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Investigations on corollaries of point spread function using Complex pupil function

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Abstract
In the case of a three-zone aperture, using Co sinusoidal, Gaussian, and Hanning filters and phase filters, a few energy-based corollaries of the point spread function with a complex pupil function for diffraction limited and defocused optical system have been theoretically investigated. In terms of amplitude filters in a suitable combination with phase filters in the second and third zone, credit provinces such as the encircled energy factor, and zonal energy increment, were examined with respect to three-zone filters. By using Hanning Gaussian, Co-sinusoidal amplitude filters, and phase filters at the second, outer zones effectively suppress first and second order side lobes of the PSF. The resolution of the optical system has been improved by using complex pupil function, which is applicable in imaging and focussing applications.
Keywords*: Point spread function, complex pupil functions, Encircled energy, excluded encircled energy, zonal energy increment resolution.*

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I. Introduction

The main corollaries quantify the portion of the total energy contained in the PSF of a particular radius in the optical imaging systems' focus plane. The primary corollary of the point spread function and the one that can best explain the behavior of the point source diffraction image is Encircled energy [1]. Encircled energy is the radial distribution of energy in the image, and it can be studied as a measure of image quality, which is a crucial component. At the focal plane, light flux diffuses throughout a spatial region, and the actual form of this propagation is influenced by a number of variables, including the size, shape of the aperture, aberrations, and transmission tolerance [2]. Lord Rayleigh developed formulas for the diffraction pattern and was the one who first emphasised the significance of the corollaries in it. He was eager to learn more about the illumination of the different rings in the diffraction pattern [3]. The point spread function is an image of a point source formed by an optical system. To study the optical system performance, the PSF is the most complete function and also expanded to include unsuitable aperture effects, apodization, defect-of –defocus and a component outside the optical system [4]. In order to improve the resolution of the aberrated optical system, the proposed amplitude apodization with phase change is crucial for astronomical observations. A N K Reddy, M Hashemi carried out investigations like the apodization of Hanning complex pupil functions, where fundamental aberrations were only present in the central portion of the pupil and were absent from the semi-annular areas.The performance is attributed to the pupil mask itself because the aberrations considered in the preceding study were limited to the central area. As a result, it was not a precise assumption. However, the side lobes in this instance were entirely suppressed at one side, and on the other side of the PSF, the side lobes are enhanced, this results in asymmetric PSF is crucial for confocal imaging [5-7]. Encircled energy becomes zero when δ is zero and reaches 1 when δ is maximum. Different corollaries like displaced energy excluded encircled energy, and zonal energy increament, are derived and calculated with the quantity of Encircled energy. Numerous studies on PSF corollaries have been conducted by various researchers [8]. It has been used and researched to maximise the diffraction pattern by making the encircled energy maximal for a specific radius analyzed by straubel (1935) [9-10]. Lansraux offered a different explanation for the encircling energy, stating that it was the contrast between the image's background and center. He has created analytical formulas for encircling energy that is accurate for tiny aberrations. By significantly changing the tool created by Shanon and Newman to measure the transfer function, encircled energy is measured [11]. Rao, K.P. Mondal, and Seshagiri Rao studied the encircled and excluded energies for straubel apodization filters (1976) [12]. Encircled energy of spherical aberrations, effects of apodization, and defocusing, was studied by M. Keshavulu and Mondal (2003) [13].

The ideal phase and amplitude complex pupil function parameters have been determined from our analysis. With appropriate amplitude filters and increasing apodization, and defocus, the results show better information when the optical system has a significant amount of focal shift and a clear reduction in the encircled energy values. It was also discovered that as circular ring width increases, the value of the aforementioned corollaries decreases. This shows that just a small portion of the overall energy is contained in the central location in the central ring group, making the entire collection of central rings the effective image component. This research using different filters of appropriate zones with different thicknesses of apertures tries to enhance some features of the optical system's image capability or to achieve high resolution by introducing amplitude filters and phase filters with appropriate sizes of apertures. Results from our investigation have been shown graphically and as tables, and these studies are useful for altering an optical imaging system's properties. In the present research, we have studied the effect of encircled energy, and zonal energy increment on the performance of symmetric optical systems apodized and defocused by Co sinusoidal filter, Gaussian filter, Hanning filter, and with the phase change of ($-\pi/4$) and($\pi/3$).

II. Theory

We can define the Encircled energy as the amount of energy that is contained within a circle of radius δ, centered on an axial point in the image plane, which can be represented mathematically by

$$\mathrm{EE}(\delta) = \frac{\int_0^{\delta} |\mathrm{A_P}(\text{Ød},z)|^2 z\,dz}{\int_0^{\infty} |\mathrm{A_P}(\text{Ød},Z)|^2 z\,dz} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(1)$$

Here A_F (Ød, Z) is the complex amplitude and phase in the image plane at point Z units away from the diffraction head in the given plane of observation and subscript P indicates that the optical imaging system means that the optical system is apodized and defocused with given filters. We report that the optical system is apodized and defocused by the three filters in a suitable order with a phase of ($-\pi/4$) and($\pi/3$).

(The encircled energy parameter is a measure of the energy contained within a circular region of an optical image and is often used to assess the effectiveness of apodization in reducing the appearance of rings in the image. On the other hand, the displaced energy parameter compares the energy distribution in an actual optical system to that of an ideal system and can be used in photometric applications to determine the performance of the system.)

Encircled energy parameter is useful to assess apodization for suppressing the ring structure. Displaced energy is useful to compare the energy distribution in the actual optical system to its perfect counterpart. For some photometric situations, this energy corollary is useful.

$$A_P(\text{Ø}, Z) = \int_0^1 (f(x) J0\,(Zx) x dx \;\ldots\ldots\ldots\ldots(2)$$

The intensity of the diffraction light in the area of the image field is linked to the Co sinusoidal filter, Gaussian filter, Hanning filter, and phase change of ($-\pi/4$) and($\pi/3$), complex pupil function is given by:

$$A_P(\text{Ø}, Z) = 2\left[\int_0^a (f1(x) J0\,(Zx) x dx + \int_a^b (f2(x) J0\,(Zx) x dx + \int_b^1 (f3(x) J0\,(Zx) x dx\right]\ldots\ldots(3)$$

$$A_P(\text{Ø}, Z) = 2\left[\int_0^a 1 + \beta\cos(\pi r\text{^}2)/1 + \beta J0(Zx) x dx - -\pi/4 \int_a^b \exp.(-\beta r/2x\text{^}2) J0\,(Zx) x dx + \pi/3 \int_b^1 \cos(\pi\beta x) J0\,(Zx) x dx\right]\ldots\ldots(4)$$

Where f1(x) is Co-sinusoidal, f2(x) is Gaussian, and f3(x) is Hanning mask pupil functions for the amplitude apodization of the pupil transmission. And Z is the reduced dimensionless diffraction coordinate in the image plane.

And, Haning function -f1=cos(πβx) ……. (5)

Gaussian function -f2=exp.(-βr/2x^2) ……... (6)

C0-sinusoidal function -f3= 1+βcos(πr^2)/1+β. ……... (7)

A quality criterion called zonal energy increment (ZEI), which looks at the specifics of energy redistribution in a degraded PSF, has been established. The ZEI calculates the energy that is added to or withdrawn from the ideal PSF (Airy) in the region between radii δ_1 and δ_2. ZEI measures a scatter function, which accounts for both the origin and destination of the energy being scattered. When a zone's ZEI is negative, there is less energy present there than in the Airy situation.

$$\mathrm{ZEI}\ (\Delta\,\delta) = [\mathrm{EE_P}(\delta_2) - \mathrm{EE_P}(\delta_1)] - [\mathrm{EE_a}(\delta_2) - \mathrm{EE_a}(\delta_1)] \;\ldots\ldots\ldots\; (8)$$

From the above equations (8) $EE_a(\delta_2)$ represents the encircled energy for perfect lens(Airy case)in which $\beta=0$, and $EE_P(\delta)$ is the apodised optical system for which $\beta\neq0$. Here δ_2 is greater than δ_1 and $(\Delta\,\delta) = \delta_2 - \delta_1$,this is the zone between radii δ_2 and δ_1. In effect, ZEI measures a scatter function, which accounts for both the origin and destination of the energy being scattered. When a zone's ZEI is negative, there is less energy present there than in the Airy situation, in another case the ZEI is positive, there is greater energy present there than in the Airy situation.

III. RESULTS AND DISCUSSIONS

The improved resolution of the point spread function with amplitude filters and phase change has been evaluated in terms of energy corollaries for all variations. However, we reported the majority of the results for decreasing encircled energy. The results of studies on the impact of amplitude filters and phase change apodization on the PSF corollaries of optical imaging systems are presented in equations (1), and (8). With the help of the Mat lab program to study the effect of encircled energy with suitable order of apodization parameter β, defocus, and also different filters. By introducing the Gaussian quadrate numerical integration method to compute the envelope energy for different values of δ ranging from 0 to 10 in steps 0.2. with these values of δ was evaluated for apodization β with 0 to 1 in steps 0.2, and defect of defocus ranging from 0-2π with the difference of π/2. The encircled energy increases for the Airy case. By increasing apodization as well as defocus values, the encircled energy decreases, this can be observed in fig 1, fig 2, fig 3, and fig 4. From fig 1, it is observed that the encircled energy increases, which is the case for the Airy situation. Encircled energy gradually decreases with the increase of apodization and also for defocus, both cases are explained here. Fig 1, fig 2, fig 3, are shown how encircled energy decreases for different values of apodization -varies from 0-1and defocus value vanishes for fig 1, and defocus with the π value for fig 2, and for fig 3 defocus has the value of 2π. However, the amount of encircled energy is increasing with the degree of apodization for larger radii and also increases for larger values of defocus this is observed in table 1, table 2. here the optical side lobes have been completely suppressed by using appropriate filters, and apodization. Encircled energy has been evaluated for different values of δ ranging from 0 to 10 in steps 0.2. these have been studied for different values of apodization β varying from 0 to 1. Zonal energy increment was evaluated from the equation [6] and which has been shown in table 3. These have been studied for different degrees of apodization ranging from 0-1 with 0.2 difference for different defocus ranging from 0-2π. For ZEI negative value indicates that the energy is scattered out of the zone. Positive values indicate that the energy is scattered into the optical system.

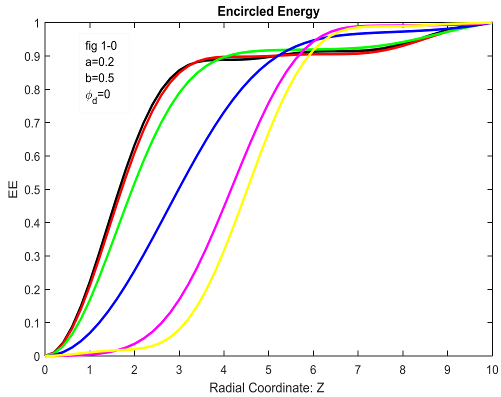


Fig 1 Variation of Encircled Energy EE(δ) for different values different
values of apodization parameter β, with Ød=π.

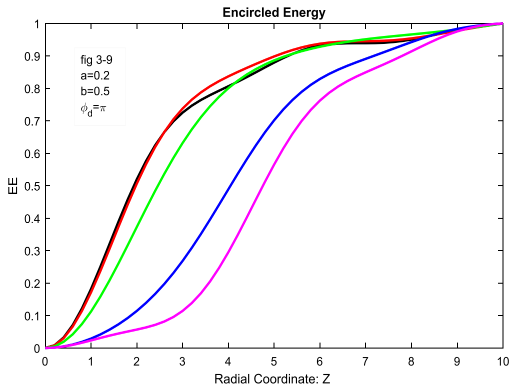
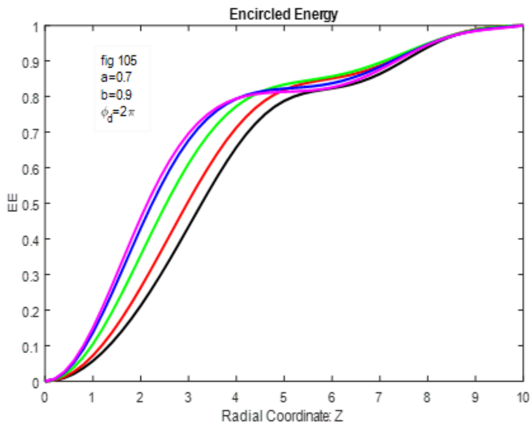


Fig 2 Variation of Encircled Energy EE(δ) for
of apodization parameter β, with. Ød=0.



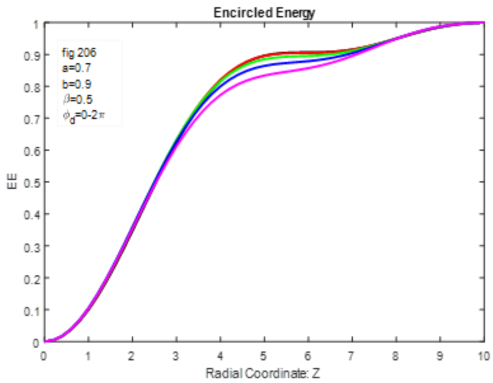


Fig 3 Variation of Encircled Energy EE(δ) for different values of apodization parameter β, with Ød=2π
Fig 4 Variation of EE(δ) for different values of defocus Ød
ranging from 0-2π, with β=0.5.

Table 1: Effect of defocus with 2π and for different values of apodization profile on encircled energy.

δ	β = 0	β=0.2	β=0.4	β=0.6	β=0.8	β=1
0	0	0	0	0	0	0
0.2	0.0023	0.0028	0.0038	0.0051	0.0061	0.0066
0.4	0.0094	0.0111	0.015	0.0202	0.0243	0.0261
0.6	0.0209	0.0248	0.0334	0.0448	0.0537	0.0576
0.8	0.0369	0.0438	0.0587	0.078	0.0932	0.0997
1	0.057	0.0677	0.0902	0.119	0.1411	0.1506
1.2	0.0811	0.0963	0.1275	0.1663	0.1956	0.208
1.4	0.109	0.1291	0.1695	0.2186	0.2547	0.2699
1.6	0.1403	0.1658	0.2157	0.2743	0.3164	0.3339
1.8	0.1749	0.206	0.2649	0.332	0.3787	0.3977
2	0.2125	0.249	0.3162	0.3901	0.4397	0.4596
2.2	0.2529	0.2945	0.3686	0.4473	0.498	0.5179
2.4	0.2958	0.3417	0.4212	0.5023	0.5524	0.5714
2.6	0.3407	0.3901	0.473	0.5542	0.6018	0.6193
2.8	0.3872	0.439	0.5231	0.602	0.6456	0.6611
3	0.4346	0.4876	0.5707	0.6453	0.6838	0.6967
3.2	0.4823	0.5351	0.6151	0.6835	0.7161	0.7264
3.4	0.5293	0.5808	0.6558	0.7166	0.7428	0.7504
3.6	0.5748	0.6239	0.6923	0.7447	0.7644	0.7693
3.8	0.6179	0.6637	0.7245	0.7678	0.7812	0.7838
4	0.6577	0.6997	0.7522	0.7864	0.794	0.7945
4.2	0.6934	0.7314	0.7755	0.801	0.8034	0.802
4.4	0.7244	0.7586	0.7947	0.8121	0.8099	0.8071
4.6	0.7505	0.7811	0.8101	0.8205	0.8143	0.8103
4.8	0.7716	0.7993	0.8221	0.8266	0.8172	0.8122
5	0.7879	0.8134	0.8315	0.8312	0.8192	0.8134
5.2	0.8	0.8239	0.8387	0.8349	0.8208	0.8144
5.4	0.8086	0.8317	0.8444	0.8382	0.8227	0.8156
5.6	0.8146	0.8376	0.8492	0.8417	0.8251	0.8177
5.8	0.8193	0.8422	0.8537	0.8457	0.8286	0.8209
6	0.8234	0.8466	0.8584	0.8506	0.8334	0.8257
6.2	0.8282	0.8513	0.8637	0.8566	0.8398	0.8322
6.4	0.8342	0.857	0.8699	0.8638	0.8478	0.8405
6.6	0.842	0.8641	0.8772	0.8721	0.8574	0.8507
6.8	0.8519	0.8727	0.8855	0.8816	0.8685	0.8625
7	0.8639	0.8828	0.8948	0.892	0.8807	0.8757
7.2	0.8776	0.8942	0.905	0.9031	0.8938	0.8898
7.4	0.8927	0.9066	0.9158	0.9145	0.9073	0.9044
7.6	0.9083	0.9196	0.9269	0.9261	0.9207	0.9188
7.8	0.9239	0.9325	0.938	0.9374	0.9337	0.9326
8	0.9388	0.945	0.9487	0.9482	0.9457	0.9453
8.2	0.9523	0.9565	0.9588	0.9582	0.9565	0.9565
8.4	0.9641	0.9668	0.968	0.9672	0.9658	0.9661
8.6	0.974	0.9756	0.9761	0.975	0.9737	0.9738

8.8	0.9817	0.9828	0.983	0.9816	0.98	0.9799
9	0.9876	0.9885	0.9886	0.987	0.985	0.9845
9.2	0.9918	0.9927	0.9929	0.9912	0.9888	0.988
9.4	0.9947	0.9957	0.996	0.9944	0.9919	0.9909
9.6	0.9968	0.9977	0.9981	0.9968	0.9946	0.9936
9.8	0.9985	0.9991	0.9994	0.9986	0.9972	0.9965
10	1	1	1	1	1	1

Table 2: Effect of apodization with the value 1 and defocus with a value of 0-2π profile on encircled energy.

δ	Ød = 0	Ød=π/2	Ød=π	Ød=3π/2	Ød=2π
0	0	0	0	0	0
0.2	2.00E-04	0.0012	0.0031	0.0054	0.0075
0.4	7.00E-04	0.0048	0.0122	0.0212	0.0296
0.6	0.0015	0.0107	0.0269	0.0467	0.0651
0.8	0.0027	0.0185	0.0464	0.0804	0.1121
1	0.0045	0.0281	0.0698	0.1206	0.168
1.2	0.007	0.0392	0.0961	0.1654	0.2301
1.4	0.0107	0.0518	0.1244	0.2129	0.2955
1.6	0.0161	0.0659	0.1539	0.2611	0.3614
1.8	0.0239	0.0819	0.1841	0.3086	0.4253
2	0.035	0.1	0.2145	0.3541	0.4851
2.2	0.0502	0.121	0.2453	0.3968	0.5393
2.4	0.0705	0.1455	0.2766	0.4363	0.5871
2.6	0.0967	0.174	0.3086	0.4727	0.628
2.8	0.1292	0.2072	0.342	0.5063	0.6623
3	0.1684	0.2452	0.3769	0.5374	0.6906
3.2	0.2141	0.2882	0.4138	0.5668	0.7137
3.4	0.2657	0.3357	0.4526	0.5949	0.7325
3.6	0.3222	0.387	0.4931	0.622	0.748
3.8	0.3821	0.4409	0.5346	0.6485	0.7611
4	0.4438	0.496	0.5765	0.6741	0.7724
4.2	0.5053	0.5508	0.6177	0.6987	0.7824
4.4	0.5645	0.6034	0.657	0.7219	0.7913
4.6	0.6197	0.6522	0.6934	0.7431	0.7992
4.8	0.6692	0.6958	0.7257	0.7618	0.806
5	0.7117	0.7331	0.7532	0.7776	0.8117
5.2	0.7465	0.7635	0.7755	0.7903	0.8161
5.4	0.7734	0.7867	0.7924	0.7999	0.8194
5.6	0.7927	0.8032	0.8043	0.8066	0.8218
5.8	0.8054	0.8137	0.8118	0.8109	0.8234
6	0.8126	0.8194	0.8159	0.8135	0.8246
6.2	0.8159	0.8217	0.8178	0.8152	0.8261
6.4	0.8169	0.8222	0.8187	0.8169	0.8282
6.6	0.8173	0.8224	0.8199	0.8194	0.8315

6.8	0.8185	0.8236	0.8224	0.8235	0.8363
7	0.8217	0.8269	0.8271	0.8296	0.8428
7.2	0.8277	0.833	0.8344	0.8381	0.8511
7.4	0.8368	0.8423	0.8446	0.8489	0.8611
7.6	0.8491	0.8546	0.8574	0.8618	0.8724
7.8	0.864	0.8695	0.8725	0.8763	0.8848
8	0.8811	0.8864	0.889	0.8918	0.8978
8.2	0.8993	0.9042	0.9063	0.9076	0.9108
8.4	0.9177	0.9222	0.9234	0.9231	0.9234
8.6	0.9354	0.9393	0.9397	0.9377	0.9354
8.8	0.9515	0.9549	0.9543	0.951	0.9465
9	0.9656	0.9683	0.9671	0.9627	0.9568
9.2	0.9773	0.9793	0.9776	0.9727	0.9662
9.4	0.9863	0.9877	0.9858	0.9812	0.9749
9.6	0.9929	0.9938	0.9921	0.9884	0.9833
9.8	0.9973	0.9977	0.9967	0.9945	0.9916
10	1	1	1	1	1

Table 3: Effect of defocus with the value 2π and variation of apodization ranging from 0-1.profile on zonal energy increment.

δ	β = 0	β=0.2	β=0.4	β=0.6	β=0.8	β=1
0	0	0	0	0	0	0
0.2	0	0.0004	-0.0014	-0.0027	-0.0038	-0.0043
0.4	0	-0.0013	-0.0042	-0.0081	-0.0111	-0.0125
0.6	0	-0.0022	-0.0069	-0.013	-0.0178	-0.0199
0.8	0	-0.003	-0.0093	-0.0173	-0.0235	-0.0261
1	0	-0.0038	-0.0114	-0.0208	-0.0278	-0.0307
1.2	0	-0.0045	-0.0131	-0.0232	-0.0304	-0.0334
1.4	0	-0.005	-0.0143	-0.0244	-0.0313	-0.034
1.6	0	-0.0054	-0.0148	-0.0244	-0.0303	-0.0326
1.8	0	-0.0055	-0.0146	-0.0231	-0.0277	-0.0293
2	0	-0.0054	-0.0137	-0.0205	-0.0234	-0.0243
2.2	0	-0.0051	-0.0121	-0.0168	-0.0179	-0.0179
2.4	0	-0.0044	-0.0097	-0.0122	-0.0115	-0.0107
2.6	0	-0.0035	-0.0069	-0.0069	-0.0045	-0.003
2.8	0	-0.0024	-0.0036	-0.0014	0.0026	0.0047
3	0	-0.0011	-0..0002	0.0042	0.0093	0.0118
3.2	0	0.0001	0.0032	0.0094	0.0153	0.018
3.4	0	0.0014	0.0064	0.0139	0.0203	0.023
3.6	0	0.0024	0.009	0.0175	0.024	0.0266
3.8	0	0.0032	0.0109	0.02	0.0262	0.0286
4	0	0.0038	0.0121	0.0211	0.027	0.0291
4.2	0	0.004	0.0124	0.0211	0.0264	0.0282

4.4	0	0.0039	0.0119	0.0199	0.0245	0.026
4.6	0	0.0035	0.0107	0.0178	0.0217	0.0229
4.8	0	0.0029	0.009	0.0149	0.0182	0.0192
5	0	0.0022	0.007	0.0117	0.0143	0.0151
5.2	0	0.0015	0.0049	0.0084	0.0104	0.0111
5.4	0	0.0008	0.0029	0.0053	0.0068	0.0073
5.6	0	0.0003	0.0013	0.0026	0.0036	0.004
5.8	0	0.0004	0.0001	0.00011	0.0011	0.0014
6	0	-0.0002	-0.0005	-0.0007	-0.0006	-0.0006
6.2	0	0	-0.0006	-0.0013	-0.0017	-0.0018
6.4	0	-0.0003	-0.0002	-0.0012	-0.002	-0.0023
6.6	0	0.0008	0.0006	-0.0005	-0.0018	-0.0023
6.8	0	0.0013	0.0016	0.0005	-0.0011	-0.0019
7	0	0.0019	0.0026	0.0016	-0.0003	-0.0012
7.2	0	0.0023	0.0036	0.0027	0.0007	-0.0004
7.4	0	0.0026	0.0042	0.0035	0.0015	0.0005
7.6	0	0.0027	0.0046	0.0041	0.0022	0.0012
7.8	0	0.0027	0.0045	0.0043	0.0027	0.0018
8	0	0.0024	0.0041	0.0041	0.0028	0.0022
8.2	0	0.002	0.0035	0.0036	0.0027	0.0023
8.4	0	0.0015	0.0026	0.0028	0.0024	0.0023
8.6	0	0.0001	0.0017	0.002	0.002	0.0021
8.8	0	0.0006	0.0009	0.0012	0.0014	0.0017
9	0	0.0002	0.0003	0.0005	0.0009	0.0012
9.2	0	0	-0.0001	0	0.0003	0.0007
9.4	0	0	-0.0002	-0.0003	-0.0001	0.0001
9.6	0	0.0001	0	-0.0003	-0.0006	-0.0006
9.8	0	0.0003	0.0004	-0.0002	-0.0001	-0.0013
10	0	0.0006	0.0009	0.0001	-0.0013	-0.0019

IV. CONCLUSIONS

The present study aimed to evaluate the corollaries of the PSF particularly encircled energy, Zonal energy increment. It has been observed that by using appropriate filters, at suitable zones. Here we studied the Hanning amplitude filters in the outer zone, Gaussian amplitude filters in the second zone, and Co-sinusoidal amplitude filters in the first zone. For smaller values of apodisation and defocus encircled energy is greater, which is the Airy case than that of the covered aperture. the quality of the criterion energy becomes zero as the degree of apodization and defocus increases, this explains the scattered energy increases for larger radii with suppressed sidelobes. However, this approach achieves symmetric apodization PSF with suppressed sidelobes of the optical system.

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