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Investigation of General Equation That Best Describe the Relationship between Survival Fraction and Critical Mass of Uranium (235)

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Abstract: *In this paper we investigated the general equation that best describes the survival fraction f which is the basic tool for determine the critical mass of uranium (235)isotope, Monte Carlo simulation were used to study neutron interactions in a rectangular geometry of uranium (**$U^{235)}$**. The simulation takes as inputs the ratio of dimensions, the number of random points as well as the mass, and returns the survival fraction. The critical masses of u^{235} were obtained for the different ratio of length to thickness (0.25, 0.50, 0.75, 1.00, and 1.25 inclusive).We used two different models: Cubic equation and logarithmic equation using SPSS6.0 Statistical program. Both models equations are in good agreement with observed data.Statistical analysis shows that logarithmic equation is the best equation that can describe the relationship between survival fraction and critical mass.*

Keywords: *chain reaction, critical mass, generation, survival fraction, uranium isotope,*

I. Introduction

Uranium is among the most technologically important and study element as a nuclear energy material. It is used in the nuclear reactors for the production of power, energy and research activities. For example the major source of neutrons used in neutron activation analysis (NAA) where sample are activated by neutrons defends on nuclear chain reaction [1-5].

In a nuclear fission chain reaction, neutrons which are emitted during first spontaneous nuclear fission collide with other U^{235} nuclei. The other U^{235} nuclei absorb the neutrons, which causes them to become highly unstable and very rapidly undergo fission, thereby emitting more which trigger more fission, and so on. We refer to each phase of this process as a generation.[6]

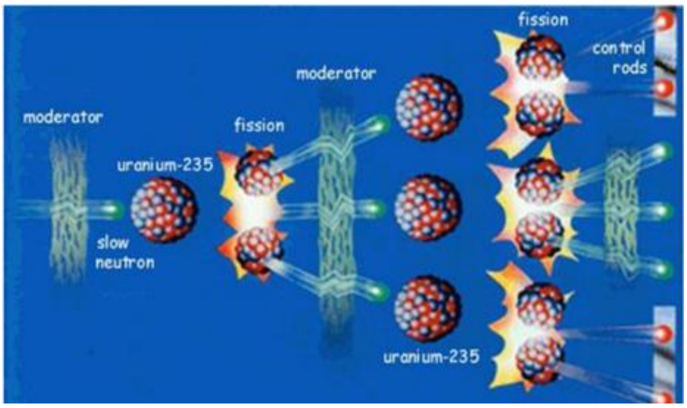

Figure 1: shows neutron induced nuclear fission chain reaction inside nuclear reactor core

If we assume that two neutrons are emitted during each fission, and that every emitted neutron induces another fission, the starting with N spontaneous fissions, there will be 2N induced fissions after one generation, 4N after two generations, and 2^n N after n generations. Thus, the number of induced fissions grows exponentially, reaching 2^{30} (about one billion) times the original number of spontaneous fissions in only 30 generation.

Due to the small size of the uranium nucleus, neutrons emitted during nuclear fission have to travel, on the average, appreciable distances before interacting with other nuclei and inducing them to fission. In the process of spontaneous fission one or possibly both neutrons may leave the piece before encountering another uranium nucleus. In that case the average number of induced fissions caused by each spontaneous fission would be a number less than or equals to two. Let us define the quantity

$$f = \frac{N_{in}}{N} \qquad (1)$$

where N_{in} is the number of fissions induced by neutrons emitted in N fissions during the preceding generation.

We shall referf as the survival fraction. Starting with N fissions in the first generation, there will be (f N) fissions in the next generation, (f^2N) fission in the one after that and f^n N fission in the n^{th} generation. Thus, the number of induced fission is growth exponential if and only if f is greater than one.

The value of f for a particular piece of uranium is determined by its mass, shape, and purity. A piece of uranium for which f = 1.0 is said to have a critical mass M_c. If a piece of uranium has a mass greater than the critical mass M_c, it will spontaneously undergo a chain reaction.

So critical mass is the smallest amount of fissile material needed for a sustained nuclear chain reaction. Several uncertainties contribute to the determination of a precise value for critical masses of uranium (235). Among thus uncertainties including detailed knowledge of cross sections and calculation of geometric effects.

The geometric effects provided significant motivation for the development of the Monte Carlo method for the determination of critical mass[7].

II. Methodology

2.1The Monte Carlo code

The Monte Carlo simulation is a computer program that proceeds to generate N random fissions, and use it to calculate the survival fraction; we need only to generate a large number of random fission (N) and keep a count of the number of neutron endpoints (N_{in}) which lie inside the block. To generate each random fission, it will be necessary to define nine random numbers $r_1, r_2, r_3 ..., r_9$, which lie between 0 and 1.

The nine quantities needed for each random fission are obtained from the random numbers r_1, r_2, r_3 ..., r_9 according to the following equations:

$$\left.\begin{aligned} X_0 &= a\left(r_1 - 1/2\right) \\ y_0 &= a\left(r_2 - 1/2\right) \\ z_0 &= b\left(r_3 - 1/2\right) \end{aligned}\right\} \text{Coordinates of the nucleus undergoing fission} \qquad (2)$$

$$\left.\begin{aligned} \Phi &= 2\pi r_4 \\ \cos\theta &= 2\left(r_5 - 1/2\right) \end{aligned}\right\} \text{are two angles for one emitted neutron} \qquad (3)$$

$$\left.\begin{aligned} \Phi^I &= 2\pi r_6 \\ \cos\theta^I &= 2\left(r_7^I - 1/2\right) \end{aligned}\right\} \text{ are two angles for the other emitted neutron} \qquad (4)$$

$$\left.\begin{aligned} d &= r_8 \\ d^I &= r_9 \end{aligned}\right\} \text{Distance traveled by each neutron} \qquad (5)$$

These formulae ensure that each of the nine parameters will lie within the proper range. For each neutron from a random fission we need to calculate the neutron endpoint from Eq. (2), and then test whether the point is inside or outside the block. The survival fraction f is then given byeq. (1)

In using the Monte Carlo method to find the survival fraction f, we are actually integrating a function F of many variables:

$$\mathbf{F} = F(x_0; y_0; z_0; \Phi; \theta ;d ;\Phi^I; \theta^I; d^I); \qquad (6)$$

This represents the number of fissions induced by two emitted neutrons for particular values of the variables x_0, y_0, ..., d_0. The value of **F** is zero, one, or two, depending on these variables.

In order to obtain the survival fraction f, we, must integrate the function F over all nine variables. The advantages of the Monte Carlo technique over conventional integration techniques are quite apparent in a case such as this.

To evaluate a nine-dimensional integral using a finite sum, each of the nine variables must be allowed to take some number of values. If only two values are used for each variable, it becomes necessary to evaluate the function 2^9 times or 512 calculations.

2.2Calculation of critical mass

The critical mass for rectangular shape of uranium U^{235} was determined by running a computer program using ratio of length to thickness$(S) = 0.25$,Number of random fission$(N) = 100$ and varying the mass from 1kg to 100kg.the values obtained were tabulated. Critical masses were found by making graphs of the calculated values of "f" against the mass "M" from the reading obtained. The critical mass is that value of M_c which correspond to $f = 1.0$.

In order to see the effect of changing the shape clearly the above procedure was repeated using different values of this ratio of length to thickness (S) such that $S = 0.50, 0.75, 1.00,$ and 1.25inclusive. The critical masses corresponding to each of these shapes were found.

III. ResultsAnd Discussion

The output files of the computation were used to deduce the tables of masses against the survival fraction and the graphs were plotted to obtain the critical mass. The critical masses were then used to obtain the minimum critical mass of uranium (235).

3.1 Graphical representation of data

The following graphs summarize the output data obtained during the Monte Carlo simulation, and are used in obtaining the critical mass of U^{235}.

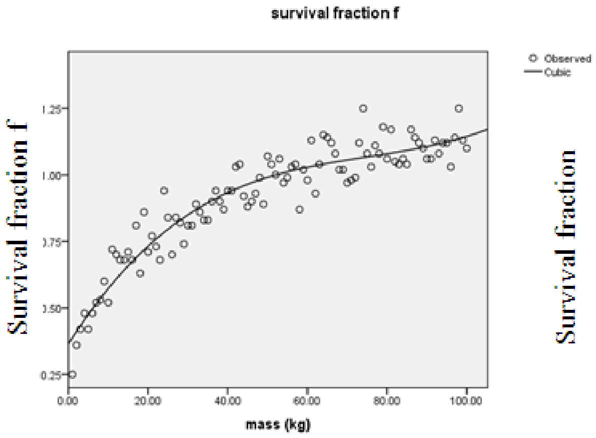

Figure 1: Graph of survival fraction against mass for observed data with model 1at ratio of length to thickness value of 0.25

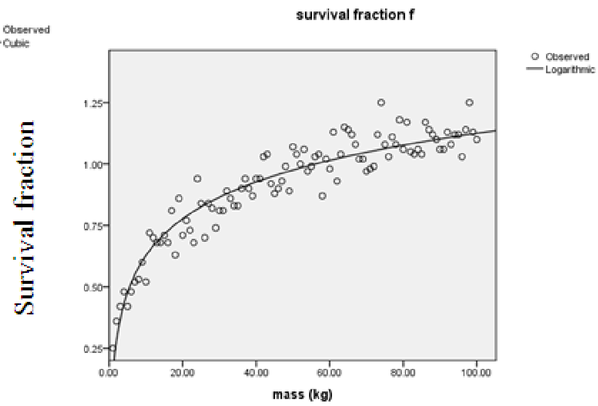

Figure 2: Graph of survival fraction against mass for observed data with model 2 at ratio of length to thickness values of 0.25

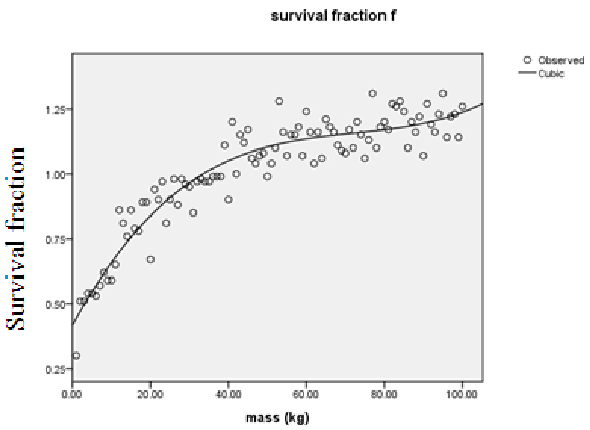

Figure 3: Graph of survival fraction against mass for observed data with model 1at ratio of length to thickness value of 0.50

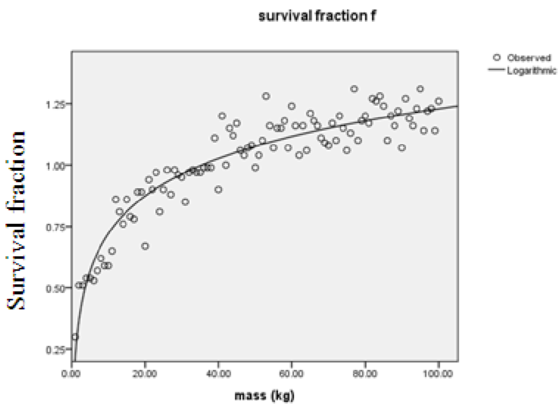

Figure 4: Graph of survival fraction against mass for observed data with model 2 at ratio of length to thickness value of 0.50

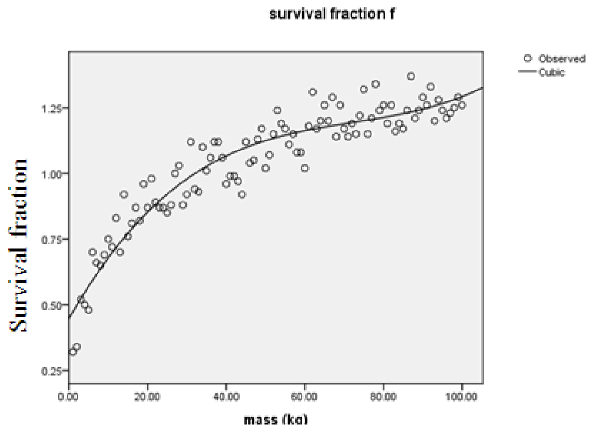

Figure 5: Graph of survival fraction against mass for observed data with model 1at ratio of length to thickness value of 0.75

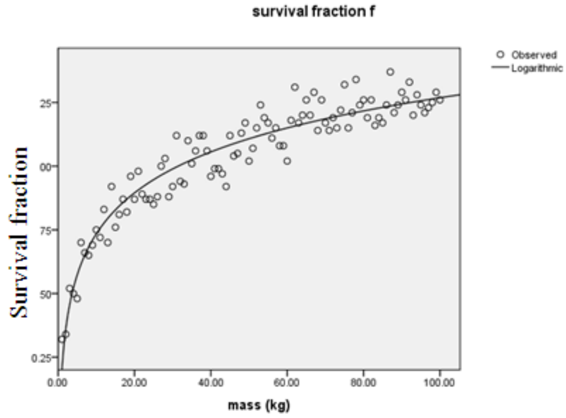

Figure 6: Graph of survival fraction against mass for observed data with model 2 at ratio of length to thickness value of 0.75

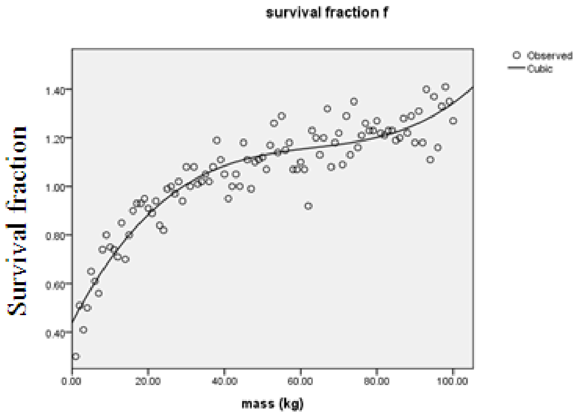

Figure 7: Graph of survival fraction against mass for observed data with model 1at ratio of length to thickness value of 1.00

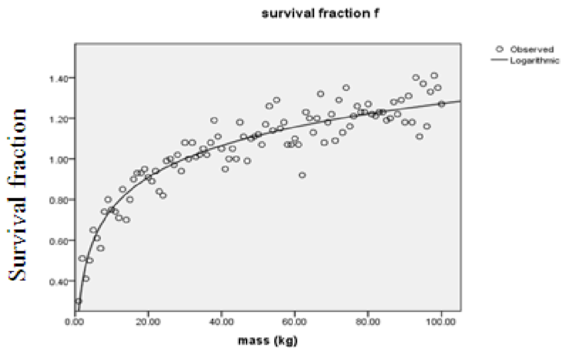

Figure 8: Graph of survival fraction against mass for observed data with model 2 at ratio of length to thickness value of 1.00

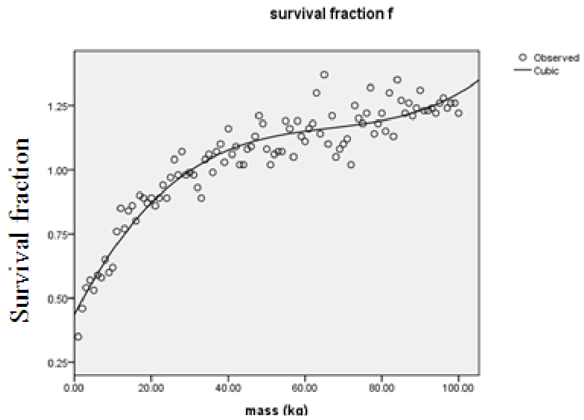

Figure 9: Graph of survival fraction against mass for observed data with model 1 of ratio of length to thickness of 1.25

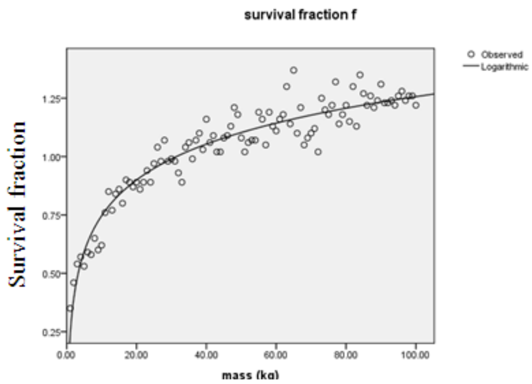

Figure 10: Graph of survival fraction against mass for observed data with model 2 of ratio of length to thickness of 1.25

Table 1: showsthe summary of the results for two model equations at the respective ratio of length to Thickness using regression analysis with SPSS 16.0

	Cubic summary					Logarithmic summary		
S	R^2	a	b_1	b_2	b_3	R^2	a	b
0.25	0.902	0.365	0.024	0.000	1.256E-6	0.902	0.129	0.216
0.50	0.897	0.417	0.028	0.000	1.679E-6	0.898	0.213	0.221
0.75	0.898	0.447	0.026	0.000	1.526E-6	0.913	0.200	0.232
1.00	0.881	0.438	0.030	0.000	2.347E-6	0.889	0.233	0.226
1.25	0.895	0.429	0.030	0.000	2.068E-6	0.913	0.220	0.227
1.50	0.912	0.436	0.029	0.000	2.045E-6	0.911	0.235	0.222

Table 2: Show critical mass of uranium (235) obtained using other method by different authors.

S/N	Method	Criticalmass (kg)
1	Monte Carlo Method (cylindrical shape) (6)	34.33
2	Apollo2 Normes (8)	46.56
3	Apollo2 Keff (8)	48.24
4	Standard Route CRISTAL (8)	46.56
5	Apollo2 TRIPOLI 14 (8)	47.31
6	Apollo MORET(2003) (8)	44.32
7	Diffusion Theory 2 (9)	45.90
8	Diffusion Theory 1 (10)	60.00
9	Neutrons Transport Equation(11)	50.00
10	Monte Carlo simulation (12)	52.18

IV. Discussion

Looking at the above graphs from figure 1 to 10,it’s difficult to notice the difference between the two models graph in describing observed data, because both equations are in good agreement with observed data, however looking at table 1.Itsobserved that the regression parameter R^2 in the logarithmic equation gives most of the values of the ratios of length to thickness closer to unity than those given in the cubic equation. This proves that the logarithmic equation gives best description of relationship between survival fraction and critical mass of uranium (235).

V. Conclusions

In this paper we investigated the general equation that best describes the survival fraction f and the critical mass of uranium (235)isotope, Monte Carlo simulation were used to study neutron interactions in a rectangular geometry of uranium ($\mathbf{U^{235)}}$. The simulation takes as inputs the ratio of dimensions, the number of random points as well as the mass, and returns the survival fraction. The critical masses of u^{235} were obtained for the different ratio of length to thickness (0.25, 0.50, 0.75, 1.00, and 1.25 inclusive). We used two different models: Cubic equation $f = a + b_1 m + b_2 m^2 + b_3 m^3$and logarithmic equation $f = a + b \ln m$ using SPSS6.0 Statistical program. Where a, b, b_1, b_2, and b_3 are constant. Both models give equations that are in good agreement with observed data. It is observed that the regression parameter R^2 in the logarithmic equation gives most of the values of the ratios of length to thickness closer to unity than those given in the cubic equation. This Statistical analysis shows that logarithmic equation is the best equation that can describe the relationship between survival fraction and critical mass.

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