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Estimation Of Diffusion Cofficients And Convective Velocities Of ^{137}cs (Radiocaesium) Within Undisturbed Soils In Southwestern Nigeria By Solving Fokker-Planck Equation Using Adomian Decomposition Method (Adm).

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Abstract: *Adomian Decomposition Method (ADM) was employed to solve the second order differential equation of diffusion-convection model for the mobility of radiocaesium in undisturbed soils of southwestern Nigeria. Applying this method the convection velocity ranged from 0.09 to 0.16 cmy^{-1} and the diffusion coefficients varied from 0.07 to 1 cm^{2}y^{-1}. These values fall in the range that are widely reported in the literature.*
Key Words: *Adomian, Caesium, Diffusion –coefficients, Convective-velocities, Soils.*

I. Introduction

The world is still battling with significant amounts of anthropogenic radionuclides (such as ^{137}Cs, ^{90}Sr to mention few) as a result of nuclear weapon tests carried out by France, Nuclear power accidents at Chernobyl 1986 and Fukushima in Japan 2011. Caesium and its various isotopes are considered as one of the major harmful anthropogenic radionuclides in the man's environment due to external exposure and through ingestion of foods (most especially plants that absorbs radiocaesium through their roots).

Researchers all over the world had studied radiocaesium migration through soil matrix and proposed different models (Rühm, et al, 1996). Notable of these models are compartment models, diffusion-convection models (Fokker-Planck) and approximation models by a function of a polynomial or exponential (Isaksson and Erlandsson, 1998, Isaksson, et al, 2001, Likar, et al, 2001, Toso and Velasco, 2001, Kanapickas, et al, 2005).

The most widely accepted model is the diffusion-convective model (Kircher and Baungartner, 1992, Konshin, 1992a, Konshin, 1992b, Mamikhin, 1995, Ivanov, et al, 1997); because it parameters are usually fitted to the available experimental data. Therefore it can be used to estimate the radionuclide migration profile at any given time (Szerbin, et al, 1999). The first and the second models had been widely reported in literature to depend on soil type, radionuclide chemistry, soil physical properties (such as structure and composition) and precipitation (Barišić, et al, 1999, Lujanienė, et al, 2002, Lee and Lee, 2002, Kanapickas, et al, 2005). Based on the aforementioned factors, there is no general model that could be applied for that analysis of the experimental radiocaesium profiles in soil without significant assumptions or corrections (Kanapickas, et al, 2005). Therefore the main goal of the present research is to estimate diffusion coefficients and convective velocities of radiocaesium by solving Fokker-Planck differential equation governing diffusion and convection activities using Adomian Decomposition Method (ADM). Experimental data was taken from the previous studied carried out by Ajayi (Ajayi,et al, 2007).

II. Modelling Procedure

Fokker-Planck equation governing diffusion and convection activities is written as:

$$\frac{\partial C}{\partial t} = D\frac{\partial^2 C}{\partial x^2} - \nu\frac{\partial C}{\partial x} - \lambda C \qquad 1$$

$$C(x,t) = C(x,1) = 1300Bqm^{-2}\ (Map, 2013)\text{and}\lambda = \frac{0.693}{t} \qquad 2$$

Where C is ^{137}Cs concentration in soil, λ is its decay constant, D is the diffusion coefficient of 137Cs, ν is convective velocity, X is the soil depth and t is time from the deposition. The term $-\lambda C$ represents the radioactive decay (Ajayi, et al, 2007).

Using ADM, we rewrite equation 1 in operator form as:

$$\frac{\partial C}{\partial t} = DL_{2x}C(x,t) - \upsilon L_x(x,t) - \lambda C \qquad 3$$

Where $\frac{\partial c}{\partial t} = L_t\ and\ \frac{\partial c}{\partial x} = L_x\ and\ \frac{\partial^2 c}{\partial x^2} = L_{2x}$

It follows that

$$L_tC(x,t) = DL_{2x}C(x,t) - \nu L_xC(x,t) - \frac{0.693}{t}C(x,t) \qquad 4$$

By applying the inverse operator to equation 4, we obtained

$$L^{-1}L_tC(x,t) = L^{-1}\left[DL_{2x}C(x,t) - \nu L_xC(x,t) - \frac{0.693}{t}C(x,t)\right] \quad 5$$

$$\Rightarrow C(x,t) = DL^{-1}L_{2x}C(x,t) - \nu L^{-1}L_xC(x,t) - \frac{0.693}{t}L^{-1}{}_tC(x,t) \quad 6$$

Imposing the initial condition $C(x,t) = C(x,1) = (1300,t)$

$$C(x,t) = C(x,1) + DL^{-1}(L_{2x}C(x,t) - \nu L^{-1}(L_xC(x,t) - 0.693L^{-1}{}_t\left(C\frac{(x,t)}{t}\right) \quad 7$$

$$= 1300Bqm^{-2} + DL^{-1}(L_{2x}C(x,t) - \nu L^{-1}(L_xC(x,t) - 0.693L^{-1}{}_t\left(\frac{C}{t}\right) \quad 8$$

From equation 8, L^{-1} is a twofold integral in L_{2x} and one fold integral in L_x and $L^{-1}{}_t\left(\frac{C}{t}\right)$; so that

$$C(x,t) = 1300Bqm^{-2} + D\iint_0^t L_{2x}C(x,t)dtdt - \nu\int_0^t L_x\,C(x,t)dt - 0.693\int_0^t C\frac{(x,t)}{t}dt \quad 9$$

From equation 9, we obtain the recurrence relations:

$$C_0(x,t) = 1300 \quad 10$$

And

$$C_{n+1}(x,t) = D\iint_0^t L_{2x}C_n(x,t)dtdt - \nu\int_0^t L_xC_n(x,t)dt - 0.693\int_0^t C_n\left(\frac{x,t}{t}\right)dt \quad 11$$

At $n = 0$, equation 11 becomes

$$C_0(x,t) = 1300Bqm^{-2} \quad *^{+}$$

Since $*^{+}$ has no variables in *x*,*t* on the RHS, the emerging solution will also not has *x* in it but will have *t* because of the integration with respect to *t* and

$$C_1(x,t) = D\iint_0^t \frac{\partial^2 C_0(x,t)}{\partial x^2}dtdt - \nu\int_0^t \frac{\partial C_0(x,t)dt}{\partial x} - 0.693\int_0^t \frac{C_0(x,t)}{t}dt \quad 12$$

$$C_1(x_0,t_0) = \frac{DC_0t^2}{2!} - \nu C_0t - 0.693\int_0^t \frac{1300}{t}dt = \frac{DC_0t^2}{2!} - \nu C_0t - 0.693C_0lnt \quad 13$$

When $n = 1$

$$C_2(x_1,t_1) = D\iint_0^t \frac{\partial^2}{\partial x^2}\left(\frac{DC_0t^2}{2!} - \nu C_0t - 0.693C_0lnt\right)dtdt - \nu\int_0^t \frac{\partial}{\partial x}\left(\frac{DC_0t^2}{2!} - \nu C_0t - 0.693C_0lnt\right)dt - 0.693\int_0^t \frac{1}{t}\left(\frac{DC_0t^2}{2!} - \nu C_0t - 0.693C_0lnt\right)dt \quad 14$$

$$C_2(x_1,t_1) = \frac{D^2C_0t^4}{4!} - \frac{D\nu C_0t^3}{3!} - 0.693DC_0lnt - \frac{D\nu C_0t^3}{3!} + \frac{\nu^2C_0t^2}{2!} + 0.693\nu\frac{C_0}{t} - 0.693\left[\frac{DC_0t^3}{3!} - \nu C_0t - 0.693C_0\int_0^t \frac{lnt}{t}dt\right] \quad 15$$

$$C_2(x_1,t_1) = \frac{D^2C_0t^4}{4!} - \frac{1}{3!}(2\nu - 0.693)DC_0t^3 + \frac{\nu^2C_0t^2}{2!} + 0.693\nu C_0t + \frac{0.693\nu C_0}{t} + \frac{(0.693)^2C_0}{2t^2} - 0.693DC_0lnt \quad 16$$

$$C_3(x_2,t_2) = D\iint_0^t \frac{\partial^2}{\partial x^2}\left(\frac{D^2C_0t^4}{4!} - \frac{1(2\nu-0.693)DC_0t^3}{3!} + \frac{\nu^2C_0t^2}{2!} + 0.693\nu C_0t + \frac{0.693\nu C_0}{t} + \frac{(0.693)^2C_0}{2t^2} - 0.693DC_0lnt\right)dtdt$$
$$- \nu\int_0^t \frac{\partial}{\partial x}\left(\frac{D^2C_0t^4}{4!} - \frac{1}{3!}(2\nu-0.693)DC_0t^3 + \frac{\nu^2C_0t^2}{2!} + 0.693\nu C_0t + \frac{0.693\nu C_0}{t} + \frac{(0.693)^2C_0}{2t^2} - 0.693DC_0lnt\right)dt$$
$$- 0.693\int_0^t \frac{1}{t}\left(\frac{D^2C_0t^4}{4!} - \frac{1}{3!}(2\nu-0.693)DC_0t^3 + \frac{\nu^2C_0t^2}{2!} + 0.693\nu C_0t + \frac{0.693\nu C_0}{t} + \frac{(0.693)^2C_0}{2t^2} - 0.693DC_0lnt\right)dt \quad 17$$

$$= \frac{D^3C_0t^6}{6!} - \frac{t^5}{5!}[(2\nu-0.693)D^2C_0 + D^2\nu C_0] + \frac{t^4}{4!}\left[\nu^2DC_0 + (2\nu-0.693)D\nu C_0 - \frac{0.693D^2C_0}{4}\right] + \frac{t^3}{3!}\left[0.693\nu DC_0 + \frac{0.693}{3}(2\nu-0.693)DC_0 - \nu^3C_0\right] - \frac{t^2}{2!}\left[0.693\nu^2C_0 + \frac{0.693\nu^2C_0}{2}\right] - (0.693)^2\nu C_0t + \frac{1}{t}(0.693^2\nu C_0 + 0.693^2C_0D - 0.693D\nu C_0) + \frac{0.693^2\nu C_0}{2t} + \frac{1}{t^2}\left(0.693\nu DC_0 + \frac{0.693^3C_0}{2}\right) - \left(\frac{0.693^2}{2}DC_0 - 0.693D^2C_0 - 0.693\nu^2C_0\right)lnt \quad 18$$

$$= \frac{D^3C_0t^6}{6!} - \frac{t^5}{5!}[2\nu - 0.693 + \nu]D^2C_0 + \frac{t^4}{4!}\left[\nu^2 + (2\nu-0.693)\nu - \frac{0.693D}{4}\right]DC_0 + \frac{t^3}{3!}\left[0.693\nu D + \frac{0.693}{3}(2\nu-0.693)D - \nu^3\right]C_0 - \frac{t^2}{2!}\left[0.693 + \frac{0.693}{2}\right]\nu^2C_0 - (0.693)^2\nu C_0t + \frac{1}{t}(0.693^2\nu + 0.693^2D - 0.693D\nu)C_0 + \frac{(0.693)^2\nu C_0}{2t} + \frac{1}{t^2}\left(0.693\nu D + \frac{0.693^3}{2}\right)C_0 - \left(\frac{0.693^2}{2}D - 0.693D^2 - 0.693\nu^2\right)C_0lnt \quad 19$$

$$C_4(x_3,t_3) = D\iint_0^t \frac{\partial^2}{\partial x^2}\left(\frac{D^3C_0t^6}{6!} - \frac{t^5}{5!}[2\nu-0.693+\nu]D^2C_0 + \frac{t^4DC_0}{4!}(\nu^2 + (2\nu-0.693)\nu) - \frac{0.693D}{4}\right)dtdt$$
$$+ D\iint_0^t \frac{\partial^2}{\partial x^2}\left(\frac{t^3}{3!}\left[0.693\nu D + \frac{0.693}{3}(2\nu-0.693)D - \nu^3\right]C_0\right)dtdt$$
$$- D\iint_0^t \frac{\partial^2}{\partial x^2}\frac{t^2}{2!}\left[\left(0.693 + \frac{0.693}{2}\right)\right]\nu^2C_0dtdt - D\iint_0^t \frac{\partial^2}{\partial x^2}((0.693)^2\nu C_0t)dtdt$$
$$+ \iint_0^t \frac{\partial^2}{\partial x^2}\left[\frac{1}{t}(0.693^2\nu + 0.693^2D - 0.693D\nu)C_0dtdt\right] + \iint_0^t \frac{\partial^2}{\partial x^2}\frac{(0.693)^2}{2t}\nu C_0dtdt$$
$$+ D\iint_0^t \frac{\partial^2}{\partial x^2}\left(\frac{1}{t^2}\left(0.693\nu D + \frac{(0.693)^3}{2}\right)C_0\right)dtdt$$
$$- D\iint_0^t \frac{\partial^2}{\partial x^2}\left(\frac{0.693^2}{2}D - 0.693D^2 - 0.693\nu^2\right)C_0lntdtdt$$
$$- \nu\left[\int_0^t \frac{\partial}{\partial x}\right]\left[\frac{D^3C_0t^6}{6!} - \frac{t^5}{3!}[3\nu - 0.693]D^2C_0 + \frac{t^4}{4!}\left[3\nu - 0.693\nu - \frac{0.693D}{4}\right]DC_0\right]dtdt$$
$$+ \cdots \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad 20$$

Neglecting the higher order terms, using the average concentration (one concentration) as seen in Table 1 (Ajayi,et al, 2007) subsequently C_n can be generalized as follows:

$$C_n = \frac{D^n C_0 t^{2n}}{2n!} - \frac{(nv - 0.693)D^{n-1} C_0 t^{2n-1}}{(2n-1)!} + \cdots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots .21$$

III. Result And Discussion

The concentration from each (Ajayi, et al, 2007) was fitted into equation 21 above by using fotran 95 to software to obtain diffusion coefficient D and the Convective velocity v. The results obtained are shown in Table 1.

Table 1. 137Cs Concentrations, Diffusion Coefficients and Convective velocities (all values are from Ajayi et al, 2007 except +)

Location	Concentration (Bqm^{-2})	D (cm^{2}year^{-1})	V (cmyear^{-1})	
Akure 1	27.30	0.72	0.07	0.09$^+$
Akure 2	63.45	1.02	0.12	0.11$^+$
Ado-Ekiti 1	53.35	1.00	0.13	0.14$^+$
Ado-Ekiti 2	30.60	1.00	0.16	0.16$^+$
Ikogosi 1	90.30	1.00	0.10	0.12$^+$
Ikogosi 2	82.80	1.00	0.16	0.16$^+$
Igbeti	29.40	1.00	0.10	0.11$^+$
Ogbomoso	15.60	1.00	0.01	0.01$^+$
Eruwa	24.90	1.00	0.02	0.03$^+$

Comparing the present study with the actual (Ajayi, et al, 2007), there are small variations. These are shown in figure 1. Although the shapes of the graphs follow the same trend, there are little variations in the points which may be due to assumptions and corrections made during modeling. Figure 1 confirmed that Diffusion-Convection model equation can be solved using Adomian Decomposition Method. The convective velocity obtained by ADM of solution on the equation of Diffusion-Convection model ranged from 0.09 to 0.16 cmy^{-1}. This in agreement with 0.07 to 0.16 cm y^{-1} obtained in the literature using Szerbin solution for the equation (Ajayi, et al, 2007).

IV. Conclusion

Adomian Decomposition Model (ADM) allows us to obtain Diffusion coefficients and Convective velocities of radiocaesium in soil profiles by solving the equation of Diffusion-Convection model. The values obtained are in good agreement with measurement results.

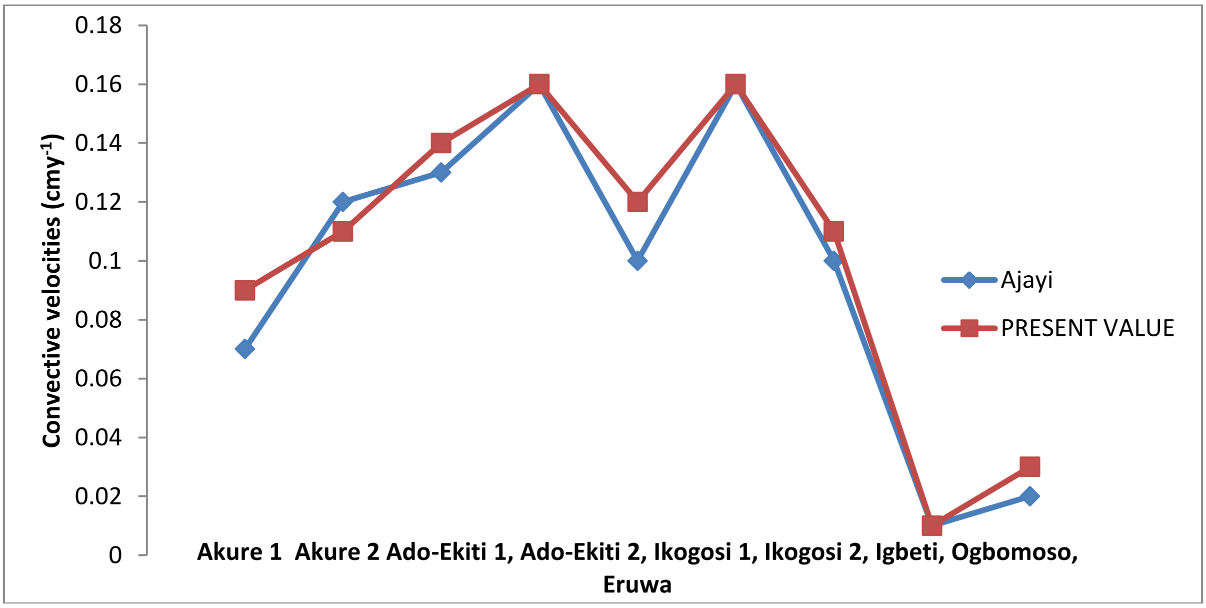

Fig 1.Observed convective velocities in various cities.

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