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# Velocity and Acceleration in Second Torridal Coordinates

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***Abstract:*** *We had established the velocity and acceleration in spheroidals and parabolic coordinates for applications in mechanics. In this paper, we proceed to derived expression for the instantaneous velocity and acceleration in Second Torroidal Coordinates for application in mechanics.*
***Keywords:*** *Second Terroidal Coordinates, velocity, Accelerations and Mechanics.*

## I. Introduction

The instantaneous velocity and acceleration in orthogonal curvilinear coordinates had been established in Cartesian, circular cylindrical, spherical, oblate spherical, prolate spheroidal and parabolic cylindrical coordinates [1, 2, 3, 4]. We therefore continue to derive the expression for instantaneous velocity an instantaneous acceleration in Second Torroidal Coordinates for application in Mechanics.

The Second Torroidal Coordinates $(\mathrm{R}, \theta, \phi)$ are defined in terms of the Cartesian coordinates $(x, y, z)$ By [4];

$$x = (a - R\cos\theta)\cos\phi \tag{1}$$
$$y = (a - R\cos\theta)\sin\phi \tag{2}$$
$$z = R\sin\theta \tag{3}$$

Where a is a constant parameter and $R < a$ ;

$$0 \leq \theta \leq \pi ; \quad 0 \leq \phi \tag{4}$$

Consequently, by definition, the Second Torroidal Metrical Coefficients are given by:

$$h_R = 1 \tag{5}$$
$$h_\theta = R \tag{6}$$

and

$$h_\phi = (a - R\cos\theta) \tag{7}$$

These metrical coefficients define units vectors line element, volume element, gradient, divergence, curl and Laplacian operations in Second Torroidal coordinates according to the theory of orthogonal curvilinear coordinates [5, 6, 7]. These quantities are necessary and sufficient for the derivation of the fields of all second Torrodal distribution of mass, charge and current. Now for the derivation of the equations of motion for test particles in these fields, we shall derive the expression for instantaneous velocity and acceleration in Second Torroidal coordinates.

## II. Mathematical Analysis

The Cartesian unit vectors are related to the parabolic coordinates unit vector as:

$$\hat{R} = -\cos\theta\cos\phi\, i - \cos\theta\sin\phi\, j + \sin\theta\, k \tag{8}$$
$$\hat{\theta} = \sin\theta\cos\phi\, i + \sin\theta\sin\phi\, j + \cos\theta\, k \tag{9}$$

and

$$\hat{\phi} = -\sin\phi\, i + \cos\phi\, j \tag{10}$$

The inversion is given by:

$$\hat{\imath} = -\cos\theta\cos\phi\,\hat{R} + \sin\theta\sin\phi\,\hat{\theta} - \sin\theta\hat{\phi} \tag{11}$$
$$\hat{\jmath} = -\cos\theta\sin\phi\,\hat{R} + \sin\theta\sin\phi\,\hat{\theta} + \cos\theta\hat{\phi} \tag{12}$$

and

$$\hat{k} = \sin\theta\,\hat{R} + \cos\theta\,\hat{\theta} \tag{13}$$

Hence denoting one time differentiating by a dot, it follows from (8), (9) and (10) and some manipulation that:

$$\dot{\hat{R}} = \dot{\theta}\hat{\theta} - \dot{\hat{\phi}}\cos\theta\hat{\phi} \tag{14}$$

Similarly, if follows from (9), (8) and (10) that:

$$\dot{\hat{R}} = -\dot{\theta}\hat{\theta} + \dot{\phi}\cos\theta\hat{\phi} \tag{15}$$

and consequently, from (10), (11) and (12):

$$\dot{\hat{\phi}} = \dot{\phi}\left[\cos\theta\,\hat{R} - \sin\theta\hat{\theta}\right] \tag{16}$$

Now it follows from definition of instantaneous position vector $\underline{r}$ as;

$$\underline{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k} \quad (17)$$

And (1)-(3) and (11)-(13) that the instantaneous position vector may be expressed entirely in terms of second torroidal coordinates as:

$$\underline{r} = (-a\cos\theta + R)\hat{R} + a\sin\theta\,\hat{\theta} \quad (18)$$

It now follows from definition instantaneous velocity vector, $\underline{u}$ as:

$$\underline{u} = \dot{\underline{r}} \quad (19)$$

and (18), (14) and (15) that the instantaneous velocity vector may be expressed entirely in terms of second torroidal coordinates as:

$$\underline{u} = u_R\hat{R} + u_\theta\hat{\theta} + u_\phi\hat{\phi} \quad (20)$$

where

$$u_R = \dot{R} \quad (21)$$

$$u_\theta = R\dot{\theta} \quad (22)$$

and

$$u_\phi = (a - R\cos\theta)\dot{\phi} \quad (23)$$

Similarly, it follows from definition of instantaneous acceleration, $\underline{a}$ as:

$$\underline{a} = \dot{\underline{u}} \quad (24)$$

and (20), (14)-(16) that the instantaneous acceleration may be expressed entirely in terms of Second Torroidal Coordinates as;

$$\underline{a} = a_R\hat{R} + a_\theta\hat{\theta} + a_\phi\hat{\phi} \quad (25)$$

where

$$a_R = \ddot{R} - R\dot{\theta} + \dot{\phi}^2\cos\theta\,(a - R\cos\theta) \quad (26)$$

$$a_\theta = R\ddot{\theta} - 2\dot{R}\dot{\theta} - \dot{\phi}^2\sin\theta\,(a - R\cos\theta) \quad (27)$$

and

$$a_\phi = \ddot{\phi}(a - R\cos\theta) - 2\dot{R}\dot{\phi}\cos\theta + 2R\dot{\theta}\dot{\phi}\sin\theta \quad (28)$$

This is the completion of the Second Torroidal Coordinates system.

## III. Result and Discussion

In this paper we derived the component of velocity and acceleration in Second torriadal coordinates as (20)-(23), paper are necessary and sufficient for expressing all mechanical quantities (Linear momentum, Kinetic energy Lagrangian and Hamiltonian) in terms of Second Torridal Coordinates.

## IV. Conclusion

The velocity and acceleration equations (21), (22), (23),(26), (27),(28) obtained in this paper paves a way for expressing all dynamical laws of motion (Newton’s Law, Lagrenge’s Law Hamiltonian’s Law, Einstein’s special relativities Law of motion and Schrödinger’s Law quantum mechanics) entirely in terms of Second Torroidal Coordinates.

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