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Entropy Generation on MHD Viscoelastic Nanofluid over a Stretching Surface

M. Zakaria, Azza M. Moshen
Department of Mathematics, Faculty of Science, Al-Baha University, Al-Aqiq, P.O. Box 1988, Kingdom of Saudi Arabia.

Abstract: *This paper deals with the double-diffusive boundary layer flow of non-Newtonian nano-fluid over a stretching sheet. In this model, where binary nano-fluid is used, the Brownian motion and thermophoresis are classified as the main mechanisms which are responsible for the enhancement ofthe convection features of the nanofluid. By means of the successive approximation method the governing equations for momentum and energy have been solved. Nevertheless, these boundary-value problems can be solved correctly using expansions solutions, and we present the formulation that makes this possible (in essence, an application of Duhamel's principle). The solutions obtained by this new approach are shown to agree identically with those obtained by using numerical method solutions. Graphical display of the numerical examine are performed to illustrate the influence of various flow parameters on the velocity, temperature, concentration, reduced Nusselt, reduced Sherwood and reduced nano-fluid Sherwood number distributions. The present study has many applications in coating and suspensions, movement of biological fluids, cooling of metallic plate, melt-spinning, heat exchangers technology, and oceanography. Someimportant findings reported in this work reveals that expansion solutions effect of momentum equations have significant impact in controlling the rate of velocity in the boundary layer region.*

I. Introduction

Boundary layer behaviour over a moving continuous solid surface is an important type of flow occurring in a number of engineering processes. To be more specific, heat-treated materials travelling between a feed roll and a wind-up roll, aerodynamic extrusion of plastic sheets, glass fiber and paper production, cooling of an infinite metallic plate in a cooling path, manufacturing of polymeric sheets are examples for practical applications of continuous moving flat surfaces. Since the pioneering work of Sakiadis [1], various aspects of the problem have been investigated by many authors. Mass transfer's analyses at the stretched sheet were enclosed in their studies by Erickson et al. [2] and relevant experimental results were reported by Tsou et al. [3] regarding several aspects for the flow and heat transfer boundary layer problems in a continuously moving sheet. Crane [4] and Gupta and Gupta [5] have analyzed the stretching problem with constant surface temperature while Soundalgekar [6] investigated the Stokes problem for a viscoelastic fluid. This flow was examined by Siddappa and Khapate [7] for a special class of non-Newtonian fluids known as second-order fluids which are viscoelastic in nature.

Crane [8] considered the laminar boundary layer flow of a Newtonian fluid caused by a flat elastic sheet whose velocity varies linearly with the distance from the fixed point of the sheet. Rajagopal et al. [9] and Chang [10] presented an analysis on flow of viscoelastic fluid over stretching sheet. Heat transfer cases of these studies have been considered by Dandapat and Gupta [11] and Vajravelu and Rollins [12], while flow of viscoelastic fluid over a stretching surface under the influence of uniform magnetic field has been investigated by Andersson [13].

Heat transfer is a widespread phenomenon in the nature which exists due to temperature difference between objects or within the same body. Fourier's law of heat conduction law [14] is successfully for understanding the phenomena of heat transfer. However, the main shortcoming of the Fourier's laws contradicts the principle of causality. The propagation speed of heat disturbance is finite by introducing the thermal relaxation time in the Fourier's model [15]. Moreover, Christov [16] modified the time derivative in the Maxwell-Cattaneo model with the Oldroyd upper-convected derivative which successful preserves the material-invariant formulation. For the incompressible fluid with the Cattaneo-Christov heat conduction model, Tibullo and Zampoli [17] proved the uniqueness of the solution. Han et al. [18] studied coupled flow and heat transfer in viscoelastic fluid with Cattaneo-Christov heat flux model. He successfully studied the effects of the relevant parameters on the temperature and velocity. Mustafa [19] explored the rotating flow of Maxwell fluid over a linear stretching sheet with the CattaneoChristov heat flux. Hayat et al. [20,21] studied the flow over a stretching sheet with variable thickness and MHD flow of Oldroyed-B fluid with homogeneous-heterogeneous reactions with Cattaneo-Christov heat flux, respectively. Recently, the interest has been extended to the problem

of the flow and heat transfer over sheet by utilizing CattaneoChristov heat flux model. However, the model has seldom been used to study coupled flow and heat transfer of boundary layer in viscoelastic fluid.

Heat and mass transfers simultaneously affecting each other will also cause a cross-diffusion effect.The mass transfer caused by the temperature gradient is called the Soret effect, while the heat transfer caused by the concentration gradient is called the Dufour effect. Due to much attention in many fields that includes solid_state physics, chemical engineering, geophysics, and oceanography, the research on the double_diffusive convective boundary layer has become a topic of great interest. Pop and Ingham [21] gave the detailed review about the topic of thermo diffusion effect. Investigators [22–24] have reported and studied results for the fluids which have very light molecular weight due to the importance of thermal diffusion and diffusion thermo effects of these fluids. Mansour et al. [25] presented the effects of chemical reaction and thermal stratification on free convective heat and mass transfer over a vertical stretching surface embedded in a porous media considering the Soret and Dufour numbers in the presence of MHD.The heat and mass transfer on Hiemenz flow through porous medium onto a stretching surface was investigated by Tsai and Huang [26] for Soret and Dufour effects. The numerical solution of MHD free convective flow under Soret and Dufour effects over a stretching sheet immersed in a porous medium was analyzed by Bég et al. [27]. Kuznetsov and Nield [28] presented the analysis of the double diffusion free convection boundary layer flow of a nanofluid in porous media. Later on, Khan and Aziz [29] employed the same idea to study the double_diffusive natural convection from a vertical plate under prescribed surface heat, solutal and nanoparticle fluxes. W.A. Khan et al. [30] studied the thermo diffusion effect on free convective boundary layer flow of a nanofluid past a convectively heated vertical plate in the presence of momentum slip. Recently, F.G. Awad et al. [31] examined numerically the thermodiffusion effects on magneto fluid flow over a stretching sheet.

In real situations in nano-fluids, the base fluid do not satisfy the properties of Newtonian fluids, henceit is more justified to consider them as viscoelastic fluids, e.g., Ethylene glycol-Al_2O_3, Ethylene glycol-CuO and Ethylene glycol-ZnO are some examples of viscoelastic nano-fluids. In the present study weextend the work of Khan and Pop [32] using a modification of the model of Buongiorno [33] to the case ofa binary nano-fluid. With the idea of creating a detailed comparison with cross-diffusion effects for the regular fluids and the cross-diffusion effects peculiar to nano-fluids, a new approach in nano-fluids to cross-diffusion is made and also when the base fluid of the nano-fluid contains solute (e.g. salt) as well as nano-particles investigation of the interaction between these effects is carried out at the same time. The outcomeis that we are investigating a sort of triple-diffusion problem involving heat, the nanoparticles and the solute. Here, the base fluid is taken as second grade fluid. To our best of knowledge, no studies have far beeninvestigated to analyze the thermo diffusion effects on the boundary layer flow of viscoelastic nano-fluidover a permeable stretching sheet. The solutions of the resulting non-linear equations are solved by using the successive approximation method. The effects of the embedded flow controlling parameters on the fluid velocity,temperature, and nanoparticle concentration, heat transfer rate, and the nanoparticlevolume fraction rate have been demonstrated graphically and discussed. A comparative study is also given.

II. Formulation Of The Problem

We consider two-dimensional incompressible boundary layer flow optically dense nanofluid over a convectively heated permeable stretching sheet located at y = 0. The sheet is stretched through two equal and opposite forces along x-axis by keeping the origin fixed with the velocity $U_o = ax^m$, at $t = 0$, moving wedge and flat plate. A uniform magnetic field of strength B_0 is assumed to be applied normal to the stretching surface. The magnetic Reynolds numberis taken to be small and therefore the induced magnetic field and electric field are neglected. Heat transfer analysis based for fixed volume fraction of nanoparticles is carried out in the presence of thermal radiation, Brownian motion, thermophoresis and viscous dissipation effects. The sheet surface is heated by a hot fluid with uniform temperature T_f and the convective heat transfer coefficient $h_f$as shown in Fig. 1:

Following the Zakaria [34], the unsteady boundary layer equations governing the two-dimensional incompressible stagnation point flow of nanofluid can be written as

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{1}$$

$$\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = \upsilon\frac{\partial^2 u}{\partial y^2} - \frac{\sigma B_o^2}{\rho_f}u - \frac{\nu}{k}u,$$
$$-K_o\left(\frac{\partial^3 u}{\partial t\partial^2 y} + u\frac{\partial^3 u}{\partial x\partial^2 y} + v\frac{\partial^3 u}{\partial^3 y} + \frac{\partial u}{\partial x}\frac{\partial^2 u}{\partial y^2} - \frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial x\partial y}\right), \tag{2}$$

$$\left(\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right) = \alpha\frac{\partial^2 T}{\partial y^2} + \tau\left[D_B\frac{\partial N}{\partial y}\frac{\partial T}{\partial y} + \frac{D_T}{T_\infty}\left(\frac{\partial T}{\partial y}\right)^2\right], \tag{3}$$

$$\left(\frac{\partial N}{\partial t} + u\frac{\partial N}{\partial x} + v\frac{\partial N}{\partial y}\right) = D_B\frac{\partial^2 N}{\partial y^2} + \frac{D_T}{T_\infty}\frac{\partial^2 T}{\partial y^2}, \quad (4)$$

where u and v are the component of velocity along the x and y directions,ρ_f is the density of the fluid $K_o$is the coefficient of elastic velocity of fluid, T is the temperature of the fluid in the boundary layer, N is the nanoparticle volume fraction,$\rho_p$is the nanoparticle density, μ is the absolute viscosity of the base fluid, υ is the kinematic viscosity of the base fluid, $\alpha = k/(\rho C)_f$is the thermal diffusivity of the fluid, $\tau(=(\rho C)_p/(\rho C)_f)$ is the ratio of effective heat capacity of the nanoparticle material T_o heat capacity of the fluid, $D_{TC}$and $D_{CT}$are the Dufour and Soret type diffiusivity, $D_S$is the solutal diffiusivity, $D_B$and $D_T$are the Brownian diffusion coefficient and the thermophoresis diffusion coefficient, T_∞ the free stream temperature, $C_p$is the specific heat at constant pressure, and g, k are the accelerationdue to gravity, the thermal conductivity of the fluid respectively, $B_o$is the applied magnetic field along y direction, $k$is porosity parameter, and $\sigma$is the electrical conductivity.
The boundary and initial conditions imposed on Eqs. (1)–(4), are

$$\begin{cases} y = 0 & u = ax^m e^{\omega t}, & v = -amx^{m-1} - V_o e^{\omega t}, & t \geq 0, \\ y = 0, & T - T_\infty = T_o x^m e^{\omega t}, & & t \geq 0, \\ y = 0, & N - N_\infty = N_w x^m e^{\omega t}, & & t \geq 0, \\ y \to \infty, & u \to 0, & T \to T_\infty, & N \to N_\infty \end{cases} \quad (5)$$

where a, and ω are constants, V_o is the velocity condition at the surface, T_o isthe mean temperature of the surface, T_∞ is the temperature condition far awayfrom the surface, and m is the power law exponent, N_o is the mean nanoparticle volume fraction.
We introduce the following non-dimensional quantities

$$\begin{cases} x^* = \sqrt{\frac{a}{\nu}}x, \quad y^* = \sqrt{\frac{a}{\nu}}y, \quad t^* = at, \quad M^* = \sqrt{\frac{\sigma B_o^2}{a\rho_f}}, \quad f_\omega = \frac{V_o}{\sqrt{a\nu}}, \\ u^* = \frac{u}{\sqrt{a\nu}}, \quad v^* = \frac{v}{\sqrt{a\nu}}, \quad P = \frac{\rho C_p \nu}{\lambda}, \quad k^* = \frac{ak}{\nu}, \quad \alpha^* = \frac{\alpha}{\sqrt{a\nu}}, \\ \omega^* = \frac{\omega}{a}, \quad K_o^* = \frac{a}{\nu}, \quad N^* = \frac{(N - N_\infty)}{N_o}, \quad T^* = \frac{(T - T_\infty)}{T_o}, \end{cases}$$

where f_ω is the mass transfer, and P is the Prandtl number$k^*$is the porosity parameter,$M^*$is the Hartmann number.
The mass transfer parameter f_ω is positive for injection and negative for suction.

Invoking the non-dimensional quantities above, Eqs. (1)-(5), are reduced to the non-dimensional equations, dropping the asterisks for convenience,

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \quad (6)$$

$$\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = \frac{\partial^2 u}{\partial y^2} - M^2 u - \frac{1}{k}u,$$
$$-K_o\left(\frac{\partial^3 u}{\partial t \partial^2 y} + u\frac{\partial^3 u}{\partial x \partial^2 y} + v\frac{\partial^3 u}{\partial^3 y} + \frac{\partial u}{\partial x}\frac{\partial^2 u}{\partial y^2} - \frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial x \partial y}\right), \quad (7)$$

$$\left(\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right) = \frac{1}{P}\frac{\partial^2 T}{\partial y^2} + N_b\frac{\partial N}{\partial y}\frac{\partial T}{\partial y} + N_t\left(\frac{\partial T}{\partial y}\right)^2, \quad (8)$$

$$\left(\frac{\partial N}{\partial t} + u\frac{\partial N}{\partial x} + v\frac{\partial N}{\partial y}\right) = \frac{1}{L_e}\frac{\partial^2 N}{\partial y^2} + \frac{N_t}{L_e N_b}\frac{\partial^2 T}{\partial y^2}, \quad (9)$$

$L_e = \nu/D_B$is the Lewis number, $N_b = \tau D_B N_o/\nu$is Brownian motion parameter and $N_t = \tau D_T T_o/T_\infty \nu$is the thermophoresis parameter

Their corresponding boundary conditions are

$$\begin{cases} y = 0 & u = x^m e^{\omega t}, & v = -x^{m-1} e^{\omega t} f_\omega t \geq 0, \\ y = 0, & T = x^m e^{\omega t}, & t \geq 0, \\ y = 0, & N = x^m e^{\omega t}, & t \geq 0, \\ y \to \infty, & u \to 0, & T \to 0, \quad N \to 0 \end{cases} \quad (10)$$

III. The Method Of Successive Approximations

A process of successive approximations [35] will integrate the unsteady boundary layer equations (6)–(9). Selecting a system of coordinates which is at rest with respect to the plate and that the magnetohydrodynamic flow of fluid moves with respect to the plane surface, we can assume that the velocities u, v, the temperature T, and the nanoparticle volume fraction$N$ possess a series solutions of the form:

$$\begin{aligned} u(x,y,t) &= \sum_{i=0}^{\infty} u_i(x,y,t), & v(x,y,t) &= \sum_{i=0}^{\infty} v_i(x,y,t) \\ T(x,y,t) &= \sum_{i=0}^{\infty} T_i(x,y,t), & N(x,y,t) &= \sum_{i=0}^{\infty} N_i(x,y,t) \end{aligned} \tag{11}$$

where $u_i(x,y,t) = O(\varepsilon^i)$, i-integer and ε is a small number.
Each term in the series (11) must satisfy the boundary continuity equation (10)

$$\frac{\partial u_i}{\partial x} + \frac{\partial v_i}{\partial y} = 0, \quad i = 1,2. \tag{12}$$

Substituting the series (11) into Eqs. (7)-(9) and setting equal to zero terms of the same order, one obtains equations for finding components of the series (11)

$$\frac{\partial^2 u_o}{\partial y^2} - K_o \frac{\partial^3 u_o}{\partial t \partial^2 y} - \frac{\partial u_o}{\partial t} - M^2 u_o - \frac{1}{k} u_o = 0, \tag{13}$$

$$\frac{\partial^2 T_o}{\partial y^2} = P \frac{\partial T_o}{\partial t}, \tag{14}$$

$$\frac{\partial^2 N_o}{\partial y^2} + \frac{N_t}{N_b} \frac{\partial^2 T_o}{\partial y^2} - L_e \frac{\partial N_o}{\partial t} = 0, \tag{15}$$

$$\begin{aligned} \frac{\partial^2 u_1}{\partial y^2} - -K_o \frac{\partial^3 u_1}{\partial t \partial^2 y} - \frac{\partial u_1}{\partial t} - M^2 u_1 - \frac{1}{k} u_1 &= \left(u_o \frac{\partial u_o}{\partial x} + v_o \frac{\partial u_o}{\partial y} \right), \\ &+ K_o \left(u_o \frac{\partial^3 u_o}{\partial x \partial^2 y} + v_o \frac{\partial^3 u_o}{\partial^3 y} + \frac{\partial u_o}{\partial x} \frac{\partial^2 u_o}{\partial y^2} - \frac{\partial u_o}{\partial y} \frac{\partial^2 u_o}{\partial x \partial y} \right), \end{aligned} \tag{16}$$

$$\frac{\partial^2 T_1}{\partial y^2} - P \frac{\partial T_1}{\partial t} = P \left[u_o \frac{\partial T_o}{\partial x} + v_o \frac{\partial T_o}{\partial y} - N_B \frac{\partial N_o}{\partial y} \frac{\partial T_o}{\partial y} - N_t \left(\frac{\partial T_o}{\partial y} \right)^2 \right], \tag{17}$$

$$\frac{\partial^2 N_1}{\partial y^2} + \frac{N_t}{N_b} \frac{\partial^2 T_1}{\partial y^2} - L_e \frac{\partial N_1}{\partial t} = L_e \left[u_o \frac{\partial N_o}{\partial x} + v_o \frac{\partial N_o}{\partial y} \right], \tag{18}$$

Combining Eq. (11) and Eq. (10), we have the corresponding boundary conditions

$$\begin{cases} y = 0 & u_o = x^m e^{\omega t}, & u_i = 0, & i = 1,2,.. & t \geq 0, \\ y = 0 & v_o = x^{m-1} e^{\omega t} f_\omega, & v_i = 0, & i = 1,2,.. & t \geq 0, \\ y = 0, & T_o = x^m e^{\omega t}, & T_i = 0, & i = 1,2,.. & t \geq 0, \\ y = 0, & N_o = x^m e^{\omega t}, & N_i = 0, & i = 1,2,.. & t \geq 0, \\ y \to \infty, & u \to 0, & T \to 0, & & N \to 0. \end{cases} \tag{19}$$

In the following analysis, the first two terms in the series solution (11) will beretained. It is known fact [35] that such solution is satisfactory in the phases of the non-periodic motion after it has been started from rest (till the moment when the first separation of boundary layer occurs) and in the case of periodic motion when the amplitude of oscillation is small. Higher-order approximations u_3, can be obtained easy in principle. However, the complexity of the method of successive approximations increases rapidly as higher approximations are considered. It is also known that third and higher terms series solutions give small changes in the results compared with the two terms series solutions.

IV. Solution Of The Problem

Let us suppose that the exact solutions of the differential equations (13)-(15), is of the form:

$$u_o(x,y,t) = x^3 e^{\omega t} f_o'(y) \tag{20}$$

$$T_o(x,y,t) = x^2 e^{\omega t} \theta_o(y) \tag{21}$$

$$N_o(x,y,t) = x^2 e^{\omega t} \phi_o \tag{22}$$

Using Eq. (12)

$$v_o(x,y,t) = -3x^2 e^{\omega t} f_o(y), \tag{23}$$

from Eqs. (13)- (15) and using Eqs. (20)–(22) one obtains the differential equations of the unknown functions $f_o(y), \theta(y) and \phi(y)$ and the corresponding boundary conditions

$$f_o''' - k_1^2 f_o' = 0, \quad (24)$$

$$\theta_o'' - \omega P \theta_o = 0, \quad (25)$$

$$\phi'' - \omega L_e \phi = \frac{N_t}{N_b}\theta'', \quad (26)$$

$$\begin{cases} y = 0 & f_o = -f_\omega, \quad f_o' = 1, \quad t \geq 0, \\ y = 0, & \theta_o = 1, \quad \phi_o = 1, \quad t \geq 0 \\ y \to \infty, & f_o' \to 0, \quad \theta_o \to 0, \quad \phi_o \to 0 \end{cases} \quad (27)$$

where $k_1^2 = \frac{(\omega + M^2 + 1/k)}{1 - \omega K_o}$.

The solutions of the system (24)–(26) are of the form, taking into account the boundary conditions (27)

$$f_o(y) = \frac{1}{k_1}(1 - e^{-k_1 y}) - f_\omega \quad (28)$$

$$\theta_o(y) = e^{-\sqrt{\omega P}\, y} \quad (29)$$

$$\phi_o(y) = \ell_1 e^{-\sqrt{\omega L_e}\, y} + (1 - \ell_1) e^{-\sqrt{\omega P}\, y}, \quad (30)$$

Where $\ell_1 = \frac{N_b(P - L_e) - N_t}{N_b \omega (P - L_e)}$.

Assuming the solutions of the differential equation (16) is of the form

$$u_1(x, y, t) = x^5 e^{2\omega t} f_1'(y). \quad (31)$$

We can obtain an exact solution of equation (17) and (18) if we consider the case $m = 4$

$$T_1(x, y, t) = x^4 e^{2\omega t} \theta_1(y) \quad (32)$$

$$N_1(x, y, t) = x^4 e^{2\omega t} \phi_1 \quad (33)$$

and using Eqs. (31)–(33) one obtains from Eqs. (16)-(18) the differential equations for $f_1, \theta_1 and \phi_1$and the correspondingboundary conditions

$$f_1'' - k_3^2 f_1' = \frac{3}{1 - 2\omega}\left[f_o'^2 - f_o f_o'' + K_o(2 f_o' f_o''' - f_o f_o'''' - (f_o'')^2)\right], \quad (34)$$

$$\theta_1'' - 2\omega P \theta_1 = P[2 f_o' \theta_o - 3 f_o \theta_o' - N_B \phi_o' \theta_o' - N_t (\theta_o')^2], \quad (35)$$

$$\phi_1'' - 2\omega L_e \phi_1 = L_e[2 f_o' \phi_o - 3 f_o \phi_o'] - \frac{N_t}{N_b}\theta_1'', \quad (36)$$

where $k_3^2 = \frac{(2\omega + M^2 + 1/k)}{1 - 2\omega K_o}$ and $k_4^2 = \frac{2\omega L_e N_b}{N_b + N_t}$.

$$\begin{cases} y = 0 & f_1 = 0, \quad f_1' = 0, \quad t \geq 0, \\ y = 0, & \theta_1 = 0, \quad \phi_1 = 0, \quad t \geq 0 \\ y \to \infty, & f_1' \to 0, \quad \theta_1 \to 0, \quad \phi_1 \to 0 \end{cases} \quad (37)$$

Using Eqs. (28)-(30) one obtains the solutions of the system (34)–(37)

$$f_1'(y) = \ell_{11} e^{-k_3 y} + \ell_{12} e^{-k_1 y} + \ell_{13} e^{-2k_1 y} \quad (38)$$

$$\theta_1(y) = \ell_{21} e^{-\sqrt{2\omega P}\, y} + \ell_{22} e^{-\sqrt{\omega P}\, y} + \ell_{23} e^{-(k_1 + \sqrt{\omega P}) y} + \ell_{24} e^{-\sqrt{\omega (P + L_e)}\, y}, \quad (39)$$

$$\phi_1(y) = \ell_{31} e^{-\sqrt{2\omega L_e}\, y} + \ell_{32} e^{-\sqrt{\omega L_e}\, y} + \ell_{33} e^{-\sqrt{\omega P}\, y} + \ell_{34} e^{2\omega P} + \ell_{35} e^{-(k_1 + \sqrt{\omega L_e}) y} + \ell_{36} e^{-\sqrt{\omega (P + L_e)}\, y} + \ell_{37} e^{-(k_1 + \sqrt{\omega L_e}) y} \quad (40)$$

where

$$\ell_{11} = -(\ell_{12} + \ell_{13}), \ell_{12} = \frac{3(1 + k_1 f_\omega)[K_o k_1^2 - 1]}{(1 - 2\omega)(k_1^2 - k_3^2)}, \ell_{13} = \frac{6}{(1 - 2\omega)(4k_1^2 - k_3^2)}$$

$$\ell_{21} = -(\ell_{22} + \ell_{23} + \ell_{24}), \ell_{22} = \frac{\sqrt{\omega P}(1 - k_1 f_\omega)}{\omega k_1^2}, \ell_{23} = \frac{P\sqrt{\omega P}}{k_1(k_1^2 + 2k_1\sqrt{\omega P} - \omega P)},$$

$$\ell_{24} = \frac{P N_B \ell_1 \sqrt{P L_e}}{L_e - P}, \ell_{31} = -(\ell_{31} + \ell_{32} + \ell_{33} + \ell_{34} + \ell_{35} + \ell_{36} + \ell_{37})$$

$$\ell_{32} = \frac{3\ell_1 \sqrt{L_e}(k_1 f_\omega - 1)}{k_1 \sqrt{\omega}}, \ell_{33} = \frac{N_t P \ell_{22}}{N_b(2L_e - P)}, \ell_{34} = \frac{N_t P \ell_{21}}{N_b(L_e - P)} \ell_{35} = \frac{N_t \ell_{23}\left(k_1 + \sqrt{\omega P}\right)^2}{N_b(2\omega L_e - k_1 - \sqrt{\omega P})}, \ell_{36} = \frac{N_t \ell_{23}(P + L_e)}{N_b(L_e - P)},$$

$$\ell_{37} = \frac{\ell_1 L_e\left(2k_1 - 3\sqrt{\omega L_e}\right)}{k_1(2\omega L_e - k_1 - \sqrt{\omega L_e})}.$$

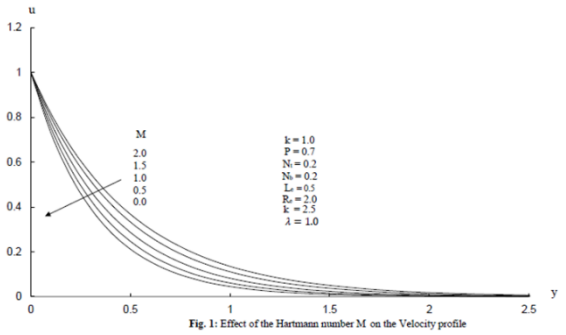

Fig. 1: Effect of the Hartmann number M on the Velocity profile

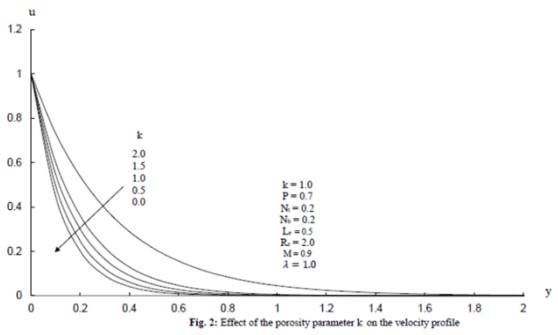

Fig. 2: Effect of the porosity parameter k on the velocity profile

V. Entropy Generation Analysis

For natural convection heat transfer in a fluid saturated porous medium, entropy generation rates due to heat transfer and fluid flow friction are expressed by the following expressions according to Bejan [36, 37]:

$$\dot{S}_{gen} = \dot{S}_T + \dot{S}_V + \dot{S}_C \quad (41)$$

$$\dot{S}_T = \frac{\kappa}{T_\infty^2}\left(\frac{\partial T}{\partial y}\right)^2 \quad (42)$$

$$\dot{S}_V = \frac{\mu}{T_\infty}\left(2\left(\frac{\partial u}{\partial x}\right)^2 + 2\left(\frac{\partial v}{\partial y}\right)^2 + \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)^2\right) + \frac{1}{T_\infty}\left(\sigma B^2 + \frac{\mu}{k}\right)u^2 \quad (43)$$

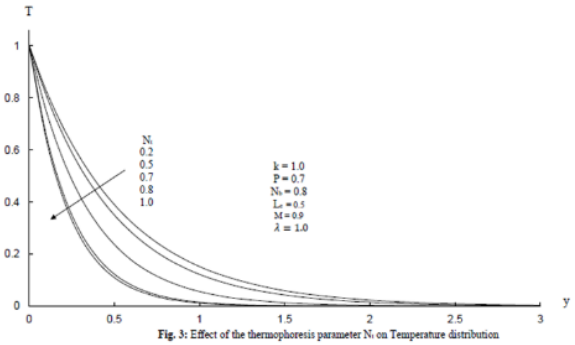

Fig. 3: Effect of the thermophoresis parameter Nt on Temperature distribution

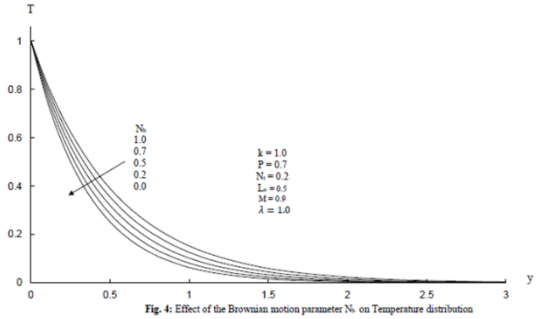

Fig. 4: Effect of the Brownian motion parameter Nb on Temperature distribution

$$\dot{S}_C = \frac{RD_B}{N_\infty}\left(\frac{\partial N}{\partial y}\right)^2 + \frac{RD_B}{T_\infty}\left(\frac{\partial T}{\partial y}\frac{\partial N}{\partial y} + \frac{\partial N}{\partial x}\frac{\partial T}{\partial x}\right), \quad (44)$$

The first term on the right hand side of Eq. (41) represents the entropy generation rate due to heat transfer irreversibility and is calculated by Eq. (42). The second term indicates the entropy generation rate due to viscous dissipation effects expressed by Eq.

(43). It's worthy to mention that the first term in Eq. (43) is important when the permeability of the medium is large and the flow tends to behave as a non-pours medium flow, while the second term in this equation is the entropy generation rate due to viscous effects for pours medium flows which is important for low speed flows. For intermediate the permeability of the medium values both of the mentioned terms in Eq. (43) are important and have to be considered for calculation of the viscous entropy generation rate. The last term in the right hand side of Eq. (41)indicates the entropy generation rate due to concentration gradients and is defined by Eq. (44).

Using the dimensionless variables defined in Section 2, Eq. (41) can be rewritten in the dimensionless form as follows:

$$\dot{S}_{gen} = Re\theta'^2 + \frac{R_e B_r}{\Omega}\left((M^2 + k)u^2 + u'^2\right) + R_e\lambda_1\left(\frac{\chi}{\Omega}\right)\left(\left(\frac{\chi}{\Omega}\right)N'^2 + \theta' N'\right) \quad (45)$$

where R_e and $B_r$are the Reynolds number, Reynolds number is used to check whether the flow is laminar or turbulent, and Brinkmann number, the Brinkman number is a dimensionless number related to heat conduction from a surface to a flowing viscoelastic fluid, respectively, Ωis thedimensionless temperature difference,R is the fluid constant,$\chi$and λ_1 are the constant parameters. These numberare given in the following form

$$R_e = \frac{aL^2}{\nu}, \quad B_r = \frac{\mu u^2}{\kappa T}, \quad \Omega = \frac{T_o - T_\infty}{T_\infty}, \quad \chi = \frac{N_o - N_\infty}{N_\infty}, \quad \lambda_1 = \frac{RD_B C_\infty}{\kappa},$$

L is the characteristic length, e.g. the diameter of the particle andu, θ and N are given by:

$$u = x^3 e^{\omega t}(u_o + x^2 e^{\omega t} u_1), \theta = x^2 e^{\omega t}(T_o + x^2 e^{\omega t} T_1)$$

$$N = x^2 e^{\omega t}(N_o + x^2 e^{\omega t} N_1)$$

VI. Results And Discussion

In this section, the numerical results of all the physical parameters of interest are plotted for velocity profile, temperature profile, nano-particle concentration and entropy generation. For this purpose Figs. 1, 2, 3, 4, 5, 6, 7, 8 and 9 aresketched for Hartmann number *(*M*)*, porous parameter *(*k*)*, Brownianmotion parameter (N_b), thermophoresis parameter(N_t), Lewis number (L_e), Reynolds number $(R_e)$and Brinkman number (B_r). The step sizeis chosen $y = 0.0001$ and $y_\infty = 3$ at their given boundary conditions. Furthermore.

Figures 1 and 2 are sketched for velocity profile. From Fig. 1 one can observe that dueto the influence of Hartmann number *(*M*)*, the velocity of the fluid accelerates, while thebehavior of velocity remains same when the porosity parameter *(*k*)* increases as shown inFig. 2. Figure 3is plotted for different values of thermophoresis parameter(N_t). From thisfigure we can scrutinize that due to the influence of thermophoresis parameter(N_t), temperature profile behaves as a decreasing function. It can be visualized from Fig. 4 that when the Brownian motion parameter N_b increases, then it helps to diminish the temperature profile.

Figures 5, 6 and 7 are drawn for nano-particle concentration. It is clear fromFig. 5 that with the

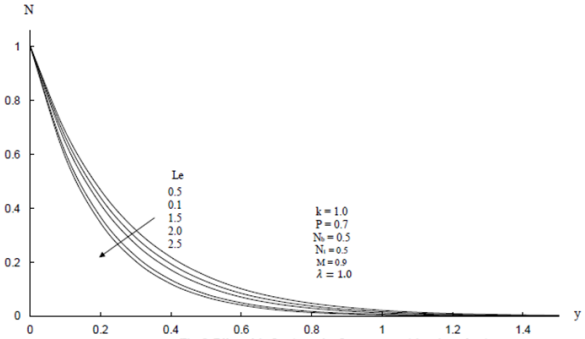

Fig. 5: Effect of the Lewis number L_e on nanoparticle volume fraction

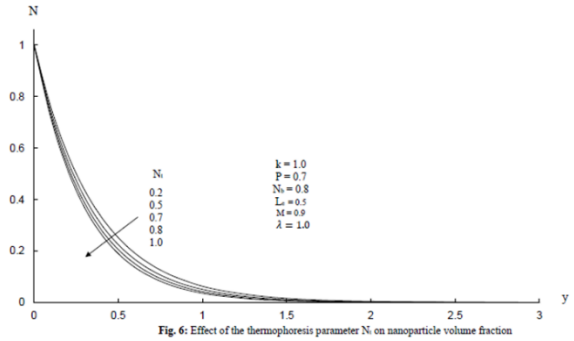

Fig. 6: Effect of the thermophoresis parameter N_t on nanoparticle volume fraction

increment in Lewis number (L_e), concentration profile decreases. It can be also observed from Figs. 6 and 7 that Brownian motion parameter and thermophoresisparameter depicts opposite behavior on concentration profile. The effect of Brinkman number (B_r)on entropy generation is illustrated in Fig. 8. It can be visualize from this figure that due to the increment in Brinkmannumber (B_r), the entropy generation number $(\dot{S_{gen}})$increases. It

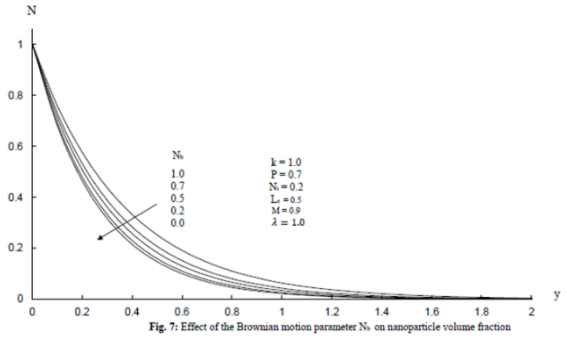

Fig. 7: Effect of the Brownian motion parameter N_b on nanoparticle volume fraction

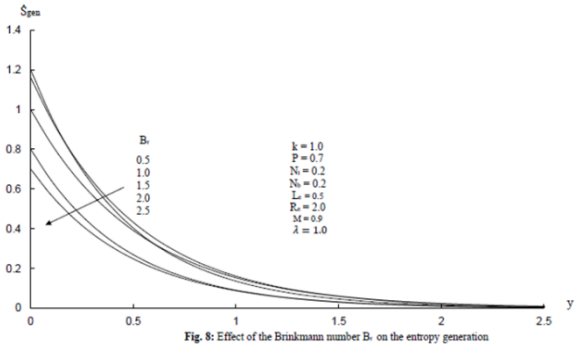

Fig. 8: Effect of the Brinkmann number B_r on the entropy generation

depicts from Fig. 9 that an increase in Reynoldsnumber (R_e), results higher entropy generation.

VII. Conclusion

In this article, MHD stagnation point flow of nanofluid over a permeable stretching surfacehas been investigated. The governing equations suchas momentum, energy, and nanoparticle concentration are reduced to ordinary differentialequations. The resulting highly coupled nonlinear ordinary differential equations are solvedusing successive approximation method (SAM).The major outcomes of the present analysis are summarized below:

- It is observed that due to influence of magnetic field, velocity profile increases.
- Brownian motion parameter and thermophoresis parameter shows similar behavior ontemperature profile.
- Nano-particle concentration profile decreases due to the increment in Lewis number.

- Entropy generation number increases due to the increment in Brinkmann number and Reynolds number.

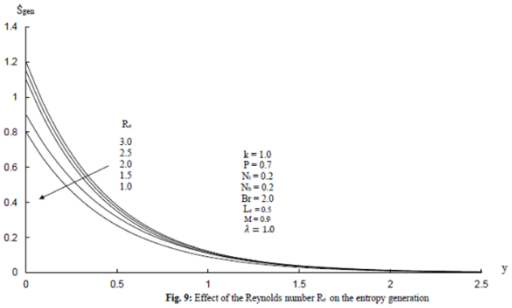

Fig. 9: Effect of the Reynolds number R_e on the entropy generation

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