tau - \cos\sigma)^2}{a^2 \sinh\tau} \left(\frac{\partial\left(F_\sigma \frac{a}{\cosh\tau - \cos\sigma}\right)}{\partial\varphi} - \frac{\partial\left(F_\varphi \frac{a\sinh\tau}{\cosh\tau - \cos\sigma}\right)}{\partial\sigma} \right)$$

$$+ \hat{e}_\varphi \frac{(\cosh\tau - \cos\sigma)^2}{a^2} \left(\frac{\partial\left(F_\tau \frac{a}{\cosh\tau - \cos\sigma}\right)}{\partial\sigma} - \frac{\partial\left(F_\sigma \frac{a}{\cosh\tau - \cos\sigma}\right)}{\partial\tau} \right)$$

(8)

IV. The Unitary Vectors $\hat{\mathbf{e}}_\sigma$, $\hat{\mathbf{e}}_\tau$ and $\hat{\mathbf{e}}_\varphi$

Also, we need the explicit expressions for the unitary mutually perpendicular basis that are represented in Fig. 1 that is:

$$\hat{e}_\sigma = \frac{1}{h_\sigma}\frac{\partial\mathbf{r}}{\partial\sigma} = \frac{(\cosh\tau - \cos\sigma)}{a}\frac{\partial\mathbf{r}}{\partial\sigma}$$

$$= \frac{1}{\cosh\tau - \cos\sigma}\Big[-\sin\sigma \sinh\tau \left(\cos\varphi\,\hat{e}_x + \sin\varphi\,\hat{e}_y\right)$$

$$+ \left(\cos\sigma \cosh\tau - 1\right)\hat{e}_z \Big]$$

(9)

$$\hat{e}_\tau = \frac{1}{h_\tau}\frac{\partial\mathbf{r}}{\partial\tau} = \frac{(\cosh\tau - \cos\sigma)}{a}\frac{\partial\mathbf{r}}{\partial\tau}$$

$$= \frac{1}{\cosh\tau - \cos\sigma}\Big[\left(1 - \cosh\tau\cos\sigma\right)\left(\cos\varphi\,\hat{e}_x + \sin\varphi\,\hat{e}_y\right)$$

$$- \sinh\tau \sin\sigma\,\hat{e}_z \Big]$$

(10)

$$\hat{e}_\varphi = \frac{1}{h_\varphi}\frac{\partial\mathbf{r}}{\partial\varphi} = \frac{\cosh\tau - \cos\sigma}{a\sinh\tau}\frac{\partial\mathbf{r}}{\partial\varphi}$$

$$= -\sin\varphi\,\hat{e}_x + \cos\varphi\,\hat{e}_y$$

(11)

V. Beam Deflection in Toroidal Symmetry

Let us take equation (2) and make in the left-hand term

$$rot\mathbf{H}_e'(\omega) = i\omega\boldsymbol{\varepsilon}'\mathbf{E}'_e(\omega)$$

(12)

So we arrive to the equation

$$i\omega \mathbf{E}_e'(\omega) = i\omega \boldsymbol{\varepsilon} e^{ih(\omega_e)} \mathbf{K}^{(\circ)}(\omega)\mathbf{E}_e(\omega)$$

(13)

Now, we suppose that the electric field points toward the unitary vector $\hat{e}_\tau$ that implies the equation (13) becomes

$$\mathbf{E}_e'(\omega) = \boldsymbol{\varepsilon} e^{ih(\omega_e)} \mathbf{K}^{(\circ)}(\omega)\, \hat{e}_\tau E_\tau(\omega)$$

(14)

Defining the permittivity tensor

$$\boldsymbol{\varepsilon}$$

(15)

In principle, there is a dependence on the frequency W but, for convenience, we bequeath this to the kernel, in order to easy look the contribution of the tensor $\boldsymbol{\varepsilon}$, which operates on the column vectors in the (σ, τ, φ) space bending the beam trajectory, then by using a similar description as the used for the Euler angles:

$$\boldsymbol{\varepsilon} = \varepsilon \begin{bmatrix} \cos 2\varphi \cos\beta & \cos\beta \sin 2\varphi & -\sin\beta \\ -\sin 2\varphi & \cos 2\varphi & 0 \\ \sin\beta \cos 2\varphi & \sin\beta \sin 2\varphi & \cos\beta \end{bmatrix}$$

(16)

The total effect is to make that from an initial point $(\sigma_\circ, \tau, \varphi_\circ)$, the $\sigma_\circ$ parameter tour to the new value $\cos\beta \cos 2\varphi\sigma_\circ + \cos\beta \sin 2\varphi\tau - \sin\beta\varphi_\circ$, $\varphi_\circ$ tour to the new value $\sin\beta \cos\varphi\sigma_\circ + \sin\beta \sin 2\varphi\tau + \cos\beta\varphi_\circ$ and τ preserves his value that is the same toroidal surface as long as:

$$\cos\varphi = \frac{1 \pm \sqrt{\frac{2\sigma_\circ}{\tau} + \left(\frac{\sigma_\circ}{\tau}\right)^2}}{1 - \frac{\sigma_\circ}{\tau}}$$

(17)

We have used the notation β for distinguish between the two successive rotations over the (σ, τ, φ) space. In terms of this last tensor, equation (14) can be written

$$\mathbf{E}_e'(\omega) = e^{ih(\omega_e)} \boldsymbol{\varepsilon} \mathbf{K}^{(\circ)}(\omega) \hat{e}_\tau E_\tau(\omega)$$

(18)

For simplicity, we propose that we have only two punctual emitters with the kernel given by

$$\mathbf{K}^{(\circ)} = \begin{bmatrix} {}^1\mathbf{K}^{(\circ)} & \mathbf{0} \\ \mathbf{0} & {}^2\mathbf{K}^{(\circ)} \end{bmatrix}$$

(19)

On matrix (19) the elements are

$$^{1,2}\mathbf{K}^{(\circ)} = \begin{bmatrix} \frac{\sin\left[(\omega-\omega_p)\delta\right]}{(\omega-\omega_p)\delta} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

(20)

As we have said, we suppose that the electric field at the two initial points only have a τ component and indeed are identical, for example

$$E_\tau(\mathbf{r}_1) = E_\tau(\mathbf{r}_2) = E_0 \cos(\omega_0)$$

(21)

In equation (12) we impose the condition that the $rot\mathbf{H}_e'(W)$ does not have σ or φ components. Also

$$H_\tau{}' = 0$$

(22)
And

$$H_{?j}{}' = 0$$

(23)
Which means that $H_\sigma{}'$ satisfy the partial differential equation:

$$\frac{\partial\left(H_\sigma{}'\frac{a}{\cosh\tau-\cos\sigma}\right)}{\partial\tau} = 0$$

(24)

$$\frac{\partial\left(H_\sigma{}'\frac{a}{\cosh\tau-\cos\sigma}\right)}{\partial\tau} =$$

$$\frac{a}{\cosh\tau-\cos\sigma}\frac{\partial H_\sigma{}'}{\partial\tau} + H_\sigma{}'\frac{\partial\left(\frac{a}{\cosh\tau-\cos\sigma}\right)}{\partial\tau} = 0$$

(25)

$$\frac{\partial H_\sigma{}'}{\partial\tau} = -\frac{\sinh\tau}{\cosh\tau-\cos\sigma}H_\sigma{}'$$

(26)

And then

$$H'_\sigma = C_0(\tau,\varphi)\frac{1}{\cosh\tau-\cos\sigma}$$

(27)

Now from equations (18-21) we can write:

$$i\omega\mathbf{E}'_e(\omega) = i\omega\boldsymbol{\varepsilon}e^{ih(\omega_e)}\mathbf{K}^{(\circ)}(\omega)\mathbf{E}_e(\omega)$$

(28)
And then it follows that:

$$\mathbf{E}'_e(\omega) = \boldsymbol{\varepsilon}e^{ih(\omega_e)}\mathbf{K}^{(\circ)}(\omega)\hat{e}_\tau E_\tau(\omega)$$

(29)
Or

$$\mathbf{E}'_e(\omega) = e^{ih(\omega_e)}\boldsymbol{\varepsilon}\mathbf{K}^{(\circ)}(\omega)\hat{e}_\tau E_\tau(\omega)$$

(30)
Remembering that we are supposing that $\mathbf{r}_1$ is very near to $\mathbf{r}_2$, so we can write that (although it is not strictly necessary):

$$E_\tau(\mathbf{r}_1) = E_\tau(\mathbf{r}_2) = E_0\cos(\omega_0)$$

(31)
And writing explicitly the kernel:

$$\mathbf{E}'_e(\omega) = \boldsymbol{\varepsilon}e^{ih(\omega_e)}\begin{bmatrix} {}^1\mathbf{K}^{(\circ)} & 0 \\ 0 & {}^2\mathbf{K}^{(\circ)} \end{bmatrix}\hat{e}_\tau E_\tau(\omega)$$

(32)

We can then write for the electric field by using equations (27-32) together with the explicit form of the curl in toroidal coordinates (8):

$$E'_{\tau}(\mathbf{r}_{1,2}) = \frac{1}{\varepsilon'}\left(\frac{\partial C_0(\tau,\varphi)}{\partial\varphi}\right)\frac{1}{\sinh\tau} = \varepsilon e^{ih(\omega_e)}\frac{\sin(\omega-\omega_p)\delta}{(\omega-\omega_p)\delta}\cos(\omega_0)E_0$$

(33)

And because E_0 is a function of (σ,τ,φ) then:

$$E_0(\sigma_{1,2},\tau,\varphi_{1,2}) = D_0(\tau,\varphi)\frac{1}{\sinh\tau}$$

(34)

And also:

$$\frac{1}{\varepsilon'}\left(\frac{\partial C_0}{\partial\varphi}\right)\frac{1}{\sinh\tau} = \varepsilon e^{ih(\omega_e)}\frac{\sin(\omega-\omega_p)\delta}{(\omega-\omega_p)\delta}\cos(\omega_0)D_0\frac{1}{\sinh\tau}$$

(35)

Finally:

$$\frac{1}{\varepsilon'}\left(\frac{\partial C_0}{\partial\varphi}\right)\frac{1}{\sinh\tau} = \varepsilon e^{ih(\omega_e)}\frac{\sin(\omega-\omega_p)\delta}{(\omega-\omega_p)\delta}\cos(\omega_0)D_0E_0$$

(36)

From equation (27) we can see that for every set (σ,τ,φ) we have a different value of C_0 and we need since equations (31-36) to establish the value of the electric field. We must remember that even the beam is moving over a torus of fixed τ , intersects a continuum of different spheres. The process is illustrated in Figure 1 where the curl trajectory is represented in yellow and the basis vectors are rotated in the space (σ,τ,φ) in accordance with the permittivity tensor ε .

VI. Conclusions

Then, we have obtained our goal for describing the electromagnetic field in the specific problem of an electromagnetic thin beam (that may be a laser beam) travelling over the surface of a torus from an initial pair of very near points to a final pair of locations in a curl trajectory. We can see that it is possible to avoid more complicated computational calculations by employing the MFE and an appropriate total permittivity tensor; we can think that also the results could be used in other current areas[6]

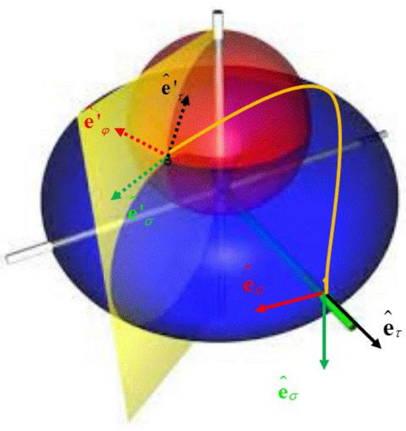

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Fig. 1. Bending of an electromagnetic beam in toroidal coordin

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