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Gravitational acceleration inside the shell

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Abstract: *Most of the investigators were astonished that, why acceleration due to gravity is zero inside the shell and how it is to be proven? Out of curiosity it has been studied but could not get such a result. It was observed different results, which were more reasonable. In the present investigation, gravitational field inside a uniform spherical shell was equal and opposite from all side, which was observed only to observer located at center as well as near to inner surface. In these two points difference was that, at the center observer observed low gravity where as at inner surface it was high. Object located at center was stable but when it displaced from center, it started accelerating towards inner surface of the shell. We have proved geometrically that, object located between center and inner surface of the shell and forces acted on that surface of the object was different in opposite places which causes acceleration.*

Keyword: *Gravity inside shell, Acceleration, Classical Mechanics, Gravitation, Euclidean Geometry,*

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I. Introduction

The aim of the present investigation was to prove that, the object is accelerating inside uniform spherical shell from center to inner surface, as explained by Kolte [1]. Their explanation is useful to other various types of natural phenomena. Gravity inside the shell was calculated and proven by geometrical proof of Newton's so-called shell theorem, also known after the work by Chandrasekhar [2] as the 'superb theorem'. The theorem states that the spherical shell having uniformly distributed mass is generating a null gravitational field inside itself.[4 − 6, 9, 10] This theorem follows the theorem in Newton's Principia [3]. All those works being done since Newton's time are something missing or incomplete calculation and conclusion; even sir Isaac Newton also not confident about this ratio [7] i.e. mass and r^2 ratio for more than two object, where as explained here. Outside object accelerating towards center with the gravity exerted by mass of shell is increasing up to outer surface and further it becomes null[4 − 6, 9, 10], in universal low of gravitation[4], gravity is related to r; and r is precise from center of shell, so shell gravity must be increased up to its center. In the present investigation we have explained that vectorial contribution of gravitational field exerted by mass of shell is related to r [3 − 6] which is precise near inner surface of shell which becomes resultant r for sphere from its center.

II. Experimental

We have assumed spherical uniform mass distributed shell, point mass and solid angle same as shell theorem, superb theorem and theorem in Newton's *Principia* [3, 5, 6] We have taken point mass at shell center, checked mass and distance square ratio [3, 5, 6] for variable solid angel at its center; and then checked on X and Y axis with variable solid angle. In this experiment we have obtained variable mass and r^2 for fixed r^2 and mass, respectively, as well as point mass, object or observer taken in variable size which has helped us to clearly apply measurements, [3, 5, 6] (vector addition and cancellation) and evaluate variable ratio easily.

III. Results And Discussion

For the sake of clarity our analysis begins by calling more reliable result for inner gravity of shell and sphere. i.e., of the following

3.1: Proof 1: Consider a point mass at a position P_1 inside a spherical shell which is at its centre O. Shell radius is R as shown in Figure 1, which is uniformly distributed mass M. Consider a pair of infinitesimal surface elements, say dS_1 and dS_2 (thick continuous), that are seen from P_1 under the same solid angle $d\Omega$. To prove the theorem, it is sufficient to show that the vectorial contributions to the gravitational field at P_1 coming from the two elements are not equal and opposite, except gravitational field acting perpendicular to related Radius. At the center gravitational field is equal and opposite for all directions of any solid angle but towards shell for same solid angle, ratio becomes strong which causes acceleration.

We have to prove that, $\frac{dS_1}{r_1^2} < \frac{dS'_1}{r_1'^2}$

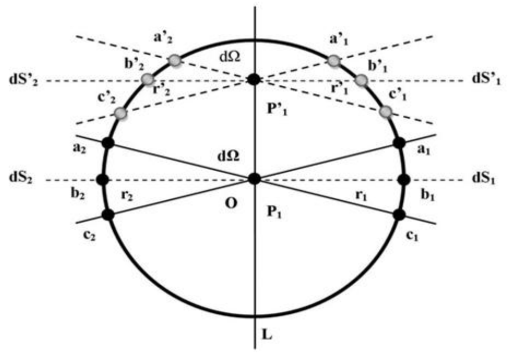

Figure1. Observation point inside the shell and Solid angle is below 2π sr.

Due to the uniform mass distribution on the sphere, same distance and same solid angle the module of such contributions turn out to be proportional to dS_1/r_1^2 and dS_2/r_2^2 correspondingly.
Where, r_1 and r_2 denote the distance from the elements to the observation point P_1.

In Figure1, P_1 is located at the center of shell so dS_1 and dS_2 are symmetric, equal mass, equal distance and gravitational field also equal and opposite; as well as by the internal symmetry (addition of vectors) its resultant force also equal and opposite. As resultant force becomes equal and opposite, object can't move anywhere (zero acceleration at center).

$$\therefore \quad \frac{dS_1}{r_1^2} = \frac{dS_2}{r_2^2} \quad \text{-----------------------------} \quad (1)$$

When point mass travel over the line L from P_1 to P'_1, gravitational force exerted by dS'_1 and dS'_2 are equal and opposite side due to its identical internal contribution.

$$\therefore \quad \frac{dS'_1}{r_1'^2} = \frac{dS'_2}{r_2'^2} \quad \text{-----------------------------} \quad (2)$$

Suppose that above ratio eqn. (1) and (2) remained constant due to decreasing mass as well as r, but it is not justifiable. Because solid angle dΩ may be variable and it will be 0 sr to 2π sr. At 0 sr, there is only one line and point mass on it. Here in same solid angle mass is same at P_1 and P'_1, so ratios are increasing with decreasing r. And other side at solid angle 2π sr, here is maximum mass of shell in calculation and result is as like above point mass, i.e. At P_1 and P'_1 mass is same, so equal and opposite forces increase with decreases in r. see fig. 02.

Here is $dS'_1 = dS_1$ and $r_1 > r'1$.

$$\therefore \left(\frac{dS_1}{r_1^2} = \frac{dS_2}{r_2^2}\right) < \left(\frac{dS'_1}{r_1'^2} = \frac{dS'_2}{r_2'^2}\right) \quad \text{-----------------------------} \quad (3)$$

$$\text{i.e} \quad \frac{dS_1}{r_1^2} < \frac{dS'_1}{r_1'^2} \quad \text{-----------------------------} \quad (4)$$

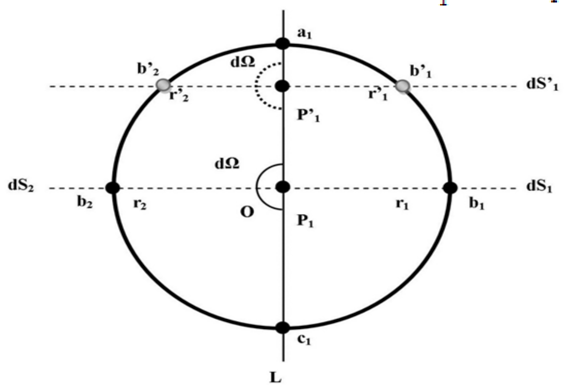

Figure2. Observation point inside the shell and Solid angle is 2π sr.

In Figure 2, at the center of shell there is equal and opposite force, that is called resultant force also. When point mass move towards shell, resultant force also moves toward shell, and equal and opposite forces become stronger. As addition of vector resultant force acts toward shell which cause acceleration. (See proof 2.)

3.2: Proof 2: As shown in Fig.3 consider uniform spherical shell having some amount thickness and radius R, as well as same thickness and same centered uniform spherical shell also called sphere, having radius r_1 is assumed here. Line or plane L passing through the center O divides both the shells in two equal parts A, B and a, b, respectively. Same as dS_1 and dS_2, that is seen from O under the same solid angle 2π sr. To prove the theorem, it is sufficient to show that the vectorial contributions to the gravitational field (resultant force) at observer coming from the two elements are not equal and opposite except center of shell.

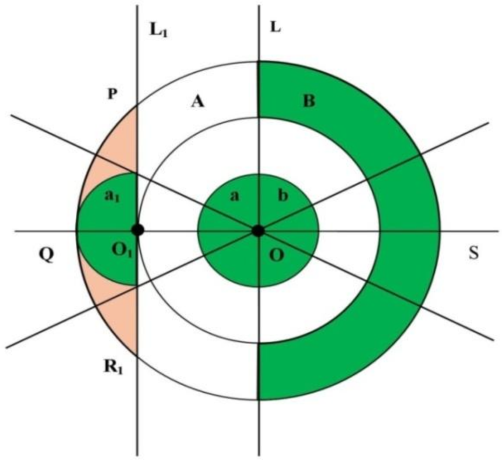

Figure3. Observation point is at center and inner surface of shell with Solid angle 2π sr.

We have to prove that, in same solid angle $\frac{\text{mass}}{r^2}$ ratio is increasing towards shell which causes acceleration. (Here r is the relative distance from mass.)

$$\text{i.e.} \quad \frac{\text{mass of shell segment } (PQR_1O_1P)}{\textit{Related } r^2} > \frac{\text{mass}(A)}{R^2}$$

By the similarity, ratio of A and B at center O is equal and opposite as well as ratio of a and b also.

$$\text{i.e.} \quad \frac{\text{mass}(A)}{R^2} = \frac{\text{mass}(B)}{R^2} \quad \text{------------------------------} \quad (5)$$

$$\frac{\text{mass}(a)}{r_1^2} = \frac{\text{mass}(b)}{r_1^2} \quad \text{------------------------------} \quad (6)$$

Point mass placed at center O observes gravity towards shell as following.

$$E(\text{mass } A) = G\frac{\text{mass}(A)}{R^2} \quad \text{------------------------------} \quad (7)$$

$$E(\text{mass } a) = G\frac{\text{mass}(a)}{r_1^2} \quad \text{--------------------------------} \quad (8)$$

Here, equation (7) and (8) are perfect examples of vectorial force imminent in solid angel $d\Omega$, i.e. mass/r^2 ratio of both is same which is explained in PROPOSITION LXX THEOREM XXX.

$$\therefore \left(\frac{\text{mass}(A)}{R^2}\right) = \left(\frac{\text{mass}(a)}{r_1^2}\right) \quad \text{------------------------------} \quad (9)$$

$$\therefore \left(E(\text{mass } A) = G\frac{\text{mass}(A)}{R^2}\right) = \left(E(\text{mass } a) = G\frac{\text{mass}(a)}{r_1^2}\right) \quad \text{-------------------------} \quad (10)$$

When point mass move from center O to O_1 (inner surface of shell), Line or plane L also move along with it. Mass separated by plane L is acting force on point mass at O_1, which is opposite to center O is as ratio of mass of segment PQR_1O_1P and its related r^2.

i.e. $$\mathbf{E}(\mathbf{PQR_1O_1P}) = G\frac{\text{mass of shell segment }(PQR_1\,O_1P)}{Related\ r^2} \text{ ------------------- (11)}$$

In figure 3, mass of segment PQR_1O_1P is divided into two parts where as a_1 and outside a_1. Part a_1 is same as part a by all means and by the similarity, ratio of mass/r^2 is same at O and O_1 respectively.

i.e. $$\frac{\text{mass }(a)}{r_1^2} = \frac{\text{mass}(a_1)}{r_1^2} \text{ ------------------------ (12)}$$

But from the equation (9) and (12)

$$\frac{\text{mass}(a_1)}{r_1^2} = \frac{\text{mass}(A)}{R^2} \text{ ----------------------------- (13)}$$

$$\therefore\ E(PQR_1O_1P) = E\left(G\frac{\text{mass}(a_1)}{r_1^2}\right) + E\left(G\frac{\text{mass of segment }(PQR_1\,O_1P)\text{ out side }a_1}{Related\ r^2}\right) \text{ ------- (14)}$$

Value of equation (13) putting in equation (14).

$$\therefore E(PQR_1O_1P) = E\left(G\frac{\text{mass}(A)}{R^2}\right) + E\left(G\frac{\text{mass of segment }(PQR_1\,O_1P)\text{ out side }a_1}{Related\ r^2}\right) \text{ ----------- (15)}$$

From the equation (15)

$$E(mass\ (PQR_1O_1P)) > E\ (mass(A)) \text{ ------------------------ (16)}$$

i.e. $$\frac{\text{mass of shell segment }(PQR_1O_1P)}{Related\ r^2} > \frac{\text{mass}(A)}{R^2} \text{ ------------------------- (17)}$$

From the equation (5) and (17) obtain that gravity exerted by mass of shell segment (PQR1O1P) at O1 and mass of (B) at O is different.

i.e. $$\frac{\text{mass of shell segment }(PQR_1O_1P)}{Related\ r^2} > \frac{\text{mass}(B)}{R^2} \text{ ---------------------------- (18)}$$

This states that when point mass moves from center to inner surface of shell, Gravity exerted by mass in same solid angle dΩ is variable and from equation (17) it is increasing toward shell.

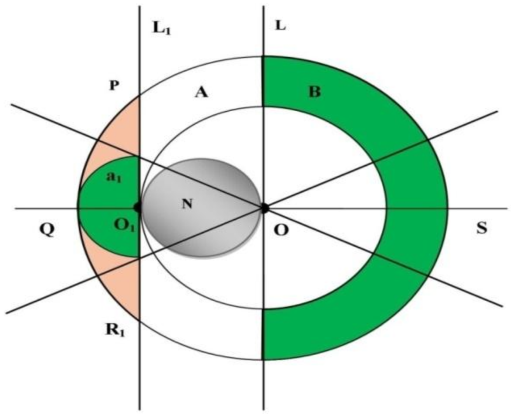

Figure 4: Object N between observation point O and O_1.

In Fig. 4 we have considered only shell and object 'N' located between O and O_1, force acting at O by half shell B, and force acting at O_1 by shell segment PQR_1O_1P in same solid angle is different, which is object N pulling opposite direction each other, see equation (18). Force acting by half shell B nulls with shell segment a_1 and mass outside a_1 exerted force which causes acceleration of the object N. Or by the difference between forces acting in opposite side shows object N in acceleration, of which direction is from center to inner surface of the shell.

IV. Conclusion

With the above proofs, acceleration due to gravity inside uniformly distributed mass of shell is not null but there is acceleration which increases from center to inner surface, whereas outer object also accelerates up to this place. Suppose outer object away from shell making solid angel Ω with shell mass and there gravity is as ratio of mass/r^2. When outer object move toward shell with same solid angel is observed that, as decreasing r, mass also decreases inside it. Decreasing mass which increase opposite side ratio but we are neglect it in routine equation (Newton's gravitational law) and considered only increasing ratio of that solid angel. (Same as inside the shell, object move from center, opposite side ratio also increase due to mass increase and we must follow above convention.) Shell theorem, superb theorem and theorem in Newton's Principia used concept of equal and opposite force from all side but here we have observed the following.

1. When point mass move on the radius from center to shell, both side (perpendicular to Radius) ratio are equal and opposite but it becomes stronger towards shell.
2. Force acting on point mass must be perpendicular to the related Radius otherwise equal and opposite force not possible.
3. The equal forces are acting only from symmetric places.
4. Forces acting perpendicular to Radius are equal and opposite but they are not resultant forces, resultant forces are equal and opposite (inside shell) only at shell center which keep point mass steady.
5. Foresees acting perpendicular to radius or r (line between center and point mass) is equal and opposite which is gravity level of sphere or shell, for outside as well as inside it, and object accelerate at gravity level difference.
6. This gravity level or equal and opposite forces become stronger up to inner surface of shell as well as center of sphere.

Sphere is just like a shell whose shell thickness is R and its acceleration due to gravity increases up to its inner surface which is center of that sphere. This result states that the maximum gravity of spherically symmetric massive body is at its center, it explain causes of why high temperature, high pressure and high density matter at center of Earth. Gravity increases with decreasing r and at that time solid angle Ω also increasing with relative mass, both these functions justify that high gravity is at center. This result also helps to find missing dark matter, unified theory (because strong force and gravity both are almost like.), explanation of various types of celestial motion and other phenomena, as well as more details about earth's gravity. Earth's gravity calculated with variable r and r is precise from center of earth but gravity equation were used for inside and outside of surface is in difference forms which is indicate that it was made for particular place. Gravity equation gives minimum error outside the sphere surface where as maximum towards center. All these things are thus clarified by this new result.

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References

[1] U. S. Kolte (*Adanyanche Vishwa – Pruthvichya kendrakade shunya gurutvakarshan?*Viraj Publication, India) (2017)
[2] S. Chandrasekhar *Newton's Principia for the Common Reader* (Oxford: Clarendon) (1995)
[3] I. Newton *Philosophiæ Naturalis Principia Mathematica* (Amsterdam: Sumptibus Societatis) (1723)
[4] Gravitation (Physics, Higher Secondary First Year Volume – I), Tamilnadu textbook corporation (2007)
[5] https://www.math.ksu.edu/~dbski/writings/shell.pdf
[6] https://en.wikipedia.org/wiki/Shell_theorem
[7] http://www.physics.arizona.edu/~varnes/Teaching/141Hspring2004/Notes/ Lecture37.pdf
[8] R. Borghi Eur. J. Phys. 35 02800 (2014)
[9] C. Schmid arXiv:1201.6534v1 [physics.class-ph] (2012)
[10] C. Schmid *Am. J. Phys.* 79 536–8 (2011)

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