

# A Geometrical Solution To Light Refraction.

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## Abstract:

We present a way to solve an arbitrary problem according to the refraction of light using a geometrical method to apply it in more complex situations.

**Background:** Physics can be described in many ways to understand nature, the core way of finding solutions is using geometrics to simulate the behavior of light through different materials. In the article “Simulation to build a solar collector upon Fresnel lens” we started to approach the light refraction on the atmosphere but at that moment we could not find an easy way to simulate this phenomenon without complex mathematics.

**Materials and Methods:** In this proposal we explain a way of finding the angle of refraction using only the index ratio or the velocities of light between materials using a geometrical method.

**Results:** We find this method can also be used to teach refraction of light for students in high school and can be another way of understanding specific situations as critical angle of refraction and reflection.

**Conclusion:** This method can be used to improve simulations to simplify iterative methods or approach to sophisticated problems.

**Key Word:** Refraction, geometrics, light

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## I. Introduction

Constructing the phenomenon of light refraction in a geometrical form can be used to solve this sort of problem in an easier way and can be a form of rectification of calculus.

This technique helps high school students understand more fluently the phenomenon of light refraction as well as obtain simple tools to solve situations or questions.

## II. Procedure

Let  $S$  be the Surface between two materials of known refraction index,  $r_i$  an incident ray with unknown angle of incidence,  $A$  is the point where light penetrate the second material, and  $N_m$  the normal of the surface as any refraction problem in Physics, as shown in Figure 1 (to help understand the method we shall use blue color to demonstrate all incidence elements and green color represents refracted elements):

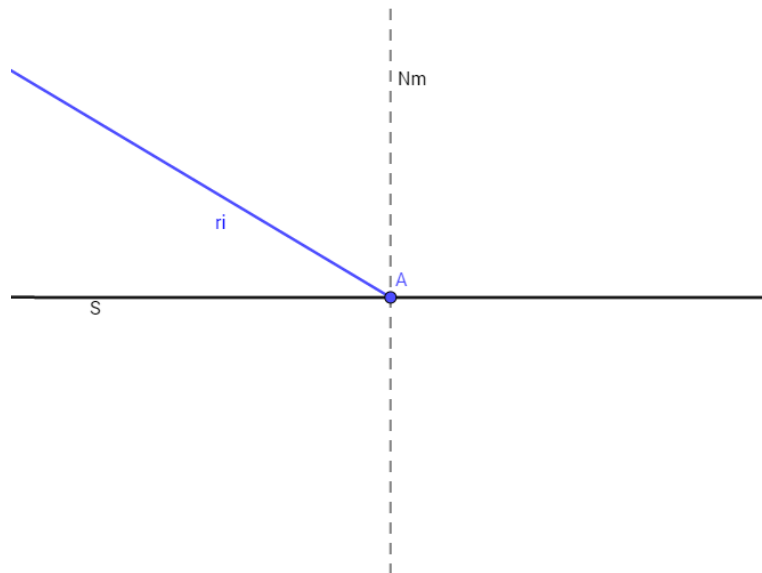
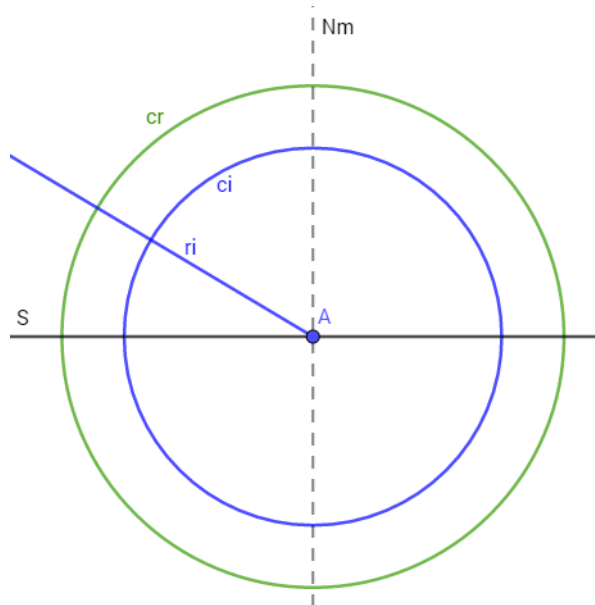


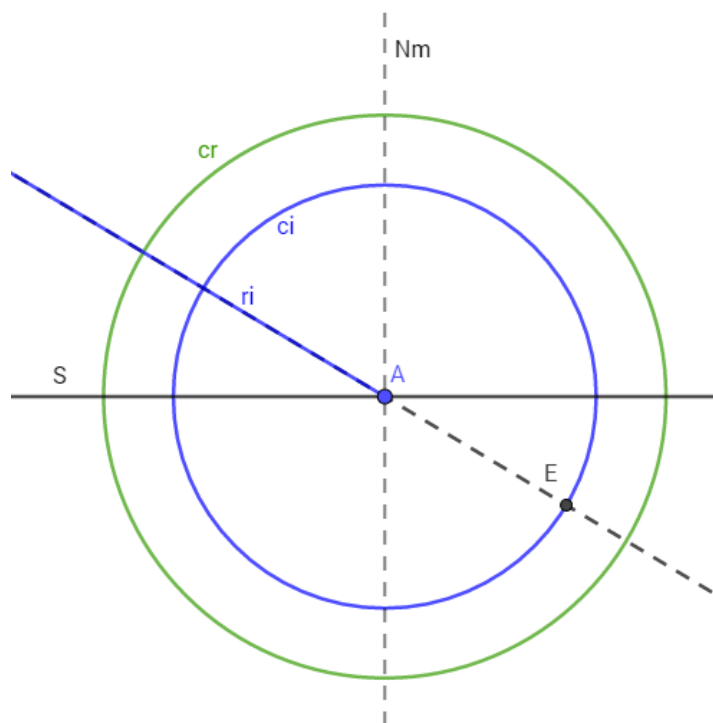
Figure 1: Refraction problem statement

Then we draw two circles, with center in A, the first one (ci) should have a radius proportional to the refraction index of the incidence material, the second one (cr) should have the radius proportional to the refraction index, in our example let us suppose the incidence material has 1 of refraction index, meanwhile the refraction index is 1.33. In our example the circle ci should have a radius of one unit as the second circle should be 1.33 times greater:



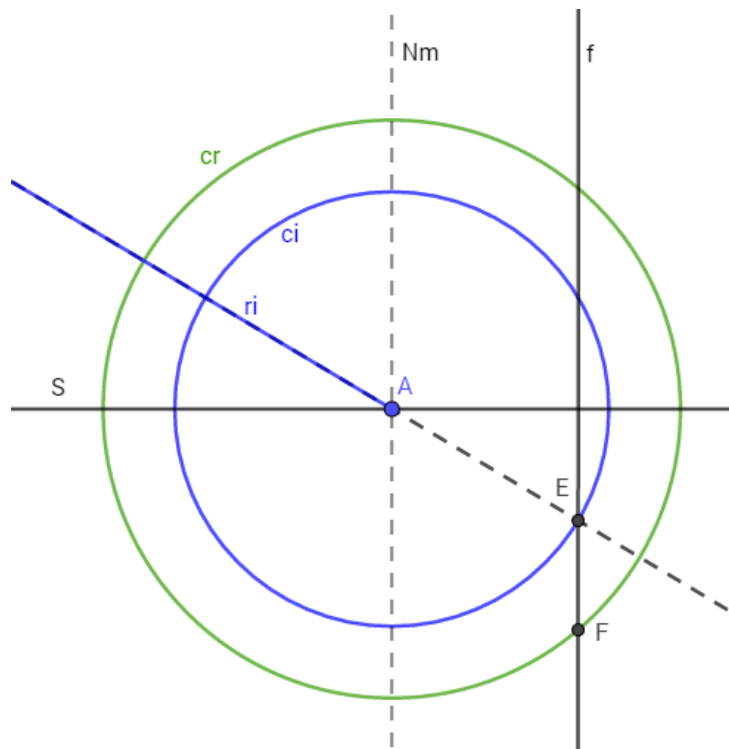
**Figure 2: Representation of refraction index of materials.**

We prolong the incidence ray (ri) to cross both circles on the other side of the surface. We need to mark the point (E) where this ray crosses the circle with incidence refraction index, we will call this point “helping point”:



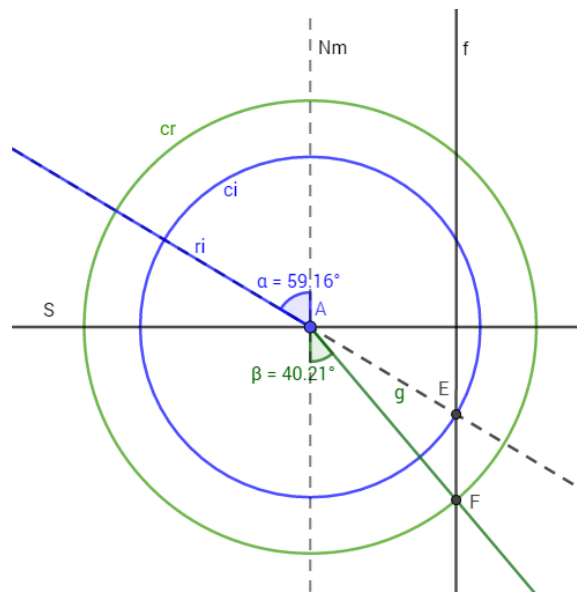
**Figure 3: Drawing the helping point E.**

Then we must draw a parallel to the normal (NM) that crosses the helping point, this line should cross the circle with the refraction index at the point F, if this line cannot cross the refraction circle, it means that light reflects, this point will be discussed in a future section:



**Figure 4: Drawing the refracted point F.**

At last, we must draw the refracted ray a semi line from A to F, this is our refracted line. We could measure the angles to corroborate with the Snell Law.



**Figure 5: Final Solution.**

Finally, we can erase the circles, the prolongation of ri, the helping points and the parallel to present the result.

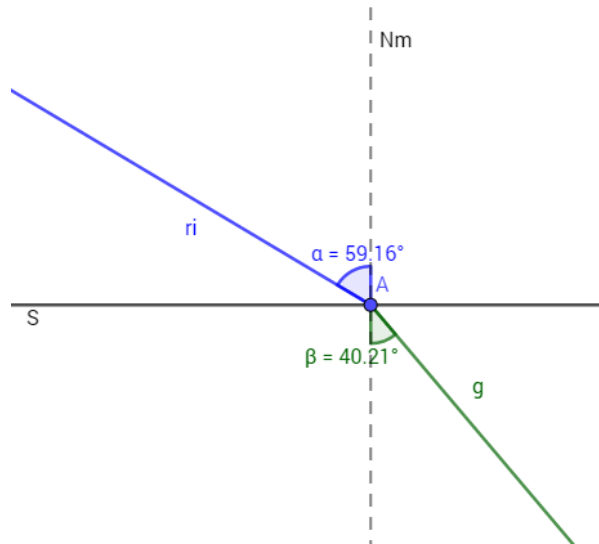


Figure 5: Formal Solution.

### III. Result

This method can be used to solve a series of problems referred to refraction of light such as critical angle of refraction, reflection and changing of refraction index.

#### Critical angle of refraction

This method also helps to find the critical angle of refraction in an easy way, first we must set the conditions: first the refraction index of the incidence material should be greater than the refraction index of the incidence material, in our example let S be the refraction surface, Nm the normal to this surface, ci the circle with radius of refraction index of the incident material meanwhile cr the radius of the refracted material as our method says, as example we will use as radius 1.33 for the incidence material and 1 for the refracted material:

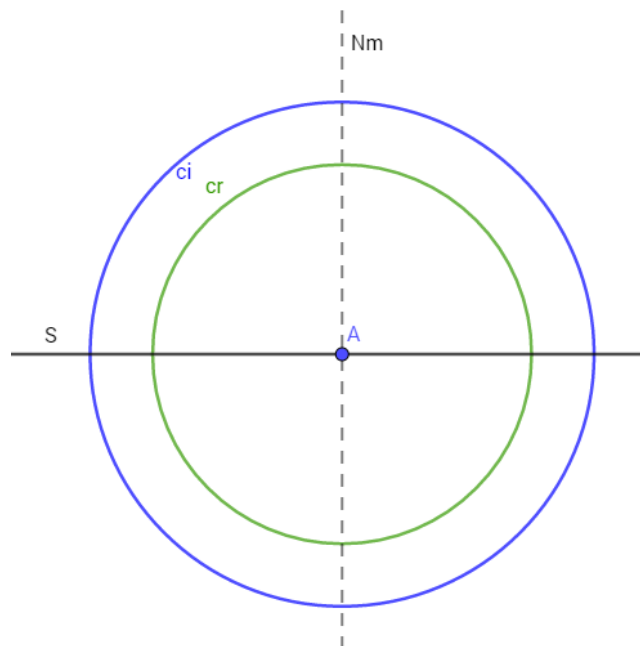
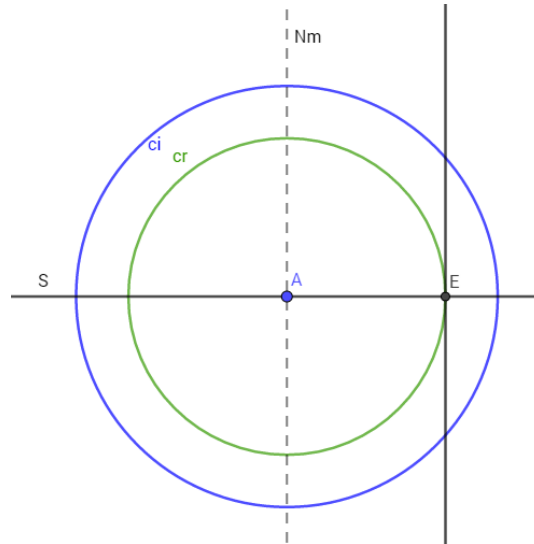


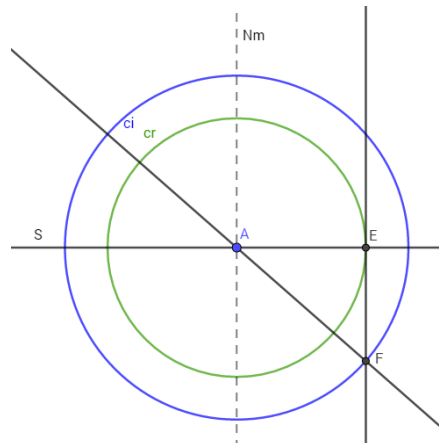
Figure 6: Problem statement of the critical angle of refraction.

Let find the intersection of the refracted material with the surface at the point E and let's draw a parallel to Nm at this point:



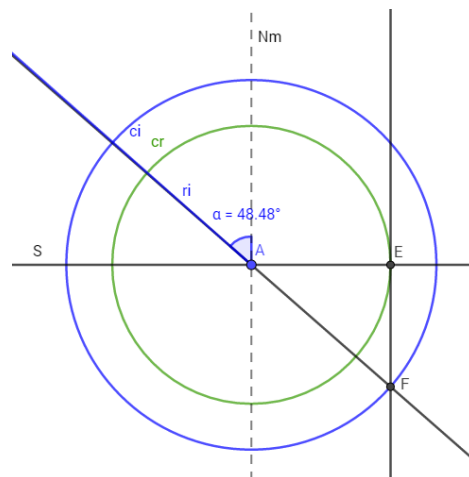
**Figure 6: Problem statement of the critical angle of refraction.**

We draw the intersection with the incidence circle at F and draw a line that crosses A and B:



**Figure 7: Drawing the solution of critical angle.**

This last line  $r_i$  represents the incidence angle that makes the critical refraction angle.



**Figure 8: Solution of critical angle.**

If we analyze this example, we find that any larger angle than the critical angle cannot build the system, the helping line will not cross the refraction circle therefore there is no refraction of light.

#### IV. Discussion

##### Snell law in this method.

This geometrical method was deduced using the Snell law, and the unit circle:

If we multiply the radius by any number, all sides of the triangle will grow at the same rate, we can use two different circles to have the same physical problem of light refraction. To get the source of the method we compare two triangles build upon different circles with same center, we need to have the same height and adequate the Snell law:

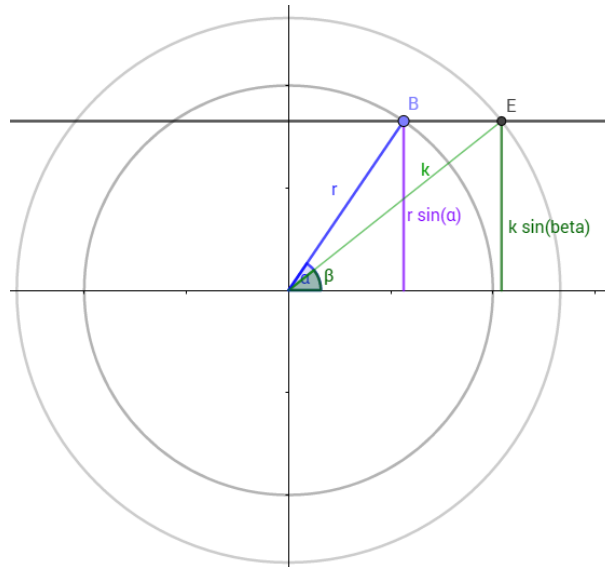


Figure 10: Compare two different triangles of same height.

With this graph we can see that:

$$r \sin(\alpha) = k \sin(\beta)$$

Since  $r$  and  $k$  can be the refraction index of the materials, we use this principle to build refraction of light in a geometrical way, using the symmetry of the circle we can draw rays at the points where we need them to find the real solution.

##### Using velocities instead of refraction index.

Another advantage of this method is that we could use velocities instead of refraction index to solve in the same way the problem, the only difference is changing the order or names of the circles and drawing the circles in a reasonable scale, we can demonstrate this idea using the Snell law and the definition of refraction index:

$$\eta_1 \sin(\theta_1) = \eta_2 \sin(\theta_2)$$

Hence

$$\eta_i = \frac{c}{v_i}$$

Where  $\eta_i$  is the refraction index of one medium,  $c$  velocity of light in vacuum and  $v_i$  velocity of light in that medium. We can rewrite the Snell law as:

$$\frac{c}{v_1} \sin(\theta_1) = \frac{c}{v_2} \sin(\theta_2)$$

Multiplying by the velocities and we get:

$$v_2 \sin(\theta_1) = v_1 \sin(\theta_2)$$

Which means we can proceed as the method just changing the circle when we draw the helping point.

##### Reflection using this method.

This method can be applied to determine the reflection since we consider both refraction index as the same, but we must draw in the same space of the light, using again the symmetry of the circle. Let us begin with the statement of an incident ray ( $r_i$ ) falling upon a surface  $S$ :

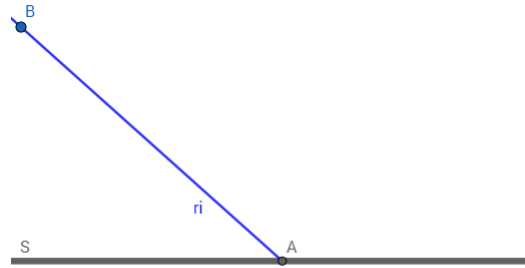


Figure 11: Reflection problem statement

We need the refraction index of both materials, since it's a reflection we can use the same circle with radius of this parameter, we draw the point where crosses the incidence ray D:

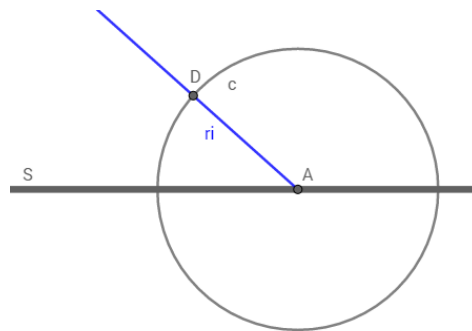


Figure 11: Reflection problem statement

To make this process faster we can draw a parallel to the surface crossing D and finding the point E where crosses again in the circle, since we have the same refraction index, and finally draw our reflection ray rx:

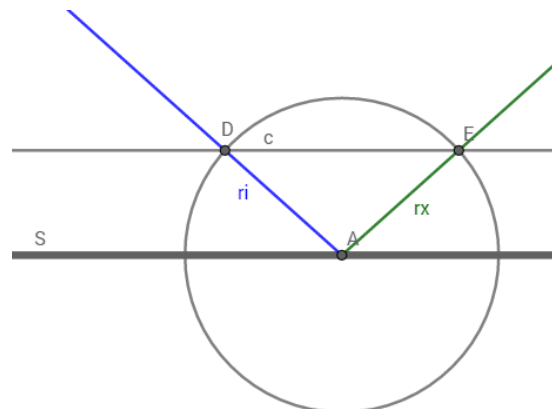


Figure 12: Solution of the reflection problem.

## V. Conclusion

This approach can be used to understand in a different way the refraction of light and can generate alternatives ways of solving a problem, this method can be applied to programs to solve a sophisticated problem involving complex surfaces of refraction in a geometrical way.

This way of solving refractive properties also indicates many properties of refraction index as critical angle of refraction and reflection.

This method also explains how light travels through different materials and is a ludic way of explaining Snell law to high school students.

An advantage of the method can be applied to 3D problems, using spheres instead of circles. This is useful in some simulations when we know the incidence ray and the normal to the surface with the refraction index.

By now, this method can solve any situation according to a flat surface with use of low computer resources, later on this method shall be applied in circular surfaces in order to find all properties and also on spherical surfaces to solve the problem of finding the Sun over the celestial vault on one point on Earth.

### **References**

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