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A Novel Wave Theory For Dielectric-Metal-Dielectric Optical Fibers

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Abstract:
A three-layer optical fiber with dielectric-core, metal-cladding, and surrounded by a dielectric analyte layer is analyzed without considering linear polarization of the fields. Eigen value equations are derived for TM modes in cylindrical coordinates using Bessel's functions. Transmittance spectrum is generated for sensor applications and compared with literature.
Key Word: *Optical Fiber; Plasmonic sensors; TE/TM Polarizer; Hybrid modes*

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I. Introduction

A three layer optical fiber is considered, consisting of a center dielectric core of radius a and refractive index (RI) n_1, surrounded by a thin metallic ring of thickness b and complex RI=$\bar{n}_m$, followed by a dielectric cladding region of RI=n_a. Such a dielectric-metal-dielectric (DMD) structure has been in use as a sensor where the outer dielectric layer is the analyte containing unknown species to be detected. This configuration supports surface plasmons (SP) at the metal-dielectric boundaries which are TM (transverse magnetic) in nature. TE modes are not supported by metal layer and hence these DMD waveguides are used in TE/TM polarizers [1]. The SP modes are very sensitive to any small changes taking place near the boundary and hence find applications in optical sensors [2]. In this research work, we have investigated a DMD structure using wave theory in contrast to the earlier published research available in the literature [3] which is based on the ray theory.

II. Theory And Mathematical Modeling

Due to the complex permittivity of the metal, the entire analysis revolves around complex Bessel functions [4], yielding a complex Eigen value equation as derived below:

The refractive index profile of an all-dielectric cylindrical optical waveguide (Fig. 1) can be written as:

$$n^2(r) = \begin{cases} n_1^2 = \epsilon_1; & r \le a \\ \bar{n}_m^2 = \bar{\epsilon}_m; & \mathrm{a} \le r \le \mathrm{b} \\ n_a^2 = \epsilon_a; & r \ge b \end{cases} \tag{1}$$

where ϵ_1 , $\bar{\epsilon}_m$, and ϵ_a are the dielectric permittivities of the dielectric-core, thin metal-ring, and the cladding respectively. The expression for $\psi(r,\varphi)$ for LP_{lm} modes is given by transfer matrix method (symbols have their usual meaning):

$$\Psi_{1m} = \begin{cases} A_1\ J_1(\kappa_1 r) + B_1\ Y_1(\kappa_1 r); & r \le a \\ A_2\ \mathrm{I}_1(\gamma_2 r) + B_2\ K_1(\gamma_2 r); & a \le r \le b \\ A_3\ I_1(\gamma_3 r) + B_3\ K_1(\gamma_3 r); & r \ge b \end{cases} \tag{2}$$

$$\kappa_1 = k_0\sqrt{{n_1}^2 - {\bar{n}_e}^2}\ , \gamma_2 = k_0\sqrt{{\bar{n}_e}^2 - {\bar{n}_m}^2}\ , \text{and } \gamma_3 = k_0\sqrt{{\bar{n}_e}^2 - {n_a}^2} \tag{3}$$

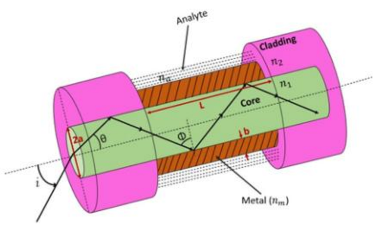

Fig.1: DMD optical fiber with coordinates (r, φ, z). A portion 'L' length of the dielectric-cladding region is removed and a thin metal layer of thickness b (complex refractive index $\bar{n}_m$) is deposited. Over the metal

region, an analyte of refractive index n_a containing the unknown species is placed, giving a DMD structure over the length L.

The finite components for the TM modes are only E_r, E_z, and H_φ out of the six field components , viz. $E_r, E_\varphi, E_z, H_r, H_\varphi, and\ H_z$ in the cylindrical coordinates. Applying boundary conditions of the continuity of E_z and H_φ for the TM modes, we get the following Eigen value equation [5–7]:

$$T_{11} = \left[\left\{\left(\gamma_3 \frac{\bar{n}_m{}^2}{n_a{}^2} K'_1(\gamma_3 b)\, I_1(\gamma_2 b) - \frac{n_a{}^2\gamma_2}{\bar{n}_m{}^2} K_1(\gamma_3 b)\, I'_1(\gamma_2 b)\right) \times \left(\gamma_2 \frac{n_1{}^2}{\bar{n}_m{}^2} K'_1(\gamma_2 a)\, J_1(\boldsymbol{\kappa}_1 a) - \frac{\bar{n}_m{}^2\boldsymbol{\kappa}_1}{n_1{}^2} K_1(\gamma_2 a)\, J'_1(\boldsymbol{\kappa}_1 a)\right)\right\} + \left\{\left(\gamma_3 \frac{\bar{n}_m{}^2}{n_a{}^2} K'_1(\gamma_3 b)\, K_1(\gamma_2 b) - \frac{n_a{}^2\gamma_2}{\bar{n}_m{}^2} K_1(\gamma_3 b)\, K'_1(\gamma_2 b)\right) \times \left(-\gamma_2 \frac{n_1{}^2}{\bar{n}_m{}^2} I'_1(\gamma_2 a)\, J_1(\boldsymbol{\kappa}_1 a) + \frac{\bar{n}_m{}^2\boldsymbol{\kappa}_1}{n_1{}^2} I_1(\gamma_2 a)\, J'_1(\boldsymbol{\kappa}_1 a)\right)\right\}\right] = 0 \quad (4)$$

Eqn. (4) can be differentiated to obtain an analytical expression for $\frac{\partial T_{11}}{\partial \bar{n}_e}$ and solved for $\bar{n}_e$ through numerical techniques employing Newton Raphson method and first order perturbation theory [8].

III. Result And Discussion

The results obtained from the solutions of the Eigen value equation are shown in Fig. 2 below. A sharp peak in the electric field profile at the metal-dielectric interface can be seen (Fig. 2 (a) and (b)). The wavelength shift in the peak of the imaginary part of the effective index (Fig. 2 (c)) with change in the analyte index can be visibly seen. This wavelength shift is calibrated to measure the refractive index of the bio-chemical species in the sensor applications. Fig. 2(d) displays the comparison of the theoretical results from the ray analysis (purple line), wave analysis proposed here (orange line) with the experimental values (green line), when such a DMD cylindrical waveguide is used as an optical sensor revealing a better approximation to the actual results from the proposed analysis.

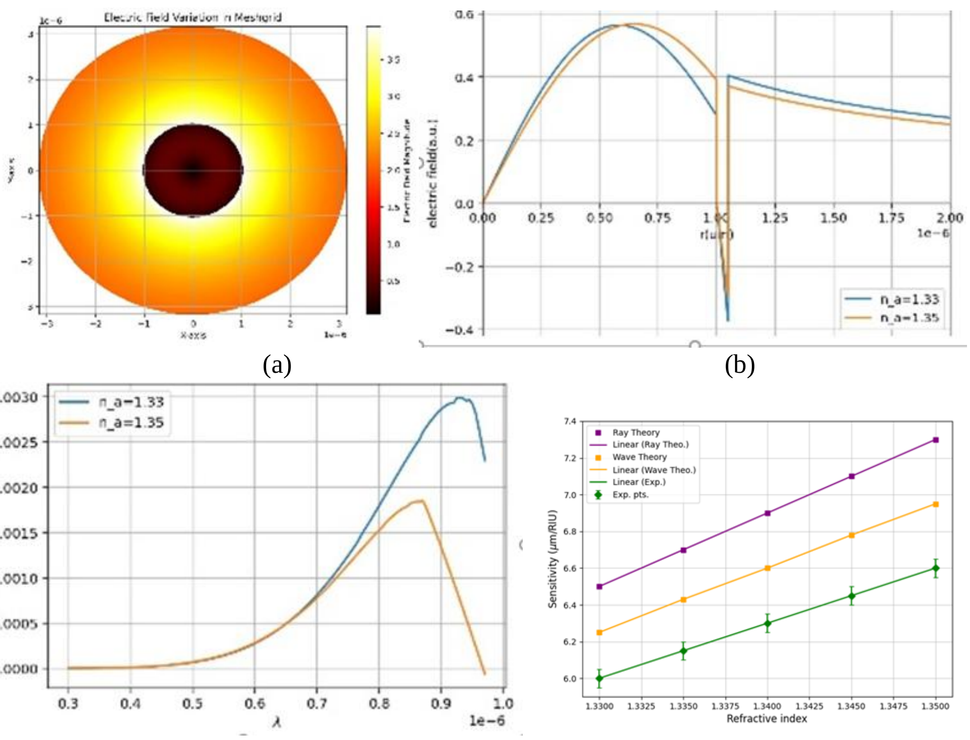

Fig. 2. (a) and (b) show the electric field profile in polar and linear coordinates at analyte index of 1.33 and 1.35, (c) shows the imaginary part of the effective index of $TM_1\ SP$ mode at analyte index of 1.33 and 1.35, and (d) shows comparison of the sensitivity of the optical fiber sensor for the experimental results with the theoretical results obtained using ray theory and wave theory.

References

[1] Chen CH, Wang L (1998) Design Of Finite-Length Metal-Clad Optical Waveguide Polarizer. IEEE J Quantum Electron, Https://Doi.Org/10.1109/3.687849.

[2] Homola J (2008) Surface Plasmon Resonance Sensors For Detection Of Chemical And Biological Species. Chem Rev 108:462–493.

[3] Gupta BD (2012) Surface Plasmon Resonance Based Fiber Optic Sensors.

[4] Watson GN (1922) A Treatise On The Theory Of Bessel Functions. The University Press.

[5] Snitzer E (1961) Cylindrical Dielectric Waveguide Modes. JOSA 51:491–498.

[6] Sharma A (1988) Constructing Linear Combinations Of LP Modes To Obtain Zeroth Order Vector Modes Of Optical Fibers. Appl Opt 27:2647–2649.

[7] Thyagarajan K, Ghatak AK, Sharma A (1989) Vector Modes Of An Optical Fiber In The Weakly Guiding Approximation. J Light Technol 7:51–54.

[8] Arun K, SI H, AK G (1981) Propagation Characteristics Of Weakly Guiding Lossy Fibres. An Exact And Perturbation Analysis, Optica Acta: International Journal Of Optics, 28:4, 559-566.