

# A Vector Mode Approach For Absorbing Optical Fibers

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**Abstract:**

Eigen value equations for surface plasmon modes excited at the dielectric-core/metallic-cladding interface of an absorbing optical fiber are derived using zeroth-order vector mode analysis. Propagation characteristics for TM modes are evaluated using perturbation analysis. The analysis will find applications in improving the performance of TE/TM polarizers and sensors.

**Key Word:** Optical Fiber; Surface plasmon modes; TE/TM Polarizer; Vector modes

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## I. Introduction

We consider a two layer optical fiber consisting of a center core region (dielectric) surrounded by a thin metallic ring. This structure has been obtained by removing the cladding over a few centimeters length of an all-dielectric optical fiber and coating the exposed core with a surface plasmon supporting metal such as gold, by the process of thermal evaporation. This configuration allows the surface plasmons (SP) which are TM (transverse magnetic) in nature to travel along the metal/dielectric boundary. These modes are very sensitive to any small changes taking place near the boundary and hence are largely used in optical sensors.

The permittivity of metal being a complex quantity, yields complex value for the propagation constant. The real part results in evolution of the SP mode and the imaginary part gives rise to attenuation as the SP mode propagates. Hence these SP modes have a finite propagation length. These sensors have obtained considerable attention from the researchers and enormous literature is available giving detailed design parameters for optical sensors using different configurations. Researchers have usually employed transfer matrix method and ray theory to understand and decipher these sensor structures and reported design parameters for various applications. For the case of optical fiber sensors, the researchers have again used ray analysis considering the meridional rays and ignoring the skew rays. These approximations indeed compromise the efficacy of these sensors. In order to optimize the performance of these SP modes to be used as sensors under considerably stringent requirements, we have reported an exact analysis of optical fibers in cylindrical coordinate systems employing Bessel’s functions. Applying Snitzers’ criteria [1] and combining the linearly polarized (LP) modes of an optical fiber, we have obtained zeroth order vector modes (hybrid modes) [2,3].

## II. Theory

The refractive index profile of a dielectric-core and metal-cladding cylindrical optical waveguide (Fig. 1) can be written as:

$$n^2(r) = \begin{cases} n_1^2 = \epsilon_1; & r < a \\ \bar{n}_m^2 = \bar{\epsilon}_m; & r > a \end{cases} \quad (1)$$

Where  $n_1$  and  $\bar{n}_m$  represent the refractive indices,  $\epsilon_1$  and  $\bar{\epsilon}_m$  are the dielectric permittivities of the dielectric-core and metal-cladding respectively, and  $a$  is the core radius.

Under weakly guiding approximation, various modes of the cylindrical waveguide are assumed to be LP modes [4-6]. The expression for  $\psi(r, \varphi)$  for  $LP_{lm}$  modes is given by [4] (symbols have their usual meaning):

$$\psi(r, \varphi) = \psi_{lm}(r, \varphi) = \begin{cases} \frac{A}{J_l(\bar{U})} J_l\left(\frac{\bar{U}r}{a}\right) \begin{bmatrix} \cos \varphi l \\ \sin \varphi l \end{bmatrix}; & r < a \\ \frac{A}{K_l(\bar{W})} K_l\left(\frac{\bar{W}r}{a}\right) \begin{bmatrix} \cos \varphi l \\ \sin \varphi l \end{bmatrix}; & r > a \end{cases} \quad (2)$$

where

$$\begin{cases} \bar{U}^2 = U^2 - iU'^2 = a^2(\kappa_0^2 n_1^2 - \beta^2) \\ \bar{W}^2 = W^2 - iW'^2 = a^2(\beta^2 - \kappa_0^2 \bar{n}_m^2) \\ \bar{V}^2 = \bar{U}^2 + \bar{W}^2 = a^2(\kappa_0^2 n_1^2 - \kappa_0^2 \bar{n}_m^2) \end{cases} \quad (3)$$

Deriving the Eigen value equation for SP TM modes, where only  $E_r$ ,  $E_z$ , and  $H_\phi$  are the finite components, of the six field components, viz.  $E_r, E_\phi, E_z, H_r, H_\phi$ , and  $H_z$  in the cylindrical coordinates [1-3]:

$$\bar{U} \frac{J_0(\bar{U})}{J_1(\bar{U})} - 1 = -\frac{n_1^4}{\bar{n}_m^4} \left( \bar{W} \frac{K_0(\bar{W})}{K_1(\bar{W})} + 1 \right) \quad (4)$$

The complex arguments of Bessel functions and modified Bessel functions are expanded by utilizing the series expansion of Bessel and modified Bessel functions [7, 8] and the following substitutions are made to simplify Eqn. (4). We obtain two coupled real transcendental equations [9] as below:

$$J_0(\bar{U}) = J_{00}(U, U') - i J_{01}(U, U') \quad (5)$$

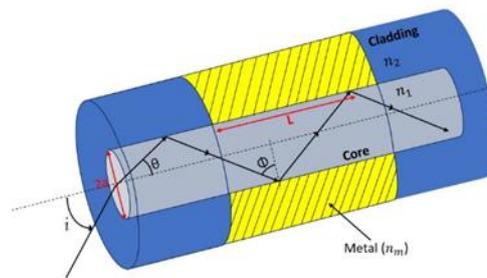
$$K_0(\bar{W}) = K_{00}(W, W') - i K_{01}(W, W') \quad (6)$$

$$\frac{J_1(\bar{U})}{\bar{U}} = J_{10}(U, U') - i J_{11}(U, U') \quad (7)$$

$$\frac{K_1(\bar{W})}{\bar{W}} = K_{10}(W, W') - i K_{11}(W, W') \quad (8)$$

$$n_2^R J_{00} K_{10} - n_2^I J_{00} K_{11} - (n_2^R + n_1^R) J_{10} K_{10} + (n_2^I) J_{10} K_{11} - n_2^R J_{01} K_{11} - n_2^I J_{01} K_{10} + (n_2^R + n_1^R) J_{11} K_{11} + (n_2^I) J_{11} K_{10} - n_1^R J_{10} K_{00} + n_1^I J_{11} K_{01} = 0 \quad (9)$$

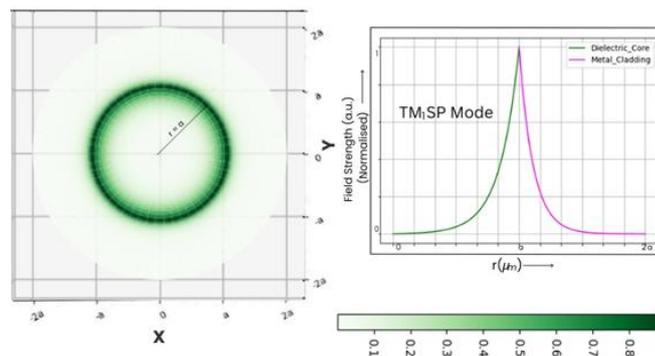
$$n_2^R J_{00} K_{11} + n_2^I J_{00} K_{10} - (n_2^R + n_1^R) J_{10} K_{11} - (n_2^I) J_{10} K_{10} + n_2^R J_{01} K_{10} - n_2^I J_{01} K_{11} - (n_2^R + n_1^R) J_{11} K_{10} + (n_2^I) J_{11} K_{11} - n_1^R J_{11} K_{00} - n_1^I J_{10} K_{01} = 0 \quad (10)$$



**Fig. 1.** Proposed design of a cylindrical optical absorbing waveguide in cylindrical coordinates  $(r, \phi, z)$ . Here  $r = a$  is the radius of the core (silica glass) of refractive index  $n_1$ . The semi-infinite cladding region is of refractive index  $n_2$ .  $\phi$  is the azimuthal angle  $(0 \leq \phi \leq 2\pi)$ , and  $z$  is the direction of the direction of propagation of electromagnetic (EM) wave through the waveguide. Various angles  $i, \theta$ , and  $\phi$  are with respect to the axis of the cylindrical waveguide along the direction of propagation and have their usual meanings. A portion ‘L’ length of the dielectric-cladding region is removed and a semi-infinite layer of a plasmon supporting metal (refractive index  $\bar{n}_m$ ) is deposited.

### III. Result

The two real simultaneous eqns. (9-10) are solved using numerical techniques to obtain the complex Eigen values of the surface plasmon modes propagating at the core-cladding boundary. Fig. 2 displays the polar and linear plots of the excited  $TM_1$  mode at red He-Ne wavelength for silica core of radius  $1 \mu m$ . The cladding is of gold metal. The refractive indices of the silica core and gold cladding are taken from references [10,11].



**Fig. 2.**  $TM_1$  SP mode of the dielectric core and metal cladding optical fiber segment of Fig. 1. The left side figure is drawn in polar coordinates and the right side figure is drawn in Cartesian coordinates. The fiber parameters are specified in the text.

#### **IV. Conclusion And Discussion**

A new technique is proposed to study the surface plasmon modes in cylindrical waveguides by utilizing the wave theory. This is expected to revolutionize the state of the art technique for the optical devices based on the phenomenon of surface plasmons.

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