

## Heat and Mass Transfer Effects on Free Convection Flow Past a Moving Vertical Isothermal Plate with Chemical Reaction

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**Abstract:** An investigation has been made to study free convective flow past a moving vertical isothermal plate in fluid medium with mass transfer and homogeneous first order chemical reaction. The dimensionless differential equations governing the flow have been solved using the Laplace transform technique. The flow phenomenon has been characterized with the help of flow parameters such as velocity, temperature and concentration profiles for different parameters like thermal Grashof number ( $G_r$ ), mass Grashof number ( $G_m$ ), Prandtl number ( $P_r$ ), Schmidt number ( $S_c$ ), Chemical reaction parameter ( $R$ ). The result obtained for vertical velocity, temperature and concentration are presented graphically for different values of physical parameters. It is observed that the Chemical reaction have significant effect on the flow pattern.

**Keywords:** Chemical reaction, Free Convection, Moving vertical isothermal plate.

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### I. Introduction

Convective heat transfer due to buoyancy is a regular phenomenon which occurs in nature as well as in many industrial, technological and geophysical areas. Mass transfer with chemical reaction is also a commonly encountered circumstance in chemical industry as well as in physical and biological sciences. Some other areas where these phenomena arise are oil industries, food processing industries, paper processing technologies, nuclear industry, etc. Similarly chemical reaction arises in these areas quite often. There are many situations where convection heat transfer phenomena are accompanied by mass transfer together with chemical reaction. Many authors already investigated a number of investigations with combined heat and mass transfer under the assumption of different physical situations. Due to wide range of industrial applications of mass transfer in free convection, considerable attention has been made and many authors have contributed their works which are available in literature. Further, due to the presence of foreign masses as impurities in fluid in the process of free convection, chemical reaction also takes place. In most cases of chemical reactions, the reaction rate depends on the concentration of the species. In many chemical engineering processes chemical reaction takes place between foreign masses and fluid. This type of chemical reaction may change the temperature and heat content of the fluid and may affect the free convection process. But if the presence of foreign mass is very low then we can assume that heat generation due to chemical reaction can be very negligible.

To study the heat and mass transfer problems one has to take the help of simplified form of Navier-Stokes equations. Since Navier-Stokes equations are highly non-linear, so they are extremely difficult to solve directly. Under certain simplifying assumptions they can be solved analytically. Because of some revolutionary works done by some pioneers in these areas more and more complex areas are gradually being covered now-a-days. Some of the pioneers are Illingworth, Siegel, Lai and Kulacki, Gebhart et. al., Soundalgekar etc. Illingworth [5] first studied the unsteady laminar flow of gas near an infinite flat plate. Soundalgekar [8] studied the free convection effects on the Stokes Problem for an infinite vertical plate and the effect of mass transfer and free convection currents on the flow past an impulsively started vertical plate was studied by Soundalgekar [9]. Lai and Kulacki [6] studied coupled heat and mass transfer by natural convection from vertical surfaces in a porous medium. Siegel [7] transient free convection from a vertical flat plate. Simultaneous heat and mass transfer in laminar free convection boundary layer flows over surfaces can be found in Gebhart et. al. [3]. Das et.al. [2] Studied the effects of mass transfer on flow past an impulsively started vertical infinite plate with constant heat flux and chemical reaction. In this paper we have investigated the effects of chemical reaction on free convective heat and mass transfer problem in a fluid medium with moving vertical plate under the assumption of first order chemical reaction.

### II. Mathematical Analysis

Here the flow of a viscous incompressible fluid past an impulsively started infinite vertical isothermal plate is considered. To visualize the flow pattern a Cartesian co-ordinate system is considered where  $x'$ -axis is taken along the infinite vertical plate, the  $y'$ -axis is normal to the plate. Initially, the fluid and the plate are kept at the same constant temperature  $T'_a$  and concentration  $C'_a$ . At time  $t' > 0$ , the plate is given an impulsive motion

in the vertical direction against the gravitational field with uniform velocity  $U_0$ , the plate temperature is raised to  $T'_w$  and the fluid concentration near the plate is  $C'_w$ . Under the above assumption and following Boussinesq approximation, the unsteady flow field is governed by the following equations.

$$\frac{\partial u'}{\partial t'} = g\beta(T' - T'_\infty) + g\beta^*(C' - C'_\infty) + \nu \frac{\partial^2 u'}{\partial y'^2} \quad (1)$$

$$\rho C_p \frac{\partial T'}{\partial t'} = k \frac{\partial^2 T'}{\partial y'^2} \quad (2)$$

$$\frac{\partial C'}{\partial t'} = D \frac{\partial^2 C'}{\partial y'^2} - K_1 C' \quad (3)$$

The following initial and boundary conditions are considered here.

$$\left. \begin{aligned} u' = 0, \quad T' = T'_\infty, \quad C' = C'_\infty \quad \text{for all } y' \text{ and } t' \leq 0 \\ u' = U_0, \quad T' = T'_w, \quad C' = C'_w \quad \text{at } y' = 0 \\ u' \rightarrow 0, \quad T' \rightarrow T'_\infty, \quad C' = C'_\infty \quad \text{as } y' \rightarrow \infty \end{aligned} \right\}, t' > 0 \quad (4)$$

To reduce the above equations in non-dimensional form we introduce the following non-dimensional quantities.

$$\begin{aligned} u = \frac{u'}{U_0}, \quad t = \frac{t' U_0^2}{\nu}, \quad y = \frac{y' U_0}{\nu}, \quad \theta = \frac{T' - T'_\infty}{T'_w - T'_\infty}, \quad \phi = \frac{C' - C'_\infty}{C'_w - C'_\infty}, \quad Pr = \frac{\mu C_p}{\nu}, \\ Gr = \frac{g\beta\nu(T'_w - T'_\infty)}{U_0^3}, \quad Gm = \frac{g\beta^*\nu(C'_w - C'_\infty)}{U_0^3}, \quad Sc = \frac{\nu}{D}, \quad R = \frac{\nu K_1}{U_0^2}. \end{aligned} \quad (5)$$

With the introduction of these non-dimensional quantities, the equations (1), (2) and (3) reduce to the following forms.

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial y^2} + Gr\theta + Gm\phi \quad (6)$$

$$\frac{\partial \theta}{\partial t} = \frac{1}{Pr} \frac{\partial^2 \theta}{\partial y^2} \quad (7)$$

$$\frac{\partial \phi}{\partial t} = \frac{1}{Sc} \frac{\partial^2 \phi}{\partial y^2} - R\phi \quad (8)$$

And the initial and boundary conditions are as follows:

$$\left. \begin{aligned} u = 0, \quad \theta = 0, \quad \phi = 0 \quad \text{for all } y \text{ and } t \leq 0 \\ u = 1, \quad \theta = 1, \quad \phi = 1 \quad \text{at } y = 0 \\ u \rightarrow 0, \quad \theta \rightarrow 0, \quad \phi \rightarrow 0 \quad \text{as } y \rightarrow \infty \end{aligned} \right\}, t > 0 \quad (9)$$

Solution of the equation (6), (7) and (8) subject to the initial and boundary conditions (9) are obtained by Laplace transform technique. Thus using Abramowitz and Stegun [1] and Hetnarski's algorithm [4] for inverse Laplace transform, we obtain the solutions as follows:

$$\theta(y, t) = \operatorname{erfc}\left(\frac{y\sqrt{Pr}}{2\sqrt{t}}\right) \quad (10)$$

$$\phi(y, t) = \frac{1}{2} \left\{ e^{-y\sqrt{ScR}} \operatorname{erfc} \left( \frac{y\sqrt{Sc}}{2\sqrt{t}} - \sqrt{Rt} \right) + e^{y\sqrt{ScR}} \operatorname{erfc} \left( \frac{y\sqrt{Sc}}{2\sqrt{t}} + \sqrt{Rt} \right) \right\} \quad (11)$$

$$u(y, t) = G_1 \left[ \left\{ \left( t + \frac{y^2}{2} \right) \operatorname{erfc} \left( \frac{y}{2\sqrt{t}} \right) - \frac{y\sqrt{t}}{\sqrt{\pi}} e^{-\frac{y^2}{4t}} \right\} - \left\{ \left( t + \frac{y^2 Pr}{2} \right) \operatorname{erfc} \left( \frac{y\sqrt{Pr}}{2\sqrt{t}} \right) - \frac{y\sqrt{t Pr}}{\sqrt{\pi}} e^{-\frac{y^2 Pr}{4t}} \right\} \right] + G_3 \operatorname{erfc} \left( \frac{y}{2\sqrt{t}} \right) + \frac{G_2}{b} \left[ \frac{e^{bt}}{2} \left\{ e^{-y\sqrt{b}} \operatorname{erfc} \left( \frac{y}{2\sqrt{t}} - \sqrt{bt} \right) + e^{y\sqrt{b}} \operatorname{erfc} \left( \frac{y}{2\sqrt{t}} + \sqrt{bt} \right) \right\} + \frac{1}{2} \left\{ e^{-y\sqrt{ScR}} \operatorname{erfc} \left( \frac{y\sqrt{Sc}}{2\sqrt{t}} - \sqrt{Rt} \right) + e^{y\sqrt{ScR}} \operatorname{erfc} \left( \frac{y\sqrt{Sc}}{2\sqrt{t}} + \sqrt{Rt} \right) \right\} - \frac{e^{bt}}{2} \left\{ e^{-y\sqrt{Sc(R+b)}} \operatorname{erfc} \left( \frac{y\sqrt{Sc}}{2\sqrt{t}} - \sqrt{Ht} \right) + e^{y\sqrt{ScH}} \operatorname{erfc} \left( \frac{y\sqrt{Sc}}{2\sqrt{t}} + \sqrt{Ht} \right) \right\} \right] \quad (12)$$

Where, 
$$b = \frac{RSc}{1 - Sc}, G_1 = \frac{Gr}{Pr - 1}, G_2 = \frac{Gm}{Sc - 1}, G_3 = 1 - \frac{G_2}{b}, H = R + b$$

### III. Result And Discussion

Exact solutions of equations (6), (7) and (8) subject to the initial and boundary conditions (9) are obtained by Laplace Transform technique and the results are presented in equations (10), (11) and (12). In order to know the effect of different physical parameters viz. thermal Grashof number, mass Grashof number, Schmidt number, Prandtl number and time on the physical flow field, computations are carried out for concentration, temperature and vertical velocity and they are presented in figures below.

Figure 1 represents the concentration profiles for different values of Schmidt number ( $Sc$ ) and chemical reaction parameter ( $R$ ). From the figure it is observed that increase in  $Sc$  and  $R$  lead to decreases in concentration.

Figure 2 represents the temperature profiles for different values of Prandtl number ( $Pr$ ). From the figure It is observed that increase in Prandtl number leads to decrease in temperature.

Figure 3(a) & 3(b) represents the velocity profile for different values of  $G_r$ ,  $G_m$ ,  $R$  and  $Gr$ ,  $G_m$ ,  $Sc$ , and  $R$  respectively. From figures 3a and 3b it is observed that increase in  $Gr$  and  $G_m$  lead to increase in velocity. Increase in  $R$  leads to decrease in velocity. Also increase in  $Sc$  leads to decrease in velocity.

IV. Figures

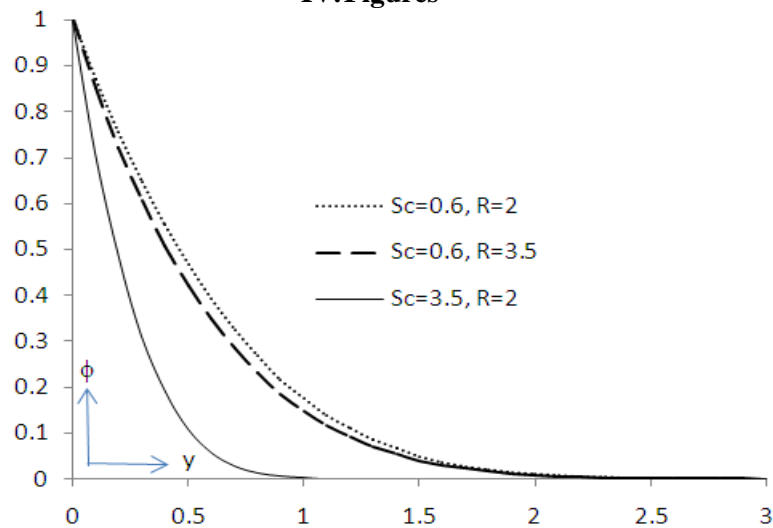


Figure-1 Effect of Sc and R on concentration.

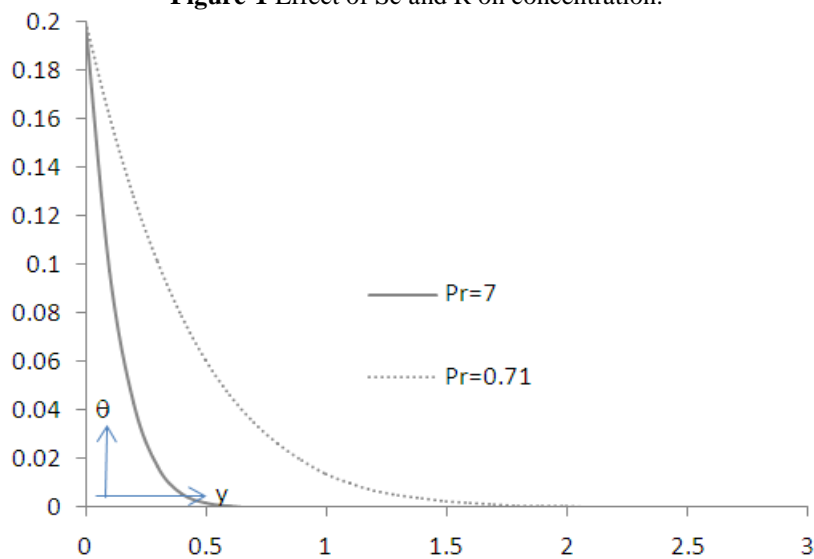


Figure-2 Effect of Pr on temperature.

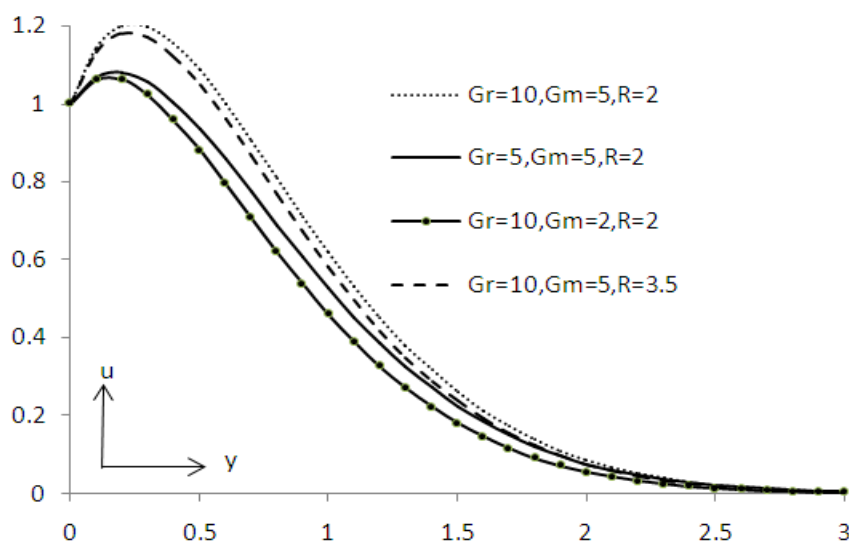


Figure-3a Effect of Gr, Gm and R on vertical velocity.

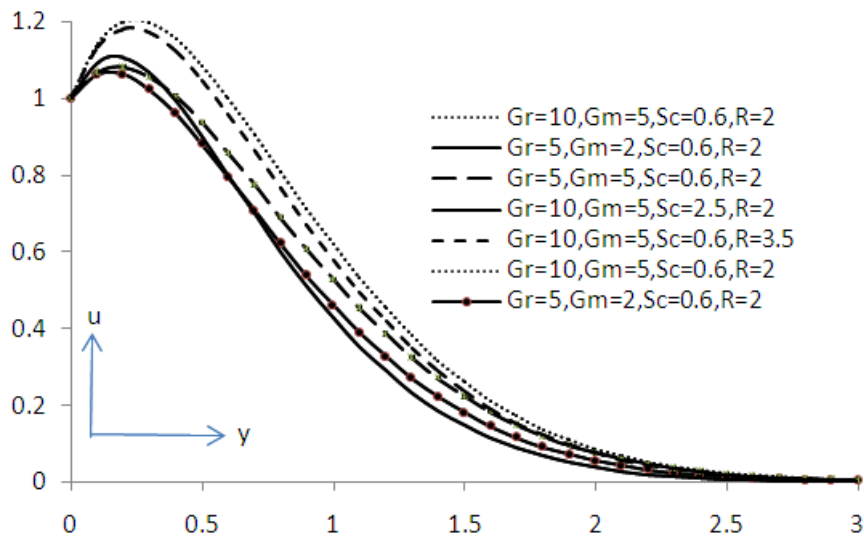


Figure-3b Effect of Gr, Gm, Sc and R on vertical velocity.

### V. Conclusions

From the figures we observe that

- Concentration decreases as Sc and R increase.
- Velocity increases with increasing Gr, Gm and decreasing Sc and R.

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