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The General Formula To Find The Straight Line Distance Of Solar Planets From Sun With The Help Of Number Theory And Classical Physics

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***Abstract**: Here a particular method is made to generate A Single Formula to find the the straight line distance of first eight planets from sun*
***Keywords**: Polynomials, Mean distance, Eccentricities, Kepler's Law, Orbital Velocity, Escape Velocity, Bode's Law*

I. Introduction

In our Solar system there are eight planets in orbit around sun. They can be classified into two different families based on the distance from sun. The outer family, major planets like Jupiter (whose radius is one tenth of that of sun, is the most massive among other planets). The outer family have widely spaced orbits; Consists mainly gaseous material. The four planets in our inner planet category contain the first four planets including our mother earth. All these planets have radius less than one tenth of the Jupiter.

In 1772 Johann Bode noticed that the planetary orbit up to Saturn were close to fairly simple mathematical progression. Which became known as Bode's Law. The law is in the form $r_n = 0.3 + 0.4\ X\ 2^n$. Where the leading term is the radius of Mercury's orbit in Astronomical Unit. And for the remaining planets n=0,1,2,3... For Venus, Earth …etc. The value we get when n=3 is assign to Asteroids.

Here I am trying to generate an equation to find the straight line distance of first eight planets from sun, via combining Classical Physics with Number Theory. The weapon we are taking from number theory is Mainly Polynomial Difference Theorem. And In Physics we are using Orbital Mechanics. Combining both these theories we can generate a single formula as said above.

Here I am going to introduce a new method, Using this method we can find the n^{th} term and sum of n terms of any different kinds of polynomial sequences. It is worth noting that using this method we can invent The General Formula to find the mean distance and eccentricity of planets as polynomial functions.

Here we are ignoring attraction of each planets one by another. And we are also ignoring the rotation effect of planets around sun.

II. The general method to find the nth term and sum of n terms of any polynomial sequences using Polynomial Difference Theorem

Polynomial Difference Theorem

Suppose the nth term of a sequence is a polynomial in n of degree m i.e.$p(n) = a_1n^m + a_2n^{m-1} + a_3n^{m-2} + a_4n^{m-3} + \ldots\ldots + a_{m+1}$

Then its mth difference will be equal and (m + 1)th difference will be zero.

2.1 Case1:- (When m=1)

Consider a polynomial sequence of power 1, i.e. p(n)=an+b

First term= p(1)=a+b

Second term=P(2)=2a+b

Third term=p(3)=3a+b

	First Term (1)	Second Term P(2)	Third Term P(3)
	a+b	2a+b	3a+b
First Difference	a	a	
Second Difference	0		

2.2 Case2:- (When m=2)

Consider a polynomial sequence of power 2, i.e. $p(n) = an^2 + bn + c$

First term= p(1)=a+b+c

Second term=P(2)=4a+2b+c

Third term=p(3)=9a+3b+c
Fourth term=p(4)=16a+4b+c

	First Term (1)	Second Term P(2)	Third Term P(3)	Fourth Term P(4)
	a+b+c	4a+2b+c	9a+3b+c	16a + 4b + c
First Difference	3a+b	5a+b	7a+b	
Second Difference	2a	2a		
Third Difference	0			

2.3 Case3:- (When m=3)
Consider a polynomial sequence of power 3, i.e. $p(n) = an^3 + bn^2 + cn + d$
First term= p(1)=a+b+c+d
Second term=P(2)=8a+4b+2c+d
Third term=p(3)=27a+9b+3c+d
Fourth term=p(4)=64a+16b+4c+d
Fifth term=p(5)=125a+25b+5c+d

	First Term P(1)	Second Term P(2)	Third Term P(3)	Fourth Term P(4)	Fifth Term P(6)
	a+b+c+d	8a+4b+2c+d	27a+9b+3c+d	64a+16b+4c+d	125a+25b+5c+d
D1	7a+3b+c	19a+5b+c	37a+7b+c	61a+9b+c	
D2	12a+2b	18a+2b	24a+2b		
D3	6a	6a			
D4	0				

From the previous experience, the basic knowledge of the polynomial difference theorem can be put in another way.

III. Inverse Polynomial Differnce theorem !

If the m^{th} difference of a polynomial sequence is equal then its n^{th} term will be a polynomial in n of degree m.

First Difference is the difference between two consecutive terms. Second difference is the difference between two neighboring first differences. So on

3.1 Problem 1: Find the nth term of the triangular number sequence1,3,6,10,15,21,28,36,45,55...
Solution
First term=t1 = 1, t2 = 3, t3 = 6 , t4 = 10 , t5 = 15 Using polynomial difference theorem, we can find the nth term,

	FirstTermt1P(1)	SecondTermt2P(2)	ThirdTermt3P(3)	FourthTermt4P(4)	FifthTermt5
	1	3	6	10	15
D1	2	3	4	5	
D2	1	1	1		
D3	0	0			

Here Second difference is Equal, which means we can represent its nth term as a polynomial of degree 2, i.e. the general representation $tn = p(n) = an^2 + bn + c$

$t_1 = p(1) = a + b + c = 1$
$t_2 = p(2) = 4a + 2b + c = 3$
$t_3 = p(3) = 9a + 3b + c = 6$
Solving this we get value of a=12; b= 12; c=0;
Nth term=p(n) = $\frac{n(n+1)}{2}$

3.2 Problem 2: the nth term of the number sequence $\sum k^4$

solution

Taking tn = $\sum k^4$

	t1	t2	t3	t4	t5	t6	t7
	1	17	98	354	979	2275	4676
D1	16	81	256	625	1296	2401	
D2	65	175	369	671	1105		
D3	110	194	302	434			
D4	84	108	132				
D5	24	24					

Let p(n) be the nth term , Since 5th difference is equal, the degree of nth term is 5

i.e.tn = p(n) = $an^5 + bn^4 + cn^3 + dn^2 + en + f$

t_1=p(1) = a+b+c+d+e+f =1
t_2=p(2) = 32a+16b+8c+4d+2e+f =17
t_3=p(3) = 243a+81b+27c+9d+3e+f =98
t_4=p(4) = 1024a+256b+64c+16d+4e+f=354
t_5=p(5) = 3125a+625b+125c+25d+5e+f=979
t_6=p(6)=7776a+1296b+216c+36d+6e+f=2275
p(2)-p(1)=3a+b=2—-(1)
p(3)-p(2)=5a+b=3—-(2)
(2)-(1)=2a=1
a=12

Solving this we get $p(n) = \frac{n(n+1)(2n+1)(3n^2+3n-1)}{30}$

$$\sum_{k=1}^{n} k^4 = \frac{n(n+1)(2n+1)(3n^2+3n-1)}{30}$$

Using this we can find

- Equation of mean distance From Sun(In 10^7 m)

$$b(n) = \frac{-144464081}{1000000}n^7 + \frac{894094629}{200000}n^6 - \frac{56448957031}{1000000}n^5 + \frac{11973677}{32}n^4 - \frac{5563041}{4}n^3 + 2864028\,n^2 - \frac{5955539}{2}n + \frac{4752953}{4}$$

- Equation of Orbital Period (In Earth days)

$$T(n) = \frac{-6194601}{500000}n^7 + \frac{376894501}{1000000}n^6 - \frac{467468457}{100000}n^5 + \frac{3810907959}{125000}n^4 - \frac{27956908203}{250000}n^3 + \frac{14593949}{64}n^2 - \frac{15086455}{64}n + \frac{5979771}{64}$$

- Equation of orbital Eccentricity

$$e(n) = \frac{151}{393750}n^7 - \frac{87493}{7200000}n^6 + \frac{1127483}{7200000}n^5 - \frac{1515287}{1440000}n^4 + \frac{28233521}{7200000}n^3 - \frac{3565267}{450000}n^2 + \frac{10873869}{1400000}n - \frac{1061}{400}$$

- Equation of Mean Orbital Velocity of planets in (km/s)

$$\frac{5969}{252000}n^7 - \frac{53069}{72000}n^6 + \frac{133961}{14400}n^5 - \frac{879793}{14400}n^4 + \frac{16029061}{72000}n^3 - \frac{15941023}{36000}n^2 + \frac{450101}{1050}n - \frac{10809}{100}$$

- Equation of Gravity at the Equator(In g)

$$-\frac{997}{63000}n^7+\frac{32633}{72000}n^6-\frac{90763}{14400}n^5+\frac{605341}{14400}n^4-\frac{11306057}{72000}n^3+\frac{11686051}{36000}n^2-\frac{710761}{2100}n+\frac{3376}{25}$$

- Equation of Escape Velocity(In km/s)

$$-\frac{10391}{36000}n^7+\frac{10883}{1200}n^6-\frac{2087581}{18000}n^5+\frac{1862227}{2400}n^4-\frac{104555069}{36000}n^3+\frac{14428591}{2400}n^2-\frac{37666903}{6000}n+\frac{250573}{100}$$

Comparison of Bodo's Equation V/s Our Equation

In 1772 Johann Bode were close to a fairy simple mathematical progression, which became known as Bode's law.

This law is illustrated in given below table

$R_n=0.4+0.3x2^n$

Where the leading term is the distance from sun to mercury

Planets	Actual Distance (In AU)	Bodes Distance	Error in Bode(%)	By above formula calculation	Error %
Mercury	0.4	0.4	0	0.4 (n=1)	0
Venus	0.7	0.7 (n=0)	0	0.7 (n=2)	0
Earth	1	1 (n=1)	0	1 (n=3)	0
Mars	1.5	1.6 (n=2)	6.67	1.5 (n=4)	0
Jupiter	5.2	5.2 (n=4)	0	5.2 (n=5)	0
Saturn	9.5	10 (n=5)	5.27	9.5 (n=6)	0
Uranus	19.6	19.6 (n=6)	0	19.6 (n=7)	0
Neptune	30	38.8(n=7)	29.3	30 (n=8)	0

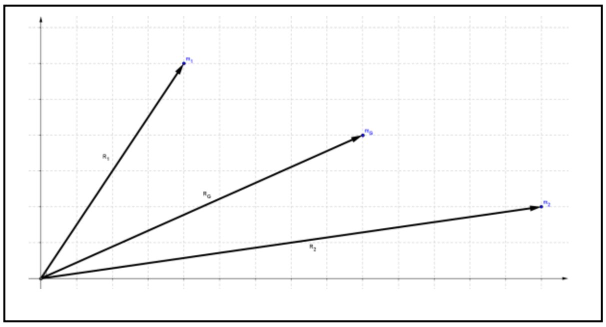
IV. Equations of Motion in an Inertial frame (Two bodies problem)

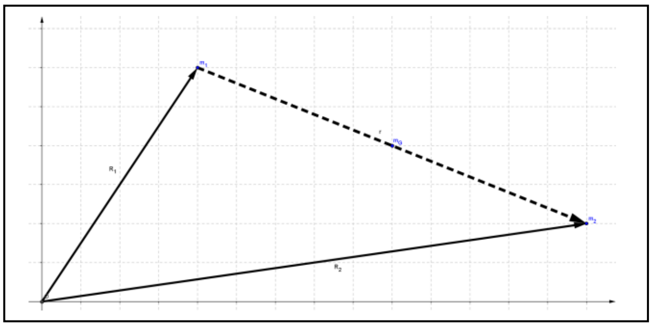
Consider two point masses in free space, the mutual attraction between them will be $\frac{-Gm_1m_2}{r^2}$ (-ve sign shows attraction) (Universal law of gravitation)

The position of center of mass relative to an inertial frame ABC is $\mathbf{R_G}=\frac{m_1\mathbf{R_1}+m_2\mathbf{R_2}}{m_1+m_2}$

Each of the body is pulled by the other $\boldsymbol{F_{1\,by\,2}}=$ Force exerted on 1 by 2.

$\boldsymbol{F_{2\,by\,1}}=$ Force exerted on 2 by 1





Absolute velocity of G (Centre of mass) is $\boldsymbol{V}_G = \frac{d\mathbf{R_G}}{dt} = \frac{m_1\frac{d\mathbf{R_1}}{dt}+m_2\frac{d\mathbf{R_2}}{dt}}{m_1+m_2}$

Absolute Acceleration of G is $\boldsymbol{a_G} = \frac{d^2\mathbf{R_G}}{dt^2} = \frac{m_1\frac{d^2\mathbf{R_1}}{dt^2}+m_2\frac{d^2\mathbf{R_2}}{dt^2}}{m_1+m_2}$

Now $\hat{\boldsymbol{n}}$ be a vector pointing from m_1 towards m_2. I.e. $\hat{\boldsymbol{n}} = \frac{\mathbf{r}}{r}$

Where **r** is the position vector of m_2 relative to m_1 . $\mathbf{r} = \mathbf{R_2}\text{-}\mathbf{R_1}$.

Thus

$\boldsymbol{F_{1\,by\,2}} = \frac{Gm_1m_2}{r^2}\ \hat{\boldsymbol{n}}$ Similarly $\boldsymbol{F_{2\,by\,1}} = -\frac{Gm_1m_2}{r^2}\ \hat{\boldsymbol{n}}$ (-ve sign indicates that the direction is opposite) .

According to Newton's second law

$\boldsymbol{F_{1\,by\,2}} = m_1\frac{d^2\mathbf{R_1}}{dt^2} = \frac{Gm_1m_2}{r^2}\ \hat{\boldsymbol{n}}$ and $\boldsymbol{F_{2\,by\,1}} = m_2\frac{d^2\mathbf{R_2}}{dt^2} = -\frac{Gm_1m_2}{r^2}\ \hat{\boldsymbol{n}}$

Therefore $\frac{d^2\mathbf{R_1}}{dt^2} = \frac{Gm_2}{r^2}\ \hat{\boldsymbol{n}}$ and $\frac{d^2\mathbf{R_2}}{dt^2} = -\frac{Gm_1}{r^2}\ \hat{\boldsymbol{n}}$ Substituting this on $\boldsymbol{a_G}$ we get $\boldsymbol{a_G} = 0$.

Thus G moves with a constant velocity.

4.1 Relative motion Equations

We have

$$\frac{d^2\mathbf{R_1}}{dt^2} = \frac{Gm_2}{r^2}\ \hat{\boldsymbol{n}}$$

$$\frac{d^2\mathbf{R_2}}{dt^2} = -\frac{Gm_1}{r^2}\ \hat{\boldsymbol{n}}$$

Therefore acceleration of m_2 w.r.t m_1 is $\frac{d^2\mathbf{r}}{dt^2} = \frac{d^2}{dt^2}(\mathbf{R_2} - \mathbf{R_1}) = -\frac{G(m_1+m_2)}{r^2}\ \hat{\boldsymbol{n}}$

Gravitational parameter $\mu = G(m_1 + m_2)$

Hence $\frac{d^2\mathbf{r}}{dt^2} = -\frac{\mu}{r^2}\ \hat{\boldsymbol{n}}$

This equation will give information about the motion of m_2 relative to m_1.

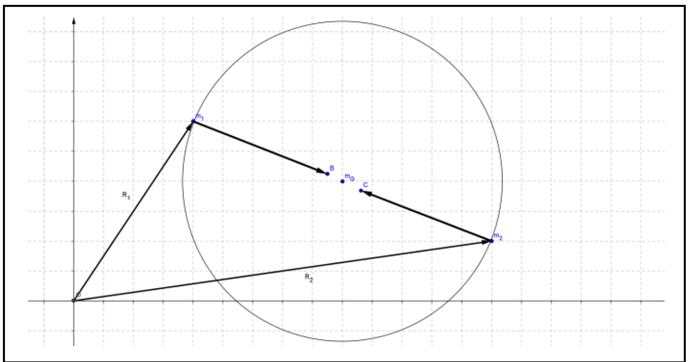
4.2 Its time for some Orbit Formulas

The Angular momentum of m_2 relative to m_1.

$\boldsymbol{L_{21}} = \boldsymbol{r}\ X\ m_2\frac{d\boldsymbol{r}}{dt}$

$\frac{d\boldsymbol{r}}{dt}$ is the velocity of m_2 with respect to m_1.

Now $\boldsymbol{l} = \frac{\boldsymbol{L_{21}}}{m_2} = \boldsymbol{r}\ X\ \frac{d\boldsymbol{r}}{dt}$



$$\frac{d\boldsymbol{l}}{dt} = \frac{d\boldsymbol{r}}{dt}\ X\ \frac{d\boldsymbol{r}}{dt} + \boldsymbol{r}\ X\ \frac{d^2\boldsymbol{r}}{dt^2} = 0 + \boldsymbol{r}\ X\ -\frac{\mu}{r^2}\ \hat{\boldsymbol{n}} = \mathbf{0} + \boldsymbol{r}\ X\ -\frac{\mu}{r^3}\mathbf{r} = 0$$

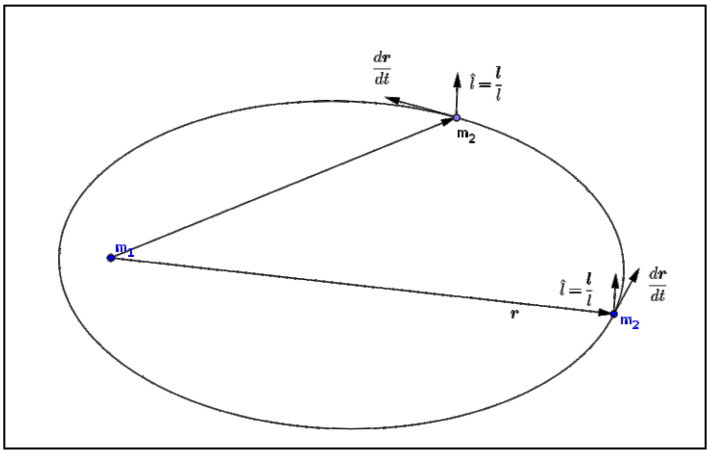
$\frac{d\boldsymbol{l}}{dt} = 0$ ➔ **l**= a constant .

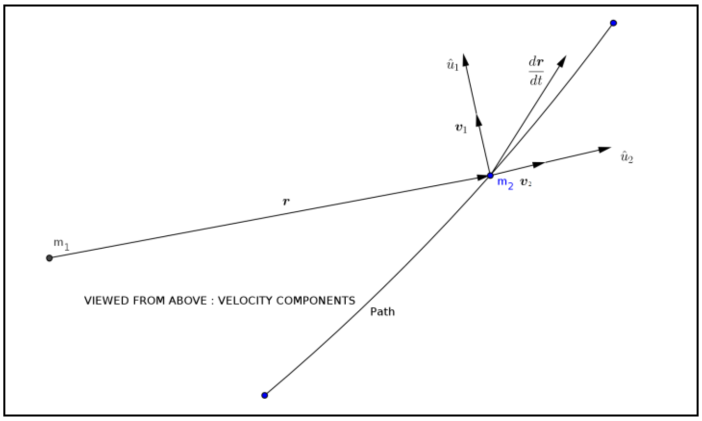
$\boldsymbol{r}\ X\ \frac{d\boldsymbol{r}}{dt}$➔ is a constant. Which means that at any given time **r** (position vector) & $\frac{d\boldsymbol{r}}{dt}$ (velocity vector) is in the same plane.

The Cross product $\boldsymbol{r}\ X\ \frac{d\boldsymbol{r}}{dt}$ is perpendicular to that plane.

$\hat{\boldsymbol{l}} = \frac{\mathbf{l}}{l}$➔ Unit vector normal to the plane.

Since it is a constant unit vector , m_2 around m_1 is in a single plane.





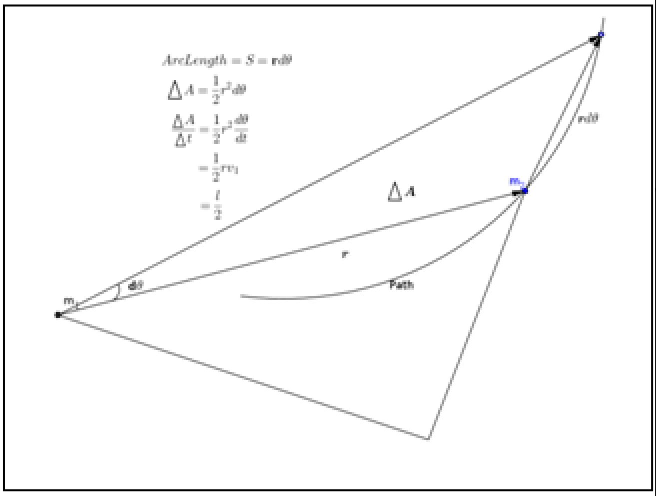
Now let us resolve $\frac{d\boldsymbol{r}}{dt}$ into two components

$$\frac{d\boldsymbol{r}}{dt} = V_2\,\widehat{\mathbf{u}_2} + V_1\,\widehat{\mathbf{u}_1}$$

$\boldsymbol{V_2}$ is along the outward radial from m_1 and $\mathbf{V}_1$ Is perpendicular to it.

Then $\boldsymbol{l} = \boldsymbol{r}\ X\ \frac{d\boldsymbol{r}}{dt} = r\widehat{\mathbf{u}_2}\ X\ (V_2\,\widehat{\mathbf{u}_2} + V_1\,\widehat{\mathbf{u}_1})$

I.e. $l = rV_1$.



<u>Identity proof</u> $\boldsymbol{r}\cdot\frac{d\boldsymbol{r}}{dt} = \mathrm{r}.\frac{d\mathrm{r}}{dt}$

We have $\mathbf{r}.\mathbf{r} = \mathrm{r}^2$

$$\frac{d}{dt}(\mathbf{r}.\mathbf{r}) = 2\mathbf{r}\frac{d\mathbf{r}}{dt}$$

$$\frac{d}{dt}(\mathrm{r}.\mathrm{r}) = 2\mathrm{r}\frac{d\mathrm{r}}{dt}$$

$$\frac{1}{\mathrm{r}^3}\,(\mathbf{r}\times\mathbf{l}) = \frac{1}{\mathrm{r}^3}\left(\mathbf{r}\times\boldsymbol{r}\times\frac{d\boldsymbol{r}}{dt}\right)$$

But A x (B xC) = B(A.C)-C(A.B)

$$\frac{1}{\mathrm{r}^3}\left\{\mathbf{r}\left(\mathbf{r}.\frac{d\mathbf{r}}{dt}\right) - \frac{d\mathbf{r}}{dt}(\mathbf{r}.\mathbf{r})\right\}$$

$$= \frac{1}{\mathrm{r}^3}\left\{\mathbf{r}\left(\mathrm{r}\frac{d\mathrm{r}}{dt}\right) - \frac{d\mathbf{r}}{dt}(\mathrm{r}^2)\right\}$$

$$= \frac{1}{\mathrm{r}^2}\left\{\mathbf{r}\frac{d\mathrm{r}}{dt} - \mathrm{r}\frac{d\mathbf{r}}{dt}\right\}$$

Let r covers an area ΔA in a time interval Δt

.

Therefore $\Delta A = \frac{1}{2}bh$

$$\frac{\Delta A}{\Delta t} = \frac{1}{2} r V_1 = \frac{l}{2}$$

4.3 Proof for $r = \frac{l^2}{\mu(1+e\cos(\theta))}$

$$\frac{d^2\boldsymbol{r}}{dt^2} \times \mathbf{l} = \frac{d}{dt}\left\{\frac{d\boldsymbol{r}}{dt} \times \mathbf{l}\right\}$$

But

$$\frac{d^2\boldsymbol{r}}{dt^2} \times \mathbf{l} = -\frac{\mu}{r^3}\mathbf{r} \times \mathbf{l}$$

$$\frac{d}{dt}\left(\frac{\mathbf{r}}{r}\right) = \frac{r\frac{d\boldsymbol{r}}{dt} - \boldsymbol{r}\frac{dr}{dt}}{r^2}$$

From the textbox it is clear that $\frac{d}{dt}\left(\frac{\mathbf{r}}{r}\right) = -\frac{1}{r^3}\ (\mathbf{r} \times \mathbf{l})$

Thus

$$\frac{d^2\boldsymbol{r}}{dt^2} \times \mathbf{l} = -\frac{\mu}{r^3}\mathbf{r} \times \mathbf{l} = \mu\left\{\frac{d}{dt}\left(\frac{\mathbf{r}}{r}\right)\right\}$$

$$\frac{d}{dt}\left\{\frac{d\boldsymbol{r}}{dt} \times \mathbf{l}\right\} = \left\{\frac{d}{dt}\left(\mu\frac{\mathbf{r}}{r}\right)\right\}$$

$$\frac{d}{dt}\left\{\frac{d\boldsymbol{r}}{dt} \times \mathbf{l} - \mu\frac{\mathbf{r}}{r}\right\} = 0$$

I.e.

$$\frac{d\boldsymbol{r}}{dt} \times \mathbf{l} - \mu\frac{\mathbf{r}}{r} = A\ constant = \boldsymbol{C}$$

Taking dot products on both sides by **l**.

$$\boldsymbol{l}.\left(\frac{d\boldsymbol{r}}{dt} \times \mathbf{l} - \mu\frac{\mathbf{r}}{r}\right) = \boldsymbol{C}.\boldsymbol{l}$$

Thus $\boldsymbol{C}.\boldsymbol{l} = \mathbf{0}$

I.e. **C** lies in the orbital plane.

$$\frac{d\boldsymbol{r}}{dt} \times \mathbf{l} - \mu\frac{\mathbf{r}}{r} = \boldsymbol{C}$$

Therefore

$$\frac{d\boldsymbol{r}}{dt} \times \mathbf{l} = \boldsymbol{C} + \mu\frac{\mathbf{r}}{r} \Rightarrow \frac{\frac{d\boldsymbol{r}}{dt} \times \mathbf{l}}{\mu} = \frac{\mathbf{r}}{r} + \frac{\mathbf{C}}{\mu}$$

$$\boldsymbol{e} = \frac{\mathbf{C}}{\mu} = Dimensionless\ eccentricity\ vector$$

Now equation becomes

$$\frac{\frac{d\boldsymbol{r}}{dt} \times \mathbf{l}}{\mu} = \frac{\mathbf{r}}{r} + \boldsymbol{e}$$

Taking dot product with **r** we get

$$\mathbf{r}.\left(\frac{\frac{d\boldsymbol{r}}{dt} \times \mathbf{l}}{\mu}\right) = \mathbf{r}.\left(\frac{\mathbf{r}}{r} + \boldsymbol{e}\right)$$

$$\boldsymbol{l}.\left(\frac{d\boldsymbol{r}}{dt} \times \mathbf{l}\right) = \begin{vmatrix} l_x & l_y & l_z \\ \frac{dx}{dt} & \frac{dy}{dt} & \frac{dz}{dt} \\ l_x & l_y & l_z \end{vmatrix} = 0$$

Since **l** is perpendicular to **r l.r=0**

Proof For $V_2 = \frac{\mu}{l} e\sin(\theta)$

Angular velocity be $\frac{d\theta}{dt}$

Then $V_1 = r\frac{d\theta}{dt}$

But $l = rV_1 = r^2\frac{d\theta}{dt}$

Thus $V_1 = \frac{l}{r}$

$$V_2 = \frac{dr}{dt} = \frac{d}{dt}\left(\frac{l^2}{\mu(1 + e\cos(\theta))}\right)$$

$$= \frac{l^2}{\mu}\left\{\frac{-e\sin\theta}{(1 + e\cos(\theta))^2}\right\}\frac{d\theta}{dt}$$

$$= \frac{l^2}{\mu}\left\{\frac{-e\sin\theta}{(1 + e\cos(\theta))^2}\right\}\frac{l}{r^2}$$

But $r = \frac{l^2}{\mu(1+e\cos(\theta))}$

Substituting this on above eqn we get

$$V_2 = -\frac{\mu}{l}e\sin\theta$$

Taking the magnitude only

$$V_2 = \frac{\mu}{l}e\sin\theta$$

$$\mathbf{l}.\left(\frac{\mathbf{r} \times \frac{d\mathbf{r}}{dt}}{\mu}\right) = \mathrm{r} + \mathbf{r}.\mathbf{e}$$

$$\mathrm{r} + \mathrm{reCos}(\theta) = \frac{\mathrm{l}^2}{\mu}$$

or

$$\mathrm{r} = \frac{\mathrm{l}^2}{\mu(1 + \mathrm{eCos}(\theta))}$$

4.4 Apse line and Periapsis

We have $r = \frac{\mathrm{l}^2}{\mu(1+\mathrm{eCos}\,(\theta))}$; $V_2 = \frac{\mu}{\mathrm{l}}\mathrm{eSin}\theta$; $V_1 = \frac{\mu}{l}(1 + \mathrm{eCos}(\theta))$

The point of closest approach lies on apse line and is called

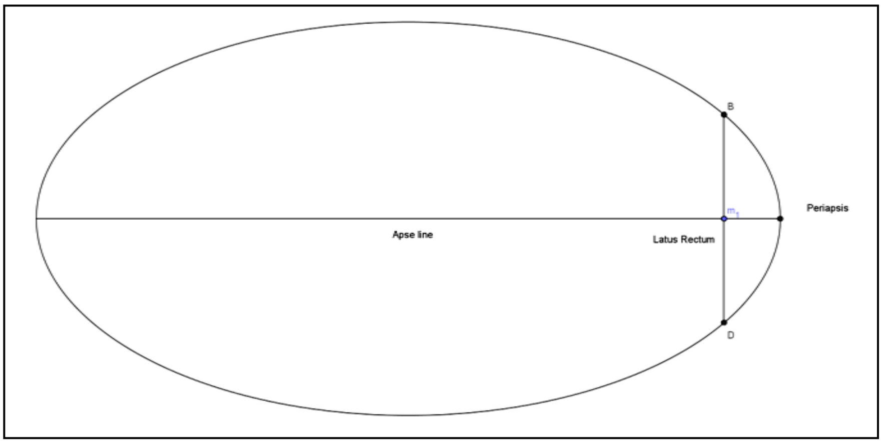
Periapsis. I.e. When $\theta = 0$ ➔ $r = \frac{\mathrm{l}^2}{\mu(1+\mathrm{e})}$ m_2 comes close to m_1 now.

The flight path angle is $Tan\gamma = \frac{V_2}{V_1} = \frac{\mathrm{eSin}\,\theta}{1+\mathrm{eCos}\,\theta}$➔The orbit equation is symmetric about its axis.

4.5 Latus Rectum

The latus rectum is the chord perpendicular to the apse line and passing through the center of attraction. By symmetry, the center of attraction divides the latus rectum into two equal parts, each of length *p*,

Where $p = \frac{\mathrm{l}^2}{\mu}$



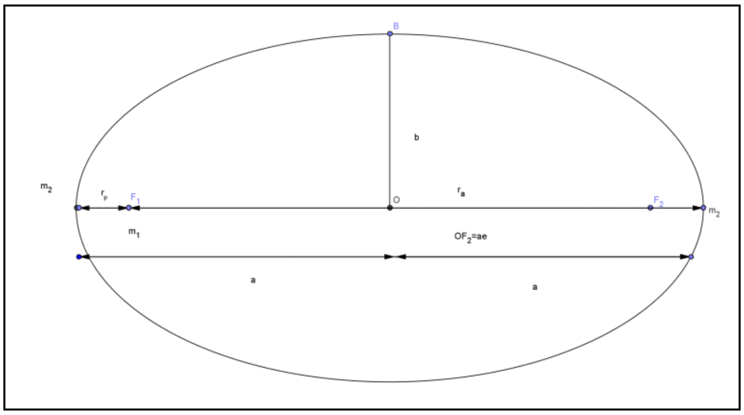
We have

$$r = \frac{\mathrm{l}^2}{\mu(1 + \mathrm{eCos}(\theta))}$$

<u>Elliptical Orbits(0<e<1)</u>

$r_a = \frac{\mathrm{l}^2}{\mu(1-\mathrm{e})}$➔Apoapsis

$r_p = \frac{\mathrm{l}^2}{\mu(1+\mathrm{e})}$➔Periapsis



From figure it is clear that $2a = r_a + r_b$

$a = \frac{\mathrm{l}^2}{\mu(1-\mathrm{e}^2)}$.

$r_a = \mathrm{a}(1 + \mathrm{e})$

4.6 Time Period

According to Kepler's law

$$\frac{dA}{dt} = A\ constant$$

In case of a complete rotation $dA = \pi ab$; $dt = T$

Therefore $\frac{\pi ab}{T} = \frac{l}{2}$

$$T = \frac{2\pi ab}{l} = \frac{2\pi a}{l} a\sqrt{1 - e^2} = \frac{2\pi a^2}{l}\sqrt{1 - e^2} = \frac{2\pi}{l}\sqrt{1 - e^2}\left\{\frac{\mathrm{l}^2}{\mu(1 - \mathrm{e}^2)}\right\}^2$$

Solving we get $T = \frac{2\pi}{\mu^2}(\frac{l}{\sqrt{1-e^2}})^3$

On further solving $= \frac{2\pi}{\sqrt{\mu}}(a)^{\frac{3}{2}}$; $T^2 = \frac{4\pi^2}{\mu}a^3$

4.7 Finding Mean distance

Now we have $r(\theta) = \frac{l^2}{\mu(1+e\text{Cos}(\theta))}$

Therefore $r_{av} = \frac{1}{2\pi}\int_0^{2\pi} r(\theta)\, d\theta = \frac{1}{2\pi}\int_0^{2\pi} \frac{a(1-e^2)}{(1+e\text{Cos}(\theta))}\, d\theta$

$r_{av} = \frac{a(1-e^2)}{2\pi}\int_0^{2\pi} \frac{1}{(1+e\text{Cos}(\theta))}\, d\theta$

$$a = \frac{l^2}{\mu(1-e^2)}$$

$$\frac{l^2}{\mu} = a(1-e^2)$$

$Put\ x = \tan\left(\frac{\theta}{2}\right)$

$\theta = 2\tan^{-1} x$

$$\text{Cos}\theta = \frac{1-\tan^2\left(\frac{\theta}{2}\right)}{1+\tan^2\left(\frac{\theta}{2}\right)} = \frac{1-x^2}{1+x^2}$$

Since the function is an even function we can write it as following

$$r_{av} = 2\frac{a(1-e^2)}{2\pi}\int_0^{\pi} \frac{1}{\left(1+e\left(\frac{1-x^2}{1+x^2}\right)\right)}\frac{2}{1+x^2}dx$$

$$\int \frac{1}{a^2+x^2}dx = \frac{1}{a}\tan^{-1}\frac{x}{a}$$

$$r_{av} = 4\frac{a(1-e^2)}{2\pi}\int_0^{\pi} \frac{1}{(1+x^2+e(1-x^2))}\, dx$$

$$r_{av} = 4\frac{a(1-e^2)}{2\pi}\int_0^{\pi} \frac{1}{x^2(1-e)+1+e}\, dx$$

$$r_{av} = 4\frac{a(1-e^2)}{2\pi(1-e)}\int_0^{\pi} \frac{1}{x^2+\frac{1+e}{(1-e)}}\, dx$$

$$r_{av} = \frac{2a(1+e)}{\pi}\int_0^{\pi} \frac{1}{x^2+\sqrt{\frac{1+e}{1-e}}^2}\, dx$$

$$r_{av} = \frac{2a(1+e)}{\pi}\sqrt{\frac{1-e}{1+e}}\left[\tan^{-1}\frac{x\sqrt{1-e}}{\sqrt{1+e}}\right]_0^{\infty}$$

$$r_{av} = \frac{2a\sqrt{(1-e^2)}}{\pi}\left[\frac{\pi}{2}\right]$$

$r_{av} = a\sqrt{(1-e^2)}$

I.e. $r_{av} = a\sqrt{(1-e^2)} = Semi\ minor\ axis = b$

Or

. $r_{av} = a\sqrt{(1-e^2)} = \sqrt{r_a r_p}$

Where $r_a = a(1+e)$; $r_p = a(1-e)$

V. Orbiting Path As A Time Function

We know that $l = r^2\frac{d\theta}{dt}$

Therefore $\frac{d\theta}{dt} = \frac{l}{r^2}$

But $r = \frac{l^2}{\mu(1+e\text{Cos}(\theta))}$

$$\frac{d\theta}{dt} = \frac{\mu^2}{l^3}\left(1 + e\mathrm{Cos}(\theta)\right)^2$$

$$\int_{t_p}^{t} \frac{\mu^2}{l^3} dt = \int_0^{\theta} \frac{1}{\left(1 + e\mathrm{Cos}(\theta)\right)^2} d\theta$$

Where t_p time at periapsis passing.

5.1 Circular Orbit

For circular orbits e=0.

$$\frac{\mu^2}{l^3}\left[t - t_p\right] = \theta$$

If we take $t_p = 0$

$$\frac{\mu^2}{l^3} t = \theta$$

On solving

$$t = \frac{r^{\frac{3}{2}}}{\sqrt{\mu}}\theta$$

For a full revolution $t = T, \theta = 2\pi$

$$T = \frac{2\pi r^{\frac{3}{2}}}{\sqrt{\mu}}$$

Therefore $\theta = \frac{2\pi}{T} t$

5.2 Elliptical Orbit (0<e<1)

$$\int_0^{\theta} \frac{1}{\left(1 + e\mathrm{Cos}(\theta)\right)^2} d\theta = \frac{1}{(1-e^2)^{\frac{3}{2}}} \; \{ \; 2\tan^{-1}[\sqrt{\frac{1-e}{1+e}}\tan\frac{\theta}{2}] - \frac{e\sqrt{1-e^2}\mathrm{Sin}\theta}{1 + e\mathrm{Cos}\theta}\}$$

$$\frac{\mu^2}{l^3} t = \frac{1}{(1-e^2)^{\frac{3}{2}}} \; \{ \; 2\tan^{-1}[\sqrt{\frac{1-e}{1+e}}\tan\frac{\theta}{2}] - \frac{e\sqrt{1-e^2}\mathrm{Sin}\theta}{1 + e\mathrm{Cos}\theta}\}$$

Rewrite it as

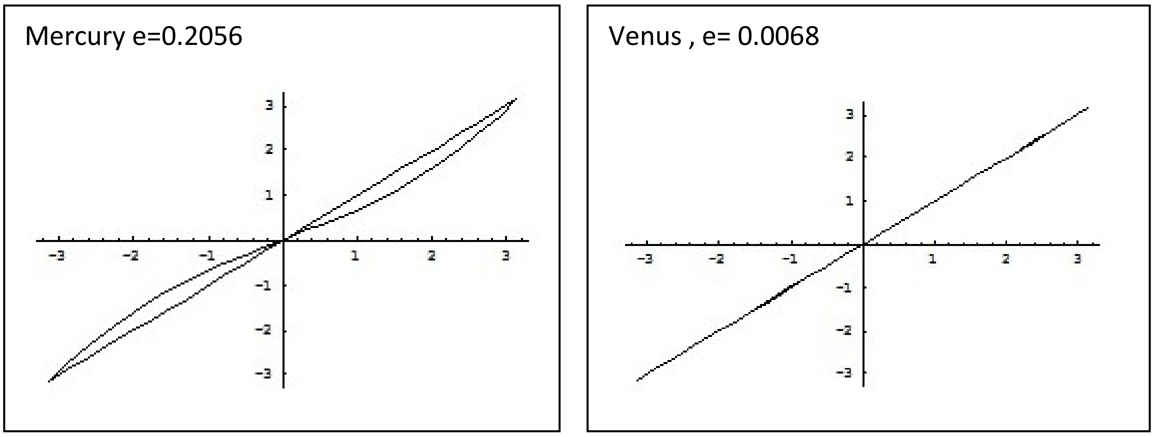
$\frac{\mu^2}{l^3} t = \frac{1}{(1-e^2)^{\frac{3}{2}}} \; \{ \; M_E \; \}$ **or** $\frac{\mu^2}{l^3}(1-e^2)^{\frac{3}{2}} t = M_E$

Now let $M_E = \frac{2\pi}{T} t = \frac{\mu^2}{l^3}(1-e^2)^{\frac{3}{2}} t$

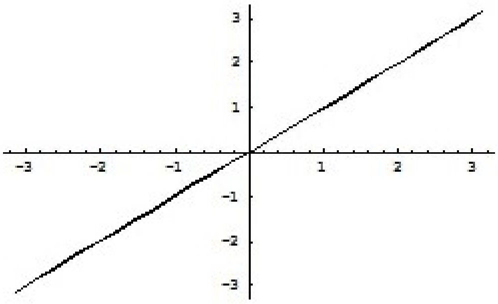
For the first Eight planets in our solar system e has a maximum value 0.20 in case of mercury. And e=0.0068 in case of venus (minimum)

If we ignore mercury then e ranges between 0.0068 and 0.05 .

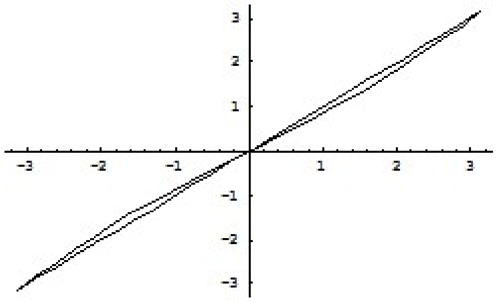
Plotting X axis =Θ and Y axis as corresponding M_E. [Straightline is : y=x=theta]



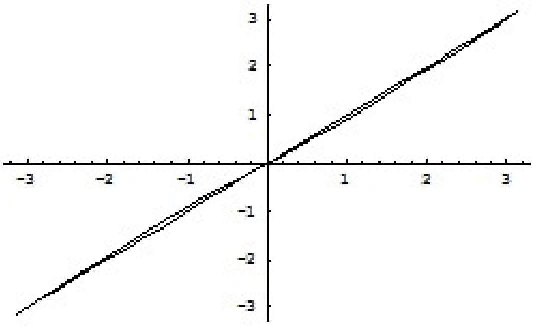
Earth e=0.0167



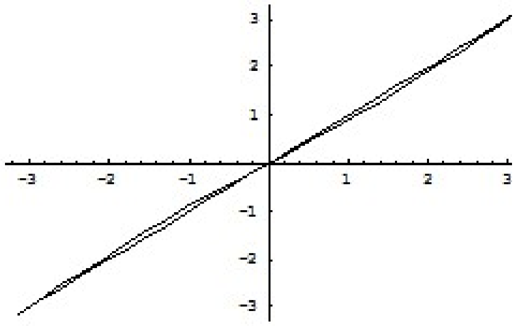
Mars e=0.0934



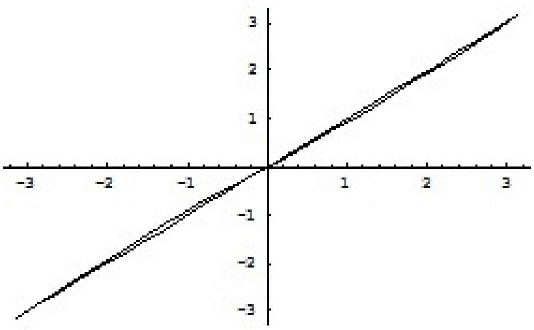
Jupiter e=0.0483



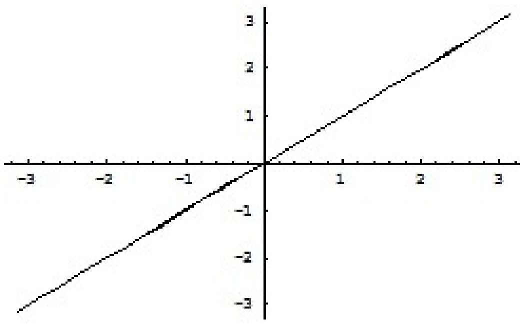
Saturn e=0.0560



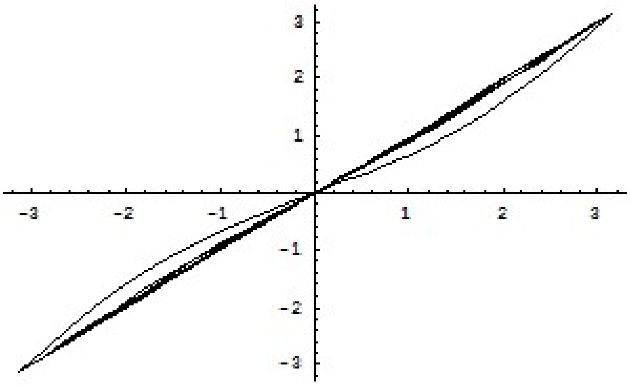
Uranus, e=0.0461



Neptune e=0.0097



AT A GLANCE



I.e. we can approximate $M_E = \frac{2\pi}{T}t$

N.B. We take t=0, at perihelion and starts counting from there.

We know that $r = \frac{l^2}{\mu(1+e\cos(\theta))}$

But

$$\frac{l^2}{\mu} = a(1-e^2)$$

$$r = \frac{a(1-e^2)}{(1+e\cos(\theta))}$$

But $b = a\sqrt{(1-e^2)}$

Therefore

$$r = \frac{b\sqrt{(1-e^2)}}{\left(1+e\cos\left(\frac{2\pi}{T}t\right)\right)}$$

VI. Combining Classical Physics to Polynomial difference theorem

$$r(n) = \frac{b(n)\sqrt{(1-e(n)^{\ 2})}}{\left(1+e(n)\ \cos\left(\frac{2\pi}{T(n)}t\right)\right)}$$

e(n)=Eccentricity of planet

T(n) =Time period of planet

Now let us recall our findings

- Equation of mean distance From Sun(In 10^7 m)

$$b(n) = \frac{-144464081}{1000000}n^7 + \frac{894094629}{200000}n^6 - \frac{56448957031}{1000000}n^5 + \frac{11973677}{32}n^4 - \frac{5563041}{4}n^3 + 2864028\,n^2 - \frac{5955539}{2}n + \frac{4752953}{4}$$

(n=1:Mercury, n=2:Venus, n=3:Earth…..etc)

- Equation of Orbital Period (In Earth days)

$$T(n) = \frac{-6194601}{500000}n^7 + \frac{376894501}{1000000}n^6 - \frac{467468457}{100000}n^5 + \frac{3810907959}{125000}n^4 - \frac{27956908203}{250000}n^3 + \frac{14593949}{64}n^2 - \frac{15086455}{64}n + \frac{5979771}{64}$$

- Equation of orbital Eccentricity

$$e(n) = \frac{151}{393750}n^7 - \frac{87493}{7200000}n^6 + \frac{1127483}{7200000}n^5 - \frac{1515287}{1440000}n^4 + \frac{28233521}{7200000}n^3 - \frac{3565267}{450000}n^2 + \frac{10873869}{1400000}n - \frac{1061}{400}$$

$$I(n) = \frac{50349}{3000000}n^7 - \frac{479215}{900000}n^6 + \frac{865181}{125000}n^5 - \frac{4268557}{90000}n^4 + \frac{91522621}{500000}n^3 - \frac{195666519}{500000}n^2 + \frac{417357269}{1000000}n - \frac{40261711}{250000}$$

Gives the angle of inclination of each planet.

(n=1:Mercury, n=2:Venus, n=3:Earth…..etc)

N.B

We have invented equation in such a way that when t=0 planet will be at perihelion and we will start counting from there onwards.

VII. Let's Make a reference point

Let us try to make an equation of t based on some date as reference point.
Here I am trying to make 1/12/1966 As a reference point.
From planet Calendar which shows distance between each planet from Earth, After some mathematical treatments (Using simple Trigonometry) We can find the position/distance of planet from sun.

In order to make a reference point function, we have applied

$$t = \frac{\mathrm{T(n)}}{2\pi} Cos^{-1}\left(\frac{\mathrm{b(n)}\sqrt{1-\mathrm{e(n)}^{\,2}} - \mathrm{d}}{\mathrm{e(n)d}}\right)$$

For each n=1,2,.. we will take corresponding planet's distance *d* based on 1/12/1966 planetary calendar.
Using that and Inverse polynomial Difference Theorem we can find a polynomial equation for this t for the specific date as

$$t(n) = \frac{-32230339}{1000000}n^7 + \frac{469816681}{500000}n^6 - \frac{5581737793}{500000}n^5 + \frac{34875074219}{500000}n^4 - \frac{15736047}{64}n^3 + \frac{15504011}{32}n^2 - \frac{15603855}{32}n + \frac{3032237}{16}$$

Now let us re write our equation as

$$\mathrm{r(n)} = \frac{\mathrm{b(n)}\sqrt{(1-\mathrm{e(n)}^{\;2})}}{1+\mathrm{e(n)}\ \mathrm{Cos}\left(\frac{2\pi}{\mathrm{T(n)}}[\mathrm{t(n)}+\Delta\mathrm{t}]\right)}$$

Δt =No of days from 1/12/1966
(n=1:Mercury, n=2:Venus, n=3:Earth…..etc)
Let us take an Example, Suppose we want to find the distance between Earth and sun on 1 may 2014
Then $\Delta \mathrm{t} = (01-05-2014) - (01-12-1966) \approx 17318.22$ Days

The distance between Earth and sun on May 1 $= \mathrm{r}(3) = \frac{\mathrm{b}(3)\sqrt{(1-\mathrm{e}(3)^{\,2})}}{1+\mathrm{e}(3)\ \mathrm{Cos}\left(\frac{2\pi}{\mathrm{T}(3)}[\mathrm{t}(3)+\Delta\mathrm{t}]\right)} = 1.016$ AU

Let us take a look on other planets and to the value we got from planetary calendar

1 May 2014

n	Name of planet	Mean distance b(n) x 10^7 m	Eccentricity e(n)	Time Period T(n)	t(n)	Value **From Formula** (In AU)	From Planetary calendar
1	MERCURY	5790.958283	0.2056	87.967214	13.029	0.31	0.30
2	VENUS	10820.00392	0.0068	224.695849	51.583032	0.73	0.73
3	EARTH	14960.32528	0.0167	365.249328	54.09746	1.02	1.01
4	MARS	22794.74907	0.0934	686.925287	306.81414	1.60	1.61
5	JUPITOR	77834.24688	0.0483	4332.53725	1211.88	5.24	5.23
6	SATURN	142941.9512	0.056	10759.08091	2753.92808	9.20	9.92
7	URANUS	287101.9634	0.0461	30684.1288	30739.04497	20.02	20.04
8	NEPTUNE	450433.9838	0.0097	60188.37532	27365.2161	30.10	30.03

Conclusion

We have shown that, through this special type method we can find the 'The General formula to find the straight line distance of solar planets from sun at any instant'.

The General formula straight line distance of n^{th} solar planets from sun At any instant

$$r(n) = \frac{b(n)\sqrt{(1 - e(n)^{\ 2})}}{\left(1 + e(n)\ \ Cos\left(\frac{2\pi}{T(n)}t\right)\right)}$$

We have invented equation in such a way that when t=0 planet will be at perihelion and we will start counting from there onwards.

Where r(n)denotes the position of n^{th} planet planet at t second.
b(n) denotes the mean distance of n^{th} planet from sun.
e(n)denotes the orbital eccentricity of n^{th} planet
T(n)denotes equation of orbital period of n^{th} planet.

$$I(n)=\frac{50349}{3000000}n^7 - \frac{479215}{900000}n^6 + \frac{865181}{125000}n^5 - \frac{4268557}{90000}n^4 + \frac{91522621}{500000}n^3 - \frac{195666519}{500000}n^2 + \frac{417357269}{1000000}n - \frac{40261711}{250000}$$

Gives the angle of inclination of each planet.
(n=1:Mercury, n=2:Venus, n=3:Earth…..etc)

But we can re-write the equation by making 1/12/1966 as a reference point as shown in the part VII Equation

I.e.

$$r(n) = \frac{b(n)\sqrt{(1 - e(n)^{\ 2})}}{1 + e(n)\ \ Cos\left(\frac{2\pi}{T(n)}[t(n) + \Delta t]\right)}$$

Δt =No of days from 1/12/1966
$t(n)$ = Reference time function

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References

[1]. Planetary Science: The Science of planets around stars (2002) By George H A Cole, Michael M Woolfson
[2]. Astrophysics of Solar System By KD Abhyankar (1999)
[3]. Orbital Mechanics for Engineering Students (2005) By Howard Curtis
[4]. The General formula to find the sum of first n Kth Dimensional S sided polygonal numbers and a simple way to find the n-th term of polynomial sequences By Arjun.K IOSR Journal of Mathematics (IOSR-JM) Volume 8 - Issue 3 **10.9790/5728-0830110**
[5]. **http://www.nightskyatlas.com/planetscal.jsp**
[6]. http://www.windows2universe.org
[7]. Classical Mechanics by Goldstein