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Tables of Values of the Nield- Kuznetsov Functions of the First- and Second Kind

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Abstract: *The Nield-Kuznetsov integral functions of the first- and second-kind are were introduced and classified in a previous article published in IOSR Journal of Applied Physics. Extensive Tables of Values of these functions are produced in this work in order to provide a vehicle for researchers to solve initial and boundary value problems involving the inhomogeneous Airy's, generalized Airy's and Weber's differential equations.*

Keywords: *Nield-Kuznetsov Integral Functions of the First and Second Kinds, Weber and Airy ODE.*

I. Introduction

In a recent article, [1], the authors introduced the Nield-Kuznetsov functions of the first and second kinds that arise in the solutions of inhomogeneous Airy's, generalized Airy's, and Weber ordinary differential equations. These ODE are listed in **Table 1**, below.

Inhomogeneous differential equations
$\frac{d^2u}{dy^2} - yu = f(y)$ (Airy's equation)
$\frac{d^2u}{dy^2} - y^n u = f(y)$ (Generalized Airy's equation)
$\frac{d^2u}{dy^2} + (\frac{y^2}{4} - a)u = f(y)$; $a \in \Re$ (Weber's equation).

Table 1: Inhomogeneous ODE

When the forcing function, $f(y)$, is a constant κ, particular and general solutions of the above ODE's are given in **Table 2**, below.

Differential Equation	**Particular solution**	**General solution**
Airy's equation	$u_p = -\pi\kappa Ni(y)$	$u = c_1 Ai(y) + c_2 Bi(y) - \pi\kappa Ni(y)$ c_1 and c_2 are arbitrary constants
Generalized Airy's equation	$(u_p)_n = -\kappa \frac{\pi}{2\sqrt{m}\sin(m\pi)} N_n(y)$	$u_n = c_{1n} A_n(y) + c_{2n} B_n(y) - \frac{\kappa\pi}{2\sqrt{m}\sin(m\pi)} N_n(y)$ c_{1n} and c_{2n} are arbitrary constants
Weber equation	$(u_p)_w = -\kappa N_w(a, y)$	$u_w = c_{1w} W(a, y) + c_{2w} W(a, -y) - \kappa N_w(a, y)$ c_{1w} and c_{2w} are arbitrary constants

Table 2: Particular and general solutions of the inhomogeneous equations: Forcing function κ

The functions $Ni(y)$, $N_n(y)$ and $N_w(a,y)$ are the Nield-Kuznetsov integral functions of the ***first kind***, defined in **Table 3**.

Integral Function	Integral Form and First Derivative
Standard Nield-Kuznetsov function of the first kind	$Ni(y) = Ai(y)\int_0^y Bi(t)dt - Bi(y)\int_0^y Ai(t)dt$ $N'i(y) = A'i(y)\int_0^y Bi(t)dt - B'i(y)\int_0^y Ai(t)dt$
Generalized Nield-Kuznetsov function of the first kind	$N_n(y) = A_n(y)\int_0^y B_n(t)dt - B_n(y)\int_0^y A_n(t)dt$ $N_n'(y) = A_n'(y)\int_0^y B_n(t)dt - B_n'(y)\int_0^y A_n(t)dt$
Parametric Nield-Kuznetsov function of the first kind	$N_w(a,y) = W(a,y)\int_0^y W(a,-t)dt - W(a,-y)\int_0^y W(a,t)dt$ $N_w'(a,y) = W'(a,y)\int_0^y W(a,-t)dt + W'(a,-y)\int_0^y W(a,t)dt$

Table 3. The Nield-Kuznetsov Functions of the First Kind

When the forcing function is a non-constant function $f(y)$, general and particular solutions of the ODE's in **Table 1** are given in **Table 4**, below.

Differential Equation	Particular solution	General solution
Airy's equation	$u_p = -\pi Ki(y)$	$u = c_1 Ai(y) + c_2 Bi(y) - \pi Ki(y)$ c_1 and c_2 are arbitrary constants
Generalized Airy's equation	$(u_p)_n = -\frac{\pi}{2\sqrt{m}\sin(m\pi)} K_n(y)$	$u_n = c_{1n} A_n(y) + c_{2n} B_n(y) - \frac{\pi}{2\sqrt{m}\sin(m\pi)} K_n(y)$ c_{1n} and c_{2n} are arbitrary constants
Weber equation	$(u_p)_w = -K_w(a,y)$	$u_w = c_{1w} W(a,y) + c_{2w} W(a,-y) - K_w(a,y)$ c_{1w} and c_{2w} are arbitrary constants

Table 4: Particular and general solutions of the inhomogeneous equations: forcing function $f(y)$

The functions $Ki(y)$, $K_n(y)$ and $K_w(a,y)$ are the Nield-Kuznetsov integral functions of the ***second kind***, defined in **Table 5**, below, where $f(y) = F'(y)$.

Integral Function	Integral Form and First Derivative
Standard Nield-Kuznetsov function of the second kind	$Ki(y) = -[Ai(y)\int_0^y F(t)B'i(t)dt - Bi(y)\int_0^y F(t)A'i(t)dt]$ $K'i(y) = -\left[A'i(y)\int_0^y F(t)B'i(t)dt - B'i(y)\int_0^y F(t)A'i(t)dt + \frac{1}{\pi}F(y)\right]$

Generalized Nield-Kuznetsov function of the second kind	$K_n(y) = -[A_n(y)\int_0^y F(t)B_n'(t)dt - B_n(y)\int_0^y F(t)A_n'(t)dt]$ $K_n'(y) = -\left[A_n'(y)\int_0^y F(t)B_n'(t)dt - B_n'(y)\int_0^y F(t)A_n'(t)dt + \frac{2\sqrt{m}\sin(m\pi)}{\pi}F(y)\right]$
Parametric Nield-Kuznetsov function of the second kind	$K_w(a,y) = W(a,-y)\int_0^y F(t)W'(a,t)dt + W(a,y)\int_0^y F(t)W'(a,-t)dt$ $K_w'(a,y) = W'(a,y)\int_0^y F(t)W'(a,-t)dt - W'(a,-y)\int_0^y F(t)W'(a,t)dt - F(y)$

Table 5. The Nield-Kuznetsov Functions of the Second Kind

II. Series And Integral Representations

In solving initial and boundary value problems involving the ODE's in **Table 1**, one requires values of the Nield-Kuznetsov functions at given values of the argument.

In what follows, we will use ascending series for $Ni(y)$ and $Ki(y)$, integrals for $N_n(y)$ and $K_n(y)$, and Maclauren series for $N_w(a,y)$ and $K_w(a,y)$ that we developed for the Nield-Kuznetsov functions to calculate and tabulate their values.

2.1. Standard Nield-Kuznetsov Functions of the First- and Second-Kind

$$N_i(y) = 2\sqrt{3}a_1a_2\left[\begin{array}{l}\left\{\sum_{k=0}^{\infty}\left(\frac{1}{3}\right)_k\frac{3^k y^{3k}}{(3k)!}\right\}\left\{\sum_{k=0}^{\infty}\left(\frac{2}{3}\right)_k\frac{3^k y^{3k+2}}{(3k+2)!}\right\}\\ -\left\{\sum_{k=0}^{\infty}\left(\frac{2}{3}\right)_k\frac{3^k y^{3k+1}}{(3k+1)!}\right\}\left\{\sum_{k=0}^{\infty}\left(\frac{1}{3}\right)_k\frac{3^k y^{3k+1}}{(3k+1)!}\right\}\end{array}\right] \quad \ldots(1)$$

$$Ki(y) = -\left(\begin{array}{l}2\sqrt{3}a_1a_2\left(\sum_{k=0}^{\infty}\left(\frac{1}{3}\right)_k\frac{3^k y^{3k}}{(3k)!}\right)\left[\sum_{k=0}^{\infty}\left(\frac{2}{3}\right)_k\int_0^y F(t)\frac{3^k t^{3k}}{(3k)!}dt\right]\\ -2\sqrt{3}a_1a_2\sum_{k=0}^{\infty}\left(\frac{2}{3}\right)_k\frac{3^k y^{3k+1}}{(3k+1)!}\left[\sum_{k=1}^{\infty}\left(\frac{1}{3}\right)_k\int_0^y F(t)\frac{3^k t^{3k-1}}{(3k-1)!}dt\right]\end{array}\right). \quad \ldots(2)$$

where

$$A_i(y) = a_1\sum_{k=0}^{\infty}\left(\frac{1}{3}\right)_k\frac{3^k y^{3k}}{(3k)!} - a_2\sum_{k=0}^{\infty}\left(\frac{2}{3}\right)_k\frac{3^k y^{3k+1}}{(3k+1)!} \quad \ldots(3)$$

$$B_i(y) = \sqrt{3}a_1\sum_{k=0}^{\infty}\left(\frac{1}{3}\right)_k\frac{3^k y^{3k}}{(3k)!} + \sqrt{3}a_2\sum_{k=0}^{\infty}\left(\frac{2}{3}\right)_k\frac{3^k y^{3k+1}}{(3k+1)!} \quad \ldots(4)$$

$$a_1 = Ai(0) = \frac{1}{\pi}\int_0^{\infty}\cos\frac{t^3}{3}dt \quad \ldots(5)$$

$$a_2 = -\frac{dAi}{dx}(0) = \frac{1}{\pi}\int_0^{\infty}t\sin\frac{t^3}{3}dt. \quad \ldots(6)$$

2.2. Generalized Nield-Kuznetsov Functions of the First- and Second-Kind

$$N_n(y) = \frac{2}{\sqrt{m}}\alpha_n\beta_n\left[g_{n1}(y)\int_0^y[g_{n2}(t)]dt - g_{n2}(y)\int_0^y[g_{n1}(t)]dt\right]$$

…(7)

$$K_n(y)=\frac{-2}{\sqrt{m}}\alpha_n\beta_n\left[g_{n1}(y)\int_0^y[F(t)g'_{n2}(t)]dt-g_{n2}(y)\int_0^y[F(t)g'_{n1}(t)]dt\right]$$
…(8)

where

$$A_n(y)=\alpha_n g_{n1}(y)-\beta_n g_{n2}(y) \quad \text{…(9)}$$

$$B_n(y)=\frac{1}{\sqrt{m}}[\alpha_n g_{n1}(y)+\beta_n g_{n2}(y)] \quad \text{…(10)}$$

$$\alpha_n=\frac{(m)^{1-m}}{\Gamma(1-m)} \quad \text{…(11)}$$

$$\beta_n=\frac{(m)^m}{\Gamma(m)} \quad \text{…(12)}$$

$$g_{n1}(y)=1+\sum_{j=1}^{\infty}m^{2j}\prod_{p=1}^{j}\frac{y^{j(n+2)}}{p(p-m)} \quad \text{…(13)}$$

$$g_{n2}(y)=y[1+\sum_{j=1}^{\infty}m^{2j}\prod_{p=1}^{j}\frac{y^{j(n+2)}}{p(p+m)}]. \quad \text{…(14)}$$

$$g_{n1}(y)=1+\sum_{j=1}^{\infty}m^{2j}\left(y^{j^2/m}\right)\prod_{p=1}^{j}\frac{1}{p(p-m)} \quad \text{…(15)}$$

$$g_{n2}(y)=y+\sum_{j=1}^{\infty}m^{2j}\left(y^{1+j^2/m}\right)\prod_{p=1}^{j}\frac{1}{p(p+m)}] \quad \text{…(16)}$$

$$g'_{n1}(y)=\sum_{j=1}^{\infty}m^{2j-1}j^2\left(y^{-1+j^2/m}\right)\prod_{p=1}^{j}\frac{1}{p(p-m)} \quad \text{…(17)}$$

$$g'_{n2}(y)=1+\sum_{j=1}^{\infty}m^{2j}(1+\frac{j^2}{m})\left(y^{j^2/m}\right)\prod_{p=1}^{j}\frac{1}{p(p+m)}] \quad \text{…(18)}$$

$$\int_0^y[g_{n1}(t)]dt=y+\sum_{j=1}^{\infty}m^{2j}\frac{1}{(1+j^2/m)}\left(y^{1+j^2/m}\right)\prod_{p=1}^{j}\frac{1}{p(p-m)} \quad \text{…(19)}$$

$$\int_0^y[g_{n2}(t)]dt=\frac{y^2}{2}+\sum_{j=1}^{\infty}m^{2j}\frac{1}{(2+j^2/m)}\left(y^{2+j^2/m}\right)\prod_{p=1}^{j}\frac{1}{p(p+m)}]. \quad \text{…(20)}$$

2.3. Parametric Nield-Kuznetsov Functions of the First- and Second-Kind

$$N_w(a,y)=2W(a,0)W'(a,0)\begin{bmatrix}\left\{\sum_{n=0}^{\infty}\delta_n(a)\frac{y^{2n+1}}{(2n+1)!}\right\}\left\{\sum_{n=0}^{\infty}\rho_n(a)\frac{y^{2n+1}}{(2n+1)!}\right\}\\-\left\{\sum_{n=0}^{\infty}\rho_n(a)\frac{y^{2n}}{(2n)!}\right\}\left\{\sum_{n=0}^{\infty}\delta_n(a)\frac{y^{2n+2}}{(2n+2)!}\right\}\end{bmatrix} \quad \text{…(21)}$$

$$K_w(a,y)=2W'(a,0)W(a,0)[\sum_{n=0}^{\infty}\rho_n(a)\frac{y^{2n}}{(2n)!}]\int_0^y[\sum_{n=0}^{\infty}\delta_n(a)F(t)\frac{t^{2n}}{(2n)!}]dt \quad \text{…(22)}$$

where

$$W(a,y)=W(a,0)\sum_{n=0}^{\infty}\rho_n(a)\frac{y^{2n}}{(2n)!}+W'(a,0)\sum_{n=0}^{\infty}\delta_n(a)\frac{y^{2n+1}}{(2n+1)!}$$
…(23)

$$W(a,-y)=W(a,0)\sum_{n=0}^{\infty}\rho_n(a)\frac{y^{2n}}{(2n)!}-W'(a,0)\sum_{n=0}^{\infty}\delta_n(a)\frac{y^{2n+1}}{(2n+1)!}$$
…(24)

$$W'(a,y)=W(a,0)\sum_{n=1}^{\infty}\rho_n(a)\frac{y^{2n-1}}{(2n-1)!}+W'(a,0)\sum_{n=1}^{\infty}\delta_n(a)\frac{y^{2n}}{(2n)!}$$
…(25)

$$W'(a,-y)=W'(a,0)\sum_{n=1}^{\infty}\delta_n(a)\frac{y^{2n}}{(2n)!}-W(a,0)\sum_{n=1}^{\infty}\rho_n(a)\frac{y^{2n-1}}{(2n-1)!}$$
…(26)

$$\rho_{n+2}=a\rho_{n+1}-\frac{1}{2}(n+1)(2n+1)\rho_n$$
…(27)

$$\delta_{n+2}=a\delta_{n+1}-\frac{1}{2}(n+1)(2n+3)\delta_n$$
…(28)

$\rho_0(a)=1$; $\rho_1(a)=a$; $\delta_0(a)=1$; $\delta_1(a)=a$.
…(29)

III. Tables of Values of the Standard Nield-Kuznetsov Functions

3.1. Tables for $Ni(y)$ from y=0 to 10, with steps of 0.1:

y	$Ni(y)$
0	0
0.1	-0.001591629009
0.2	-0.006368744579
0.3	-0.01434329156
0.4	-0.02554637147
0.5	-0.04003797119
0.6	-0.05791696555
0.7	-0.07933159847
0.8	-0.1044907107
0.9	-0.1336760520
1	- 0.1672560919

y	$Ni(y)$
1.1	-0.2057018349
1.2	-0.2496052452
1.3	-0.2997010208
1.4	-0.3568926103
1.5	-0.4222835396
1.6	-0.4972153994
1.7	-0.5833140612
1.8	-0.6825461435
1.9	-0.7972881754
2	-0.9304114376

y	$Ni(y)$
2.1	-1.085386335
2.2	-1.266410872
2.3	-1.478569201
2.4	-1.728027443
2.5	-2.022276037
2.6	-2.370430091
2.7	-2.783602260
2.8	-3.275366486
2.9	-3.862335306
3	-4.564880838

y	$Ni(y)$
3.1	-5.408035090
3.2	-6.422617643
3.3	-7.646650141
3.4	-9.127133677
3.5	-10.92228408
3.6	-13.10435354
3.7	-15.76319194
3.8	-19.01075333
3.9	-22.98680541
4	-27.86619072

y	$Ni(y)$
4.1	-33.86804425
4.2	-41.26754079
4.3	-50.41092131
4.4	-61.73464590
4.5	-75.78995952
4.6	-93.27446693
4.7	-115.0725253
4.8	-142.3074502
4.9	-176.4088668
5	-219.1991376

y	$Ni(y)$
5.1	-273.0064317
5.2	-340.8096524
5.3	-426.4279454
5.4	-534.7663385
5.5	-672.1358347
5.6	-846.6711932
5.7	-1068.878550
5.8	-1352.346236
5.9	-1714.687611
6	-2178.767510

y	$Ni(y)$
6.1	-2774.315757
6.2	-3540.108866
6.3	-4526.640329
6.4	-5800.051763
6.5	-7446.955283
6.6	-9580.777435
6.7	-12350.80556
6.8	-15953.78699
6.9	-20648.76232
7	-26776.86426

y	$Ni(y)$
7.1	- 34791.58888
7.2	- 45293.26865
7.3	- 59078.31490
7.4	- 77206.06292
7.5	-1.010984030×10^5
7.6	-1.326174479×10^5
7.7	-1.742714797×10^5
7.8	-2.295014280×10^5
7.9	-3.026808708×10^5
8	-4.008336962×10^5

y	$Ni(y)$
8.1	-5.296039888×10^5
8.2	-7.031465389×10^5
8.3	-9.336028964×10^5
8.4	-1.243636726×10^6
8.5	-1.659986057×10^6
8.6	-2.218619907×10^6
8.7	-2.973014338×10^6
8.8	-3.994789073×10^6
8.9	-5.328507495×10^6
9	-7.193803430×10^6

y	$Ni(y)$
9.1	-9.867606473×10^6
9.2	-1.327352226×10^7
9.3	-1.776169166×10^7
9.4	-2.482817113×10^7
9.5	-3.374084795×10^7
9.6	-4.679155329×10^7
9.7	-6.429859704×10^7
9.8	-8.912676817×10^7
9.9	-1.177746579×10^8
10	-1.623380420×10^8

3.2. Tables for $Ki(y)$ from y=0 to 10, with steps of 0.1, Various *F(y)*:

y	$Ki(y)$, $F(y)=y$	$Ki(y)$, $F(y)=y^2$	$Ki(y)$, $F(y)=\sin(y)$	$Ki(y)$, $F(y)=\exp(y)$
0	0	0	0	0
0.1	-0.001591629009812	-0.0001061068322202	-0.0015903031290647	-0.033479676987410
0.2	-0.006368744566574	-0.0008490527420054	-0.0063475481422588	-0.070519800352871
0.3	-0.014343291529498	-0.0028673682527963	-0.014236114752380	-0.11159922316439
0.4	-0.025546371408149	-0.0068051104251086	-0.025208130387305	-0.15732214088432
0.5	-0.040037971091158	-0.013318270089452	-0.039213457590056	-0.20843678166552
0.6	-0.057916965402628	-0.023083819496656	-0.056209786983380	-0.26585767544705
0.7	-0.079331598205150	0.036811515945290	-0.076172874230833	-0.33069240353658
0.8	-0.10449071031238	-0.055258650648998	-0.099107018135135	-0.40427390082338
0.9	-0.13367605156535	-0.079248026686369	-0.12505593769112	-0.48819959381989
1	-0.16725609184051	-0.10968956771722	-0.15411426990974	-0.58437891995633

y	$Ki(y)$ $F(y)=y$	$Ki(y)$ $F(y)=y^2$	$Ki(y)$ $F(y)=\sin(y)$	$Ki(y)$ $F(y)=\exp(y)$
1.1	-0.20570183497721	-0.14760610372346	-0.18643997948455	-0.69509109995201
1.2	-0.24960524495024	-0.19416405617104	-0.22226804844881	-0.82305544174215
1.3	-0.29970102038673	-0.25070995920833	-0.26192590215853	-0.97151696137337
1.4	-0.35689260828084	-0.31881401442251	-0.30585113135579	-1.1443507384900
1.5	-0.42228353858650	-0.40032219546504	-0.35461219385618	-1.3461892128969
1.6	-0.49721539829846	-0.49741880996300	-0.40893292993475	-1.5825776136383
1.7	-0.58331405916392	-0.61270190802527	-0.46972191068360	-1.8601639428459
1.8	-0.68254614295180	-0.74927452289630	-0.53810786834852	-2.1869314761347
1.9	-0.79728817189521	-0.91085546989636	-0.6154827442728	-2.5724836691688
2	-0.9304114343388	-1.1019143528447	-0.7035542491136	-3.0283937771756

y	$Ki(y)$ $F(y)=y$	$Ki(y)$ $F(y)=y^2$	$Ki(y)$ $F(y)=\sin(y)$	$Ki(y)$ $F(y)=\exp(y)$
2.1	-1.0853863281009	-1.3278365812082	-0.8044102809657	-3.568634528782
2.2	-1.2664108664951	-1.5951256480006	-0.9205981147451	-4.210107010985
2.3	-1.4785691949239	-1.9116517365605	-1.0552219919734	-4.973292725937

2.4	-1.7280274340271	-2.2869580141676	-1.2120636439889	-5.883058838109
2.5	-2.0222760210674	-2.7326388601818	-1.3957314245440	-6.969654281285
2.6	-2.3704300698776	-3.2628079297964	-1.611845174886	-8.26994407254
2.7	-2.7836022465700	-3.8946785818879	-1.867265779204	-9.82894144108
2.8	-3.2753664366941	-4.6492850703018	-2.170380697903	-11.70171293794
2.9	-3.8623352827599	-5.5523803587890	-2.531459728007	-13.95575146508
3	-4.5648807856508	-6.6355559161812	-2.963099011400	-16.67393733156

y	$Ki(y)$ $F(y)=y$	$Ki(y)$ $F(y)=y^2$	$Ki(y)$ $F(y)=\sin(y)$	$Ki(y)$ $F(y)=\exp(y)$
3.1	-5.4080349602722	-7.9376409566631	-3.480776120812	-19.9582395334
3.2	-6.4226174921937	-9.5064540502317	-4.103545195447	-23.9343504299
3.3	-7.6466500753117	-11.400999801835	-4.854908953978	-28.7574993947
3.4	-9.1271334241456	-13.694228629871	-5.763914476663	-34.6197581510
3.5	-10.922283881414	-16.476510170944	-6.866532560128	-41.7592366320
3.6	-13.104353434492	-19.860012598754	-8.207397040822	-50.471678876
3.7	-15.763191564930	-23.984233890176	-9.842001837516	-61.125110902
3.8	-19.010751971426	-29.023000355953	-11.83948098978	-74.178376147
3.9	-22.986804800885	-35.193337189433	-14.28613250619	-90.204631032
4	-27.866189499110	-42.766731441309	-17.28989278824	-109.921179790

y	$Ki(y)$ $F(y)=y$	$Ki(y)$ $F(y)=y^2$	$Ki(y)$ $F(y)=\sin(y)$	$Ki(y)$ $F(y)=\exp(y)$
4.1	-33.868039843207	-52.083457617864	-20.9860279033	-134.22742459
4.2	-41.267537819706	-63.570830334100	-25.5443851658	-164.25322188
4.3	-50.410915503429	-77.766500899048	-31.1786487577	-201.42060464
4.4	-61.734635568853	-95.348242801148	-38.1581735715	-247.52270036
4.5	-75.789956577692	-117.17209959035	-46.823141451	-304.82480912
4.6	-93.274448628192	-144.32132569533	-57.604005873	-376.1940844
4.7	-115.07249526921	-178.16928272050	-71.046481071	-465.2661961
4.8	-142.30743228945	-220.46040584454	-87.843711223	-576.6598836
4.9	-176.40878145421	-273.41461300706	-108.877752997	-716.2536327
5	-219.19909495779	-339.86216681845	-135.27315829	-891.543059

y	$Ki(y)$ $F(y)=y$	$Ki(y)$ $F(y)=y^2$	$Ki(y)$ $F(y)=\sin(y)$	$Ki(y)$ $F(y)=\exp(y)$
5.1	-273.00632165850	-423.41816856365	-168.46630334	-1112.103327
5.2	-340.80943749483	-528.70871235317	-210.29524216	-1390.188476
5.3	-426.42750551683	-661.66447473247	-263.11635559	-1741.509470
5.4	-534.76552290062	-829.90250211979	-329.95603933	-2186.24594
5.5	-672.13465258219	-1043.2235042436	-414.70828232	-2750.36420
5.6	-846.67001590928	-1314.2606325707	-522.3924491	-3467.33648
5.7	-1068.8767002156	-1659.3273704448	-659.4901586	-4380.38827
5.8	-1352.3444634328	-2099.5275878219	-834.3862612	-5545.4395
5.9	-1714.6848366236	-2662.2103808099	-1057.9470193	-7034.9613
6	-2178.7617727547	-3382.8815265669	-1344.2794190	-8943.0418

y	$Ki(y)$, $F(y)=y$	$Ki(y)$, $F(y)=y^2$	$Ki(y)$, $F(y)=\sin(y)$	$Ki(y)$, $F(y)=\exp(y)$
6.1	-2774.3104619112	-4307.7172259587	-1711.729987	-11392.0465
6.2	-3540.0701949422	-5496.8770421716	-2184.200772	-14541.395
6.3	-4526.5992577925	-7028.8733141080	-2792.885996	-18599.155
6.4	-5799.9950787244	-9006.3487929062	-3578.567588	-23837.343
6.5	-7446.8199106147	-11563.725793999	-4594.654228	-30612.196
6.6	-9580.6324430470	-14877.363116252	-5911.211197	-39391.08
6.7	-12350.661947398	-19178.971572218	-7620.312583	-50788.10
6.8	-15953.348411751	-24773.652604469	-9843.160919	-65611.80
6.9	-20647.714283969	-32063.637947175	-12739.57300	-84928.05
7	-26775.884631869	-41580.127837640	-16520.63794	-1.101451×10^5

y	$Ki(y)$ $F(y)=y$	$Ki(y)$ $F(y)=y^2$	$Ki(y)$ $F(y)=\sin(y)$	$Ki(y)$ $F(y)=\exp(y)$
7.1	-34790.504260427	-54026.229248636	-21465.6353	-1.431257×10^5
7.2	-45291.427744034	-70333.144655837	-27944.6805	-1.863385×10^5
7.3	-59074.973755314	-91737.689971709	-36449.0886	-2.430611×10^5
7.4	-77199.943220883	$-1.1988452018417\times10^5$	-47632.1399	-3.17651×10^5
7.5	$-1.0107650955443\times10^5$	$-1.5696284359094\times10^5$	-62363.907	-4.15911×10^5
7.6	$-1.3258583928166\times10^5$	$-2.0589598090403\times10^5$	-81805.107	-5.45584×10^5
7.7	$-1.7424181708827\times10^5$	$-2.7058236279906\times10^5$	-1.07506712×10^5	-7.1701×10^5
7.8	$-2.2940881310389\times10^5$	$-3.5625028856174\times10^5$	-1.4154454×10^5	-9.4407×10^5
7.9	$-3.0259689710584\times10^5$	$-4.6990469127312\times10^5$	-1.8670139×10^5	-1.24533×10^6
8	$-3.9986246187092\times10^5$	$-6.2092982220861\times10^5$	-2.4671370×10^5	-1.64566×10^6

y	$Ki(y)$ $F(y)=y$	$Ki(y)$ $F(y)=y^2$	$Ki(y)$ $F(y)=\sin(y)$	$Ki(y)$ $F(y)=\exp(y)$
8.1	$-5.2934936780921\times10^5$	$-8.2204863378028\times10^5$	-3.2660662×10^5	-2.1785×10^6
8.2	$-7.0202859794092\times10^5$	$-1.0902844141452\times10^6$	-4.331491×10^5	-2.8894×10^6
8.3	$-9.3270416372343\times10^5$	$-1.4484032391182\times10^6$	-5.754740×10^5	-3.8389×10^6
8.4	$-1.2413764022419\times10^6$	$-1.9278245103484\times10^6$	-7.659223×10^5	-5.110×10^6
8.5	$-1.6551061824935\times10^6$	$-2.5703980394484\times10^6$	-1.0211962×10^6	-6.813×10^6
8.6	$-2.2106095174970\times10^6$	$-3.4330632646662\times10^6$	-1.363938×10^6	-9.100×10^7
8.7	$-2.9577019205289\times10^6$	$-4.5930869725274\times10^6$	-1.824888×10^6	-1.217×10^7
8.8	$-3.9641273627749\times10^6$	$-6.1564013904229\times10^6$	-2.445848×10^6	-1.630×10^7
8.9	$-5.3221071574331\times10^6$	$-8.2660392740418\times10^6$	-3.283740×10^6	-2.187×10^7
9	$-7.1577346032946\times10^6$	$1.1116994903300\times10^7$	-4.416238×10^6	-2.953×10^7

y	$Ki(y)$ $F(y)=y$	$Ki(y)$ $F(y)=y^2$	$Ki(y)$ $F(y)=\sin(y)$	$Ki(y)$ $F(y)=\exp(y)$
9.1	$-9.6426732558976\times10^6$	$-1.4973579231433\times10^7$	-5.94940×10^6	-3.9684×10^7
9.2	$-1.3011512076619\times10^7$	$-2.0188784213023\times10^7$	-8.02836×10^6	-5.3549×10^7
9.3	$-1.7588629540701\times10^7$	$-2.7342154048283\times10^7$	-1.085203×10^7	-7.2387×10^7
9.4	$-2.3816391424395\times10^7$	$-3.7003533452902\times10^7$	-1.469339×10^7	-9.799×10^7
9.5	$-3.2294087307122\times10^7$	$-5.0160191387195\times10^7$	-1.992734×10^7	-1.3292×10^8
9.6	$-4.3879775158951\times10^7$	$-6.8138878769761\times10^7$	-2.707072×10^7	-1.8055×10^8
9.7	$-5.9707255438515\times10^7$	$-9.2699359221088\times10^7$	-3.683488×10^7	-2.458×10^8
9.8	$-8.1375211041202\times10^7$	$-1.2671683708174\times10^8$	-5.020306×10^7	-3.350×10^8
9.9	$1.1107641828939\times10^8$	$-1.6922136389948\times10^8$	-6.85348×10^7	-4.573×10^8
10	$-1.5186621480764\times10^8$	$-2.3590531999088\times10^8$	-9.3709×10^7	-6.254×10^8

3.3. Tables of values of for $N_n(y)$ from y=0 to 1, with steps of 0.1

y	$N_1(y)(=Ni(y))$	$N_2(y)$	$N_3(y)$
0	0	0	0
0.1	-0.001591629008	-0.001125399147	- 0.0008367272500
0.2	-0.006368744771	-0.004501821668	- 0.003346933706
0.3	-0.01434329669	-0.01013129344	- 0.007530979168
0.4	-0.02554642263	-0.01802169498	- 0.01339089697
0.5	-0.04003826894	-0.02819352974	- 0.02093374202
0.6	-0.05791816009	-0.04068944470	- 0.03017795748
0.7	-0.07933502829	-0.05558659595	- 0.04116378680
0.8	-0.1044967395	-0.07301204525	- 0.05396883072

0.9	-0.1336725243	-0.09316151260	- 0.06873025908
1	- 0.1671679498	- 0.1163218192	- 0.08567790804

y	$N_4(y)$	$N_5(y)$	$N_{10}(y)$
0	0	0	0
0.1	- 0.0006497473468	- 0.0005220047831	- 0.0002378240305
0.2	- 0.002598992311	- 0.002088019499	- 0.0009512961219
0.3	- 0.005847802133	- 0.004698057311	- 0.002140416280
0.4	- 0.01039671775	- 0.008352266574	- 0.003805184840
0.5	- 0.01624821576	- 0.01305153560	- 0.005945608733
0.6	- 0.02341039345	- 0.01879947876	- 0.008561767496
0.7	- 0.03190451911	- 0.02560749306	- 0.01165426375
0.8	- 0.04177858228	- 0.03350563491	- 0.01522648504
0.9	- 0.05312966759	- 0.04256347834	- 0.01929364193
1	- 0.06614082173	- 0.05292832891	- 0.02391327336

3.4. Tables of values of for $K_n(y)$ from y=0 to 1, with steps of 0.1, Various *F(y)*:

NOTE: $K_n(0)=0$

y	$K_1(y)$	$K_2(y)$	$K_3(y)$
0.1	-0.001591629010	-0.001125399146	-0.0008367272512
0.2	-0.006368744764	-0.004501821666	-0.003346933708
0.3	-0.01434329669	-0.01013129344	-0.007530979168
0.4	-0.02554642265	-0.01802169499	-0.01339089698
0.5	-0.04003826897	-0.02819352976	-0.02093374203
0.6	-0.05791816009	-0.04068944472	-0.03017795748
0.7	-0.07933502829	-0.05558659595	-0.04116378680
0.8	-0.1044967396	-0.07301204525	-0.05396883068
0.9	-0.1336725244	-0.09316151255	-0.06873025912
1	-0.1671679498	-0.1163218194	-0.08567790816

Values of $K_n(y)$ with $F(y)=y$

y	$K_4(y)$	$K_5(y)$	$K_{10}(y)$
0.1	-0.0006497473460	-0.0005220047831	-0.0002378240305
0.2	-0.002598992309	-0.002088019501	-0.0009512961219
0.3	-0.005847802136	-0.004698057311	-0.002140416280
0.4	-0.01039671775	-0.008352266574	-0.003805184840
0.5	-0.01624821576	-0.01305153560	-0.005945608739
0.6	-0.02341039345	-0.01879947872	-0.008561767496
0.7	-0.03190451911	-0.02560749305	-0.01165426375
0.8	-0.04177858228	-0.03350563491	-0.01522648503
0.9	-0.05312966759	-0.04256347831	-0.01929364193
1	-0.06614082166	-0.05292832894	-0.02391327336

Values of $K_n(y)$ with $F(y)=y$

y	$K_1(y)$	$K_2(y)$	$K_3(y)$
0.1	-0.0001061068324	-0.00007502653825	-0.00005578181344
0.2	-0.0008490528673	-0.0006002337435	-0.0004462569776
0.3	-0.002867373081	-0.002026102462	-0.001506174050
0.4	-0.006805174398	-0.004804615632	-0.003570688296
0.5	-0.01331873814	-0.009392272470	-0.006976617680
0.6	-0.02308611568	-0.01625585902	-0.01206561322
0.7	-0.03681962276	-0.02588198466	-0.01919067652
0.8	-0.05527824728	-0.03879128930	-0.02872790680
0.9	-0.07926470717	-0.05555638205	-0.04109561620
1	-0.1095475388	-0.07681056585	-0.05678057772

Values of $K_n(y)$ with $F(y)=y^2$

y	$K_4(y)$	$K_5(y)$	$K_{10}(y)$
0.1	-0.00004331648958	-0.00003480031886	-0.00001585493537
0.2	-0.0003465322197	-0.0002784025902	-0.0001268394829
0.3	-0.001169557044	-0.0009396108914	-0.0004280832560
0.4	-0.002772413007	-0.002227260949	-0.001014715945
0.5	-0.005415736243	-0.004350417468	-0.001981869224
0.6	-0.009362425884	-0.007519207080	-0.003424701536
0.7	-0.01488184670	-0.01194743414	-0.005438601276
0.8	-0.02225888854	-0.01785930395	-0.008120383369
0.9	-0.03181133199	-0.02550442470	-0.01157379424
1	-0.04391997436	-0.03518808431	-0.01593053627

Values of $K_n(y)$ with $F(y) = y^2$

y	$K_1(y)$	$K_2(y)$	$K_3(y)$
0.1	-0.001590303129	-0.001124461628	-0.0008360302108
0.2	-0.006347548337	-0.004486835956	-0.003335792162
0.3	-0.01423611963	-0.01005554581	-0.007474667356
0.4	-0.02520817656	-0.01778277791	-0.01321331920
0.5	-0.03921370927	-0.02761159628	-0.02050137392
0.6	-0.05621071126	-0.03948569997	-0.02928409701
0.7	-0.07617519934	-0.05336166425	-0.03951286309
0.8	-0.09911002958	-0.06922355225	-0.05116014920
0.9	-0.1250496399	-0.08710027215	-0.06424008284
1	-0.1540536847	-0.1070890219	-0.07883898672

Values of $K_n(y)$ with $F(y) = \sin(y)$

y	$K_4(y)$	$K_5(y)$	$K_{10}(y)$
0.1	-0.0006492060705	0.0005215699240	-0.0002376259098
0.2	-0.002590340547	-0.002081068708	-0.0009481293599
0.3	-0.005804075199	-0.004662927454	-0.002124411241
0.4	-0.01025883586	-0.008241496131	-0.003754718860
0.5	-0.01591255478	-0.01278189447	-0.005822769553
0.6	-0.02271667217	-0.01824229315	-0.008307977699
0.7	-0.03062384967	-0.02457918597	-0.01118609493
0.8	-0.03960117311	-0.03175808880	-0.01443160505
0.9	-0.04965104668	-0.03977317177	-0.01802644553
1	-0.06084470900	-0.04868204345	-0.02198824773

Values of $K_n(y)$ using series (32) with

3.5. Tables of values of for $N_w(a, y)$ from y=0 to 1, with steps of 0.1

y	$N_w(a,y); a=0$	$N_w(a,y); a=0.5$	$N_w(a,y); a=1$
0	0	0	0
0.1	-0.004999995827	-0.005002079512	-0.005004163888
0.2	-0.01999973332	-0.02003308857	-0.02006648827
0.3	-0.04499696254	-0.04516595734	-0.04533545954
0.4	-0.07998293445	-0.08051760658	-0.08105513505
0.5	-0.1249349069	-0.1262419177	-0.1275598486
0.6	-0.1798056698	-0.1825197255	-0.1852664744
0.7	-0.2445101224	-0.2495456854	-0.2546639319
0.8	-0.3189089751	-0.3275118455	-0.3362995397
0.9	-0.4027896942	-0.4165877536	-0.4307617641
1	-0.4958448910	-0.5168969487	-0.5386588510

Values of $N_w(a, y)$

3.6. Tables of values of for $K_w(a, y)$ from y=0 to 1, with steps of 0.1,

Various *F(y):*

y	$K_w(a,y); a=0$	$K_w(a,y); a=0.5$	$K_w(a,y); a=1$
0	0	0	0
0.1	-0.004999979160	-0.005018751691	-0.005037569427
0.2	-0.01999866668	-0.02030009960	-0.02060443967
0.3	-0.04498481424	-0.04651972004	-0.04808799913
0.4	-0.07991469830	-0.08480415263	-0.08988313805

0.5	-0.1246747744	-0.1367281977	-0.1495144069
0.6	-0.1790298279	-0.2043047605	-0.2318029195
0.7	-0.2425575130	-0.2899605010	-0.3430722329
0.8	-0.3145710706	-0.3964886560	-0.4913833999
0.9	-0.3940333734	-0.5269685113	-0.6867835814
1	-0.4794673172	-0.6846394223	-0.9415446652

Values of $K_w(a, y)$ with $F(y) = y$

y	$K_w(a,y); a=0$	$K_w(a,y); a=0.5$	$K_w(a,y); a=1$
0	0	0	0
0.1	-0.0003333317456	-0.0003346668218	-0.0003360053955
0.2	-0.002666463495	-0.002709351797	-0.002752690054
0.3	-0.008996529010	-0.009324275765	-0.009659773517
0.4	-0.02130733755	-0.02270031466	-0.02415201443
0.5	-0.04154278194	-0.04583861754	-0.05041843662
0.6	-0.07155657245	-0.08237560275	-0.09422931145
0.7	-0.1130311074	-0.1367254211	-0.1635219131
0.8	-0.1673594862	-0.2141964089	-0.2690996788
0.9	-0.2354868836	-0.3210611281	-0.4254486950
1	-0.3177111593	-0.4645515687	-0.6516586206

Values of $K_w(a, y)$ with $F(y) = y^2$

y	$K_w(a,y); a=0$	$K_w(a,y); a=0.5$	$K_w(a,y); a=1$
0	0	0	0
0.1	-0.004995813904	-0.005014569056	-0.005033369365
0.2	-0.01993209438	-0.02023241170	-0.02053562388
0.3	-0.04464846700	-0.04617060328	-0.04772579586
0.4	-0.07885511745	-0.08367235653	-0.08867593176
0.5	-0.1221006021	-0.1338763876	-0.1463657018
0.6	-0.1737300984	-0.1981687574	-0.2247467511
0.7	-0.2328376690	-0.2781125430	-0.3288039768
0.8	-0.2982174230	-0.3753503233	-0.4645964625
0.9	-0.3683198455	-0.4914749104	-0.6392561548
1	-0.4412208948	-0.6278652542	-0.8609179616

Values of $K_w(a, y)$ with $F(y) = \sin(y)$

IV. Conclusion

In this work, we produced Tables of Values for the Nield-Kuznetsov functions of the first- and second-kind. These were defined in Reference [1]. All evaluations or series and integrals were programmed and computed using *MAPLE.* Values of the Nield-Kuznetsov functions at points not listed in the above Tables can be calculated using the methods presented above.

Reference

[1]. S.M. Alzahrani, I. Gadoura, M.H. Hamdan, Nield- Kuznetsov Functions of the First- and Second Kind. *IOSR Journal of Applied Physics. .Volume 8, Issue 3 Ver. III (May. - Jun. 2016), PP 47-56.*