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The Generalized Rienmanian Schrodinger Wave Equation for Hydrogen Atom

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Abstract: *It is well known that the Theoretical Physics from the time of Galileo to date was built upon the Euclidean geometry. In this paper we derive the Riemannian Schrodinger wave equation for the Hydrogen atom based upon the Great Metric Tensor*

Keywords*: Great Metric Tensor, Riemannian Schrodinger wave equation*

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I. Introduction

In about the last two centuries, the studies of mathematics and physics and other related sciences and technology have been founded on the Euclidean Geometry which came into existence in about 300 BC.The Theoretical Physics, built from the time of Galileo (1564-1642) till date has been built upon Euclidean geometry. The Newtonian and Einstein theories of gravitation were based mostly on the Euclidean geometry through the implication of Einstein. General Theory of Relativity is that physical space is Euclidean and Euclidean space is a good approximation for it only where the gravitationalfield is weak.Since the studies of most physical quantities have been done using the Euclidean geometry, there is the need to study these quantities using other geometries such as the Riemann geometry in which the gravitational field influence is taking into account.

II. Theory

It is well known that, the generalized great metric tensor in all gravitational fields in nature in Einstein coordinates came into existence in 2008 and are given as,[Howusu, 2010, 2013]

$$g_{11}(r,\theta,\emptyset,x^0) = (1+\frac{2}{c^2}f)^{-1} \quad (1)$$

$$g_{22}(r,\theta,\emptyset,x^0) = r^2 \quad (2)$$

$$g_{33}(r,\theta,\emptyset,x^0) = r^2 sin^2\theta \quad (3)$$

$$g_{00}(r,\theta,\emptyset,x^0) = -(1+\frac{2}{c^2}f) \quad (4)$$

$$g_{\mu\vartheta}(r,\theta,\emptyset,x^0) = 0\,;otherwise \quad (5)$$

Where f is the gravitational scalar potential and t coordinate time and

$$x^0 = ct \quad (6)$$

The generalized great metric tensor contains the phenomenon of gravitational space contraction for which there is experimental evidence. It also contains the phenomenon of gravitational time dilation for which there is also experimental evidence. It reduces to the pure Euclidean metric tensor in all space time without gravitational field in all orthogonal curvilinear coordinates. The generalized great metric tensor makes the three space parts of the Riemann's Tensorial Geodesic Equation of motion for particles of nonzero rest masses in gravitational fields in nature, in all orthogonal curvilinear coordinates to reduce to the corresponding pure Newton's equation of motion in the limit of c^0.

III. The Generalized Riemann's Laplacian Operator

The Riemannian Laplacian operator$\nabla_R^2$is defined by [Spiegel, 1974, Howusu, 2009, 2013] as

$$\nabla_R^2 \;=\frac{1}{\sqrt{g}}\frac{\partial}{\partial x^\alpha}\left\{\sqrt{g}g^{\mu\beta}\frac{\partial}{\partial x^\beta}\right\} \quad (7)$$

Where $g_{\mu\upsilon}$ is the metric tensor, g is its determinant.

In the Einstein spherical coordinates the generalized Riemann's Laplacian Operator is given as [Nyam et al, 2016]

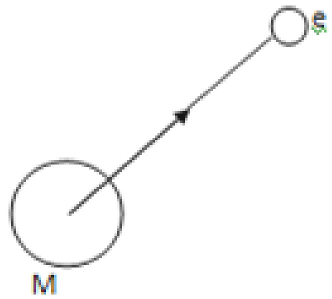
$$\nabla_R^2 = \frac{1}{r^2}\frac{\partial}{\partial r}\left[r^2\left(1+\frac{2}{c^2}f\right)\frac{\partial}{\partial r}\right] + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial r}\left[\sin\theta\frac{\partial}{\partial\theta}\right] + \frac{1}{r^2\sin^2\theta}\frac{\partial^2}{\partial\theta^2} - \frac{1}{v_p^2}\frac{\partial^2}{\partial t^2} \quad (8)$$

Where, $v_p = (1+\frac{2}{c^2}f)^{\frac{1}{2}}c$ (9)

The generalized Riemann's Laplacian Operator given by (8)satisfies the Principle of Equivalence of Physics, contains post Newton or pure Riemann's correction terms of all orders and predicts the existence ofgravitational, electrostatic and magnetic waves from some distributions of linearly accelerating mass with a velocity of propagation given by equation (9).

IV. The Generalized Riemann's Schrodinger Wave Equation

Consider the interaction between the hydrogen nucleus and an electron in a gravitational field as shown below



Electrostatic interaction of a hydrogen nucleus and an electron

The Euclidean Schrodinger wave equation for this interaction is given as [Martin, 1961]

$$\hat{H}\Psi = E\Psi \quad (10)$$

Where H is the Hamiltonian operator given by

$$\hat{H} = \frac{-\hbar^2}{2m}\nabla^2 + V \quad (11)$$

$\psi$is an energy wave function.

The Euclidean schrodinger wave equation in spherical coordinate is given explicityas [Onyenege, et al, 2017]

$$\left\{-\frac{1}{2}\left[\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\left(\sin\theta\frac{\partial^2}{\partial\phi^2}\right)\right] + V(r) - E\right\}\psi_{nlm}(r,\theta,\phi) = 0 \quad (12)$$

Whose solution is given by the wave function;

$$\psi_{nlm}(r,\theta,\phi) = r^{-1}R_n l(r)\psi_{lm}(\theta,\phi) \quad (13)$$

By replacing the Euclidean Laplacian∇^2in equation (11) with the Riemannian Laplacianwe get

$$\hat{H} = \frac{-\hbar^2}{2m}\nabla_R^2 + V \quad (14)$$

So that 14can be writing explicitly as

$$i\hbar\frac{\partial}{\partial t}\psi(x^\alpha, f) = \left\{\frac{-\hbar^2}{2m}\nabla_R^2 + V\right\}\psi(x^\alpha, f) \quad (15)$$

Substituting for the Riemannian Laplacian in (15) with some transformation and neglecting the time part we have

$$ER(r) = \frac{-\hbar^2}{2m}\left[\frac{\left(1-\frac{2k}{c^2R}\right)}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right)U(r)\right] + \frac{1}{r^2 sin\theta}\frac{\partial}{\partial\theta}\left(sin\theta\frac{\partial}{\partial\theta}\right) + \frac{1}{r^2 sin^2\theta}\frac{\partial^2}{\partial\phi^2} - \frac{E^2}{C^2\hbar^2} - U(r)\left(1-\frac{2k}{c^2R}\right)^{-1} - \frac{e^2}{r}U(r). \quad (16)$$

Equation 14 is the Generalized Riemann's Schrodinger wave equation.

Where U(r) is the transformed energy wave function.

V. Conclusion

Towards the realization of a complete migration from the well-known Euclidean Theoretical Physics [since 300BC], to the latest Riemann's Theoretical Physics, this study have been able to formulate a generalized Riemann's Schrodinger wave equation given by equation (16) and also the Generalized Riemann's Laplacian Operator in Einstein spherical coordinates and in all gravitational fields in nature which predicts the existence of gravitational, electrostatic and magnetic wave propagated by a velocity given by equation (9). It may be noted that in the limit of c^0, the Generalized Riemann's Schrodinger wave equation (16) reduce to the pure Euclidean.

References

[1] G.G.Nyam, D.I Jwanbot and M.M. Izam, 2016, GeneralisedLaplacian Operator for Hydrogen Atom Based upon Riemannian Geometry. International Journal of Basic and Applied Science, Vol 5 No 3 pp. 83-85

[2] Howusu, S.X.K., 2013, Riemannian Revolutions in Physics and Mathematics(Reprint) Jos University press Ltd, pp 69 – 106

[3] Howusu, S.X.K., 2010, The Great Metric Tensor in Orthogonal Curvilinear Coordinates, Jos University press Ltd pp1- 3

[4] Howusu, S.X.K., 2009, The Metric Tensors For Gravitational Fields and The Mathematical Principles of Riemannian Theoretical Physics, Jos University Press Ltd, pp 1 -44

[5] Martin Eisberg Robert, 1961, Fundamentals of Modern Physics, John wiley and sons, inc.pp 293 – 299.

[6] Chibueze P. Onyenegecha, Akpan N. Ikot, 2017, Analytical Solution of the Schrodinger Equation for the Ring-Shaped Multi-Parameter Expontial Type Potential, transaction of the Nigerian Association of Mathematical Physics, Vol 3, pp 223-232.

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