

Quantum Teleportation Of Four-Qubit Entangled State Via Ghz-Like State Channels

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Abstract:

Background: We Propose A Quantum Teleportation Protocol Scheme To Transmit A Quantum State, Which Is A State Of Four Entanglement Qubits Using Channels The Double Three Qubits GHZ-Like State. This Teleportation Scheme Is Supported By Bob's Two Additional Particles.

Materials And Methods: The Four-Qubit Quantum Teleportation Protocol Entangled Via GHZ-Like State Channels With Their Join States, Measurements Of Alice's Projection And Bob's Unitary Transformations In Two Stages, And Calculations Of The Probability And Fidelity.

Results: We Propose A Quantum Teleportation Protocol To Transmit The Quantum State Of Four Qubits Entangled States Via A Three-Qubit GHZ-Like State Channel. The Measurements Of Alice's Projection And Bob's Unitary Transformation Involved Two Additional Bob Particles. The Probability And Fidelity Of This Protocol Reach One.

Key Word: Fidelity, Projection Measurement, Quantum Teleportation.

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I Introduction

The development of research in the field of quantum protocols has been widely carried out. In the field of quantum cryptography, Ekert [1] uses the state of two EPR qubits (Einstein, Podolsky, Rosen) as a state tightness tester and in the Bennet communication protocol [2], this EPR is shared through one- and two-particle operators. In 1993 Bennet, et al [3] for the first time proposed a theoretical protocol of quantum teleportation of the state of one qubit over an EPR channel. Where quantum teleportation is the process of sending an arbitrary number of unrecognized qubits at a separate place between the sender (Alice) and the receiver (Bob) through the division of quantum entanglement states and classical states involving some non-local measurements. In general, non-local measurements in Alice use projective measurements, and in Bob are unitary operations. There are also protocols whose non-local measurements were implemented through the methods of Aharonov and Albert [4], non-linear interactions in experiments by Kim, et al [5] and in the work of Cardoso, et al [6] that used resonances between state source cavities and channel sources. The selection of quantum channels for the entanglement state of two arbitrary bits is obtained through Schmidt decomposition testing [23] and in multicubits, it is through the composition of the rank value of its reduced density matrix [24].

Proposed quantum teleportation protocols over GHZ-like three-qubit entanglement state channels have been widely proposed [7–12] [15] [21]. So is the use of GHZ-like state channels in the implementation of quantum communication protocols [14] [16-20] [22]. From some quantum teleportation protocols over a three-qubit channel of GHZ-like states that have been proposed [7] [10-12], it is observed that sending an arbitrary entangled n-qubit state requires an NFZ-like state channel. In the work of Tsai, et al [8] and Nandhi, et al [9] it was observed that sending a two-qubit state in a special form requires only one GHZ-like three-qubit state.

In this study, we propose a quantum teleportation protocol scheme to transmit a quantum state, which is a state of four entanglement qubits using channels twice the state of three GHZ-like qubits. This teleportation scheme is supported by Bob's two additional particles [13] and Alice's projective measurement process and Bob's unitary transformation are worked out in two stages. The protocol is expected to save quantum channels as much as one GHZ-like three-qubit state compared to Wen Yuan's work [11] and a larger number of qubit states compared to the work of Tsai, et al [8] and Nandhi, et al [9].

Furthermore, the structure of this article in part 2 is to explain the four-qubit quantum teleportation protocol entangled through GHZ-like state channels with descriptions of their joining states, measurements of Alice's projection and Bob's unitary transformations in two stages, calculations of the total chance of success and fidelity. The last section or part 3 is the conclusion of our work.

II Quantum teleportation of the entangled three-qubit state through a GHZ-like state channel

In this protocol scheme, the quantum information to be sent is four entangled qubits,

$$|\varphi\rangle_{ABCD} = (a|0000\rangle + b|0100\rangle + c|1001\rangle + d|1101\rangle)_{ABCD} \tag{2.1}$$

with $|a|^2 + |b|^2 + |c|^2 + |d|^2 = 1$. Next by implementing the CNOT operator, where particle A is the controller and particle D is the target to be controlled,

$$\begin{aligned} |\chi\rangle_{ABCD} &= CNOT|\varphi\rangle_{ABCD} \\ |\chi\rangle_{ABCD} &= (a|0000\rangle + b|0100\rangle + c|1000\rangle + d|1100\rangle)_{ABCD} \end{aligned} \tag{2.2}$$

To teleport information $|\chi\rangle_{ABCD}$ will use a double quantum channel of state three qubits GHZ-like and two additional qubits (*ancillary*) $|0\rangle_7|0\rangle_8$,

$$|\psi\rangle_{123} = \frac{1}{2}(|000\rangle + |011\rangle + |101\rangle + |110\rangle)_{123} \tag{2.3}$$

$$|\psi\rangle_{456} = \frac{1}{2}(|000\rangle + |011\rangle + |101\rangle + |110\rangle)_{456} \tag{2.4}$$

with the measurement bases used are orthogonal bases in the category of the GHZ-like three-qubit states

$$|\eta^\pm\rangle_{ijk} = \frac{1}{2}[(|000\rangle + |011\rangle) \pm (|101\rangle + |110\rangle)]_{ijk} \tag{2.5}$$

$$|\xi^\pm\rangle_{ijk} = \frac{1}{2}[(|100\rangle + |111\rangle) \pm (|001\rangle + |010\rangle)]_{ijk} \tag{2.6}$$

Next, the joint states are written between the quantum state of the four entangled qubits and the quantum channel of the double GHZ-like state

$$\begin{aligned} |\Gamma\rangle_{ABC1234567} &= |\chi\rangle_{ABCD} \otimes |\psi\rangle_{123} \otimes |\psi\rangle_{456} \otimes |0\rangle_7 \otimes |0\rangle_8 \\ &= \frac{1}{4}|\eta^+\rangle_{A12}|\eta^+\rangle_{B45}(a|000000\rangle + b|010000\rangle + c|100000\rangle + d|110000\rangle)_{3678CD} \\ &+ \frac{1}{4}|\eta^+\rangle_{A12}|\eta^-\rangle_{B45}(a|000000\rangle - b|010000\rangle + c|100000\rangle - d|110000\rangle)_{3678CD} \\ &+ \frac{1}{4}|\eta^-\rangle_{A12}|\eta^+\rangle_{B45}(a|000000\rangle + b|010000\rangle - c|100000\rangle - d|110000\rangle)_{3678CD} \\ &+ \frac{1}{4}|\eta^-\rangle_{A12}|\eta^-\rangle_{B45}(a|000000\rangle - b|010000\rangle - c|100000\rangle + d|110000\rangle)_{3678CD} \\ &+ \frac{1}{4}|\xi^+\rangle_{A12}|\eta^+\rangle_{B45}(a|100000\rangle + b|110000\rangle + c|000000\rangle + d|010000\rangle)_{3678CD} \\ &+ \frac{1}{4}|\xi^+\rangle_{A12}|\eta^-\rangle_{B45}(a|100000\rangle - b|110000\rangle + c|000000\rangle - d|010000\rangle)_{3678CD} \\ &+ \frac{1}{4}|\xi^-\rangle_{A12}|\eta^-\rangle_{B45}(-a|100000\rangle + b|110000\rangle + c|000000\rangle - d|010000\rangle)_{3678CD} \\ &+ \frac{1}{4}|\eta^+\rangle_{A12}|\xi^+\rangle_{B45}(a|010000\rangle + b|000000\rangle + c|110000\rangle + d|100000\rangle)_{3678CD} \\ &+ \frac{1}{4}|\eta^+\rangle_{A12}|\xi^-\rangle_{B45}(-a|010000\rangle + b|000000\rangle - c|110000\rangle + d|100000\rangle)_{3678CD} \\ &+ \frac{1}{4}|\eta^-\rangle_{A12}|\xi^+\rangle_{B45}(a|010000\rangle + b|000000\rangle - c|110000\rangle - d|100000\rangle)_{3678CD} \\ &+ \frac{1}{4}|\eta^-\rangle_{A12}|\xi^-\rangle_{B45}(-a|010000\rangle + b|000000\rangle + c|110000\rangle - d|100000\rangle)_{3678CD} \\ &+ \frac{1}{4}|\xi^+\rangle_{A12}|\xi^+\rangle_{B45}(a|110000\rangle + b|100000\rangle + c|010000\rangle + d|000000\rangle)_{3678CD} \\ &+ \frac{1}{4}|\xi^+\rangle_{A12}|\xi^-\rangle_{B45}(-a|110000\rangle + b|100000\rangle - c|010000\rangle + d|000000\rangle)_{3678CD} \\ &+ \frac{1}{4}|\xi^-\rangle_{A12}|\xi^+\rangle_{B45}(-a|110000\rangle - b|100000\rangle + c|010000\rangle + d|000000\rangle)_{3678CD} \\ &+ \frac{1}{4}|\xi^-\rangle_{A12}|\xi^-\rangle_{B45}(a|110000\rangle - b|100000\rangle - c|010000\rangle + d|000000\rangle)_{3678CD} \end{aligned} \tag{2.7}$$

III Results and Discussion

Quantum Measurements on Alice Particles and Information Recovery on Bob Particles

As an example of measurement steps, review Alice's projected measurements on the $|\eta^+\rangle_{A12}|\eta^+\rangle_{B45}|0\rangle_c|0\rangle_D$ bases in the first term on equation (2.7)

$$|\eta^+\rangle_{A12}|\eta^+\rangle_{B45}(a|00000\rangle + b|01000\rangle + c|10000\rangle + d|11000\rangle)_{3678CD} \tag{2.8}$$

and state of particle pair (3,6,7, 8) are obtained

$$(a|0000\rangle + b|0100\rangle + c|1000\rangle + d|1100\rangle)_{3678} \tag{2.9}$$

Next, Bob performed information recovery on particle 3678 using $I_3 \otimes I_6 \otimes I_7 \otimes I_8$ operator and simultaneously apply the CNOT operator, where particle 6 is the controller and particle 8 is the target, so obtained

$$(a|0000\rangle + b|0100\rangle + c|1001\rangle + d|1101\rangle)_{3678}. \tag{2.10}$$

Furthermore, evaluations for the other fifteen terms are analogies with equations (2.8) to (2.10). The complete results of Alice and Bob's measurement are listed in Table 2.1

Table. 2.1. Alice and Bob's measurements

Alice's measurements	Classic bits	Bob's states	Bob's unitary transformation
$ \eta^+\rangle_{A12} \eta^+\rangle_{B45} 0\rangle_c 0\rangle_D$	0000	$(a 0000\rangle + b 0100\rangle + c 1000\rangle + d 1100\rangle)_{3678}$	$I \otimes I \otimes I \otimes I$
$ \eta^+\rangle_{A12} \eta^-\rangle_{B45} 0\rangle_c 0\rangle_D$	0001	$(a 0000\rangle - b 0100\rangle + c 1000\rangle - d 1100\rangle)_{3678}$	$I \otimes Z \otimes I \otimes I$
$ \eta^-\rangle_{A12} \eta^+\rangle_{B45} 0\rangle_c 0\rangle_D$	0010	$(a 0000\rangle + b 0100\rangle - c 1000\rangle - d 1100\rangle)_{3678}$	$Z \otimes I \otimes I \otimes I$
$ \eta^-\rangle_{A12} \eta^-\rangle_{B45} 0\rangle_c 0\rangle_D$	0011	$(a 0000\rangle - b 0100\rangle - c 1000\rangle + d 1100\rangle)_{3678}$	$Z \otimes Z \otimes I \otimes I$
$ \xi^+\rangle_{A12} \eta^+\rangle_{B45} 0\rangle_c 0\rangle_D$	0100	$(a 1000\rangle + b 1100\rangle + c 0000\rangle + d 0100\rangle)_{3678}$	$X \otimes I \otimes I \otimes I$
$ \xi^+\rangle_{A12} \eta^-\rangle_{B45} 0\rangle_c 0\rangle_D$	0101	$(a 1000\rangle - b 1100\rangle + c 0000\rangle - d 0100\rangle)_{3678}$	$X \otimes Z \otimes I \otimes I$
$ \xi^-\rangle_{A12} \eta^+\rangle_{B45} 0\rangle_c 0\rangle_D$	0110	$(-a 1000\rangle - b 1100\rangle + c 0000\rangle + d 0100\rangle)_{3678}$	$XZ \otimes I \otimes I \otimes I$
$ \xi^-\rangle_{A12} \eta^-\rangle_{B45} 0\rangle_c 0\rangle_D$	0111	$(-a 1000\rangle + b 1100\rangle + c 0000\rangle - d 0100\rangle)_{3678}$	$XZ \otimes Z \otimes I \otimes I$
$ \eta^+\rangle_{A12} \xi^+\rangle_{B45} 0\rangle_c 0\rangle_D$	1000	$(a 0100\rangle + b 0000\rangle + c 1100\rangle + d 1000\rangle)_{3678}$	$I \otimes X \otimes I \otimes I$
$ \eta^+\rangle_{A12} \xi^-\rangle_{B45} 0\rangle_c 0\rangle_D$	1001	$(-a 0100\rangle + b 0000\rangle - c 1100\rangle + d 1000\rangle)_{3678}$	$I \otimes XZ \otimes I \otimes I$
$ \eta^-\rangle_{A12} \xi^+\rangle_{B45} 0\rangle_c 0\rangle_D$	1010	$(a 0100\rangle + b 0000\rangle - c 1100\rangle - d 1000\rangle)_{3678}$	$Z \otimes X \otimes I \otimes I$
$ \eta^-\rangle_{A12} \xi^-\rangle_{B45} 0\rangle_c 0\rangle_D$	1011	$(-a 0100\rangle + b 0000\rangle + c 1100\rangle - d 1000\rangle)_{3678}$	$Z \otimes XZ \otimes I \otimes I$
$ \xi^+\rangle_{A12} \xi^+\rangle_{B45} 0\rangle_c 0\rangle_D$	1100	$(a 1100\rangle + b 1000\rangle + c 0100\rangle + d 0000\rangle)_{3678}$	$X \otimes X \otimes I \otimes I$
$ \xi^+\rangle_{A12} \xi^-\rangle_{B45} 0\rangle_c 0\rangle_D$	1101	$(-a 1100\rangle + b 1000\rangle - c 0100\rangle + d 0000\rangle)_{3678}$	$X \otimes XZ \otimes I \otimes I$
$ \xi^-\rangle_{A12} \xi^+\rangle_{B45} 0\rangle_c 0\rangle_D$	1110	$(-a 1100\rangle - b 1000\rangle + c 0100\rangle + d 0000\rangle)_{3678}$	$XZ \otimes X \otimes I \otimes I$
$ \xi^-\rangle_{A12} \xi^-\rangle_{B45} 0\rangle_c 0\rangle_D$	1111	$(a 1100\rangle - b 1000\rangle - c 0100\rangle + d 0000\rangle)_{3678}$	$XZ \otimes XZ \otimes I \otimes I$

Calculation of probability of success and fidelity of quantum teleportation protocol

The calculation of the probability of successful transmission of information from Alice to Bob is calculated as follows:

$$P_i = |\langle \chi_{in} | \chi_{(out)_i} \rangle|^2, i = 1, \dots, 16 \tag{2.11}$$

with $|\chi_{in}\rangle$ is the information sent by Alice and $|\chi_{(out)_i}\rangle$ is the information recovered by Bob in the i-th term of the equation of the joint state. Review one of the calculation examples on the first term of the equation (2.10),

$$\begin{aligned} |\varphi_{(out)_1}\rangle &= \frac{1}{4}(a|0000\rangle + b|0100\rangle + c|1001\rangle + d|1101\rangle)_{3678}, \\ P_1 &= \left| \frac{1}{4} (a^* \langle 0000| + b^* \langle 0100| + c^* \langle 1001| + d^* \langle 1101|)_{3678} \right. \\ &\quad \left. \times \frac{1}{4} (a|0000\rangle + b|0100\rangle + c|1001\rangle + d|1101\rangle)_{3678} \right|^2, \end{aligned}$$

$$= 1/16 (|a|^2 + |b|^2 + |c|^2 + |d|^2)^2 = 1/16 |1|^2$$

$$P_1 = 1/16. \tag{2.12}$$

The calculation of the probability of success for the other fifteen terms of the equation (2.7) is analogous to the description of the equation (2.12). The results of the calculations for these sixteen terms give the total probability of successful transmission of information from Alice to Bob is $P_{Success} = (1/16) \times 16 = 1$. Calculation of channel fidelity in the successful transmission of information,

$$\mathcal{F}_i(\rho_{in}, \rho_{(out)_i}) = \left[\text{tr} \left(\sqrt{\sqrt{\rho_{in}} \rho_{(out)_i} \sqrt{\rho_{in}}} \right) \right]^2 \tag{2.13}$$

with, the density of information sent by Alice $\rho_{in} = |\chi_{in}\rangle\langle\chi_{in}|$ and the density of information recovered by Bob $\rho_{(out)_i} = |\chi_{(out)_i}\rangle\langle\chi_{(out)_i}|$ in the i-th term. Equation (2.13) can be simplified through the trace matrix properties i.e.,

$$\begin{aligned} \mathcal{F}_i(\rho_{in}, \rho_{(out)_i}) &= \left[\text{tr} \left(\sqrt{\sqrt{\rho_{in}} \rho_{(out)_i} \sqrt{\rho_{in}}} \right) \right]^2 \\ &= \left[\text{tr}(\sqrt{\rho_{in} \rho_{(out)_i}}) \right]^2 = \left[\text{tr} \left(\sqrt{(|\chi_{in}\rangle\langle\chi_{in}|)(|\chi_{(out)_i}\rangle\langle\chi_{(out)_i}|)} \right) \right]^2 \\ &= \left[\text{tr} \left(\sqrt{\langle\chi_{in}|(|\chi_{(out)_i}\rangle\langle\chi_{(out)_i}|)|\chi_{in}\rangle} \right) \right]^2 = \left[\text{tr} \left(\sqrt{|\langle\chi_{in}|\chi_{(out)_i}\rangle|^2} \right) \right]^2 \\ \mathcal{F}_i(\rho_{in}, \rho_{(out)_i}) &= |\langle\chi_{in}|\chi_{(out)_i}\rangle|^2 \end{aligned} \tag{2.14}$$

The fidelity in equation (2.14) is the same as the probability of success in equation (2.11), so the total fidelity for this protocol is $\mathcal{F}_{Total}(\rho_{in}, \rho_{(out)}) = (1/16) \times 16 = 1$

IV Conclusion

We have demonstrated that the double of three GHZ-like state can be used as a quantum channel to realize the sending four entangled qubits states. The measurement of Alice's projection and Bob's unitary transformation on this protocol was assisted by Bob's two additional particles. The main result of this protocol is that the probability and the fidelity of the channel reach one and satisfy the criteria of perfect quantum teleportation. We hope that these two protocols can be realized in experiments that use photons as their resources, as well as in the realization of experiments for EPR state resources.

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