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Interaction between two closed-loop supply chains with game theory approach

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Abstract: *In this paper, try to present competition modeling and comparison between two three-level closed-loop supply chain that each of them includes one manufacturer, one retailer and one third part to collect used customer's products with considering concepts of game theory under two different scenarios include a collector for each chain and a collector common to both chains. In the forward supply chain manufacturer produces new products using the new parts or reproduced used products that have collected from consumers then these products wholesale to retailer and retailers also have to the consumer sells. In reverse supply chain two different scenarios considered for collecting used products to manufacturer.With using definitions and concepts of game theory to modeling these closed-loop supply chains in the form of Stackelberg game try to obtain the optimal value on wholesale price,retail price and optimal remanufacturing return rate of products for collector and the optimal value for the profit of each members of the chain with considering two supply chains competition. Finally, try to create a vision of managerial and economical behavior based on solving models by several numerical examples.*

Keywords: *Backward method, Closed-loop supply chain, Game theory, Two supply chains competition, backward method*

I. Introduction

In recent years due to environmental issues, law enforcement and economic benefits caused by reproduction activities, the attention has turned toward inverse logistic in form of closed ring supply chain, either in industry or in scientific research. A chain that is a combination of production and reproduction is called closed-loop supply chain. A closed-loop supply chain is simultaneously a forward and direct supply chain and an inverse supply chain. Inverse supply chain could be defined as logistic activities for recollecting used products used by customers for reproduction and its reusability in the market. A closed-ring inverse supply chain, in fact, could consist of supplier (whole-seller), distributor (retailer) and a part responsible to recollect the used products which all are interconnected with transportation, information and financial infrastructure. Reviewing supply chain literature, we find out that coordination between members of closed-loop supply chain improves the overall performance of a closed-loop supply chain. Now, in real world, there a plenty of other coordinated chains which should not be ignored regarding to its effect on the competition between chains. One can finds many examples of competition between two supply chains just by having a look at real world related problems. For instance, two closed-loop chains of automobile which compete against each other not only in group demands of customers domain but also in cost reduction of used products recollection and their reproduction domain. According to a report by Delvite Consulting Agency in 1999, "from now on, the competition between companies turns to be the competition between the supply chains".

Game theories has a wide use after its introduction in 1940 so that the economy Noble prize of 1994, 2002 and 2005 have been awarded to the activities alike and is one of the most developed topics in supply chain management, cooperation/coordination and competition between members of supply chain channel, hence, the game theories is one of the most practical tools in problem estimation of the supply chain management. The purpose of this study is to obtain the equilibrium wholesale and retail prices of products and optimal product returns rates for collector in closed-loop three-level chain that each contain a manufacturer (which is in addition to the production of a new product also remanufactures the product), retailer and the third party as collector of return product. These equilibrium points are calculated and compared under two different scenarios in a Stackelberg game where the manufacturer is the leader, so that each chain is a collection of separate or a joint collector of both chain. In the paper, we will review the literature in Section 2, problem and its assumptions are defined In Section 3, and mathematical modeling and calculating the equilibrium points will be reviewed under two aforementioned scenarios. Practicality of the model is examined through two numerical examples and computational results will be presented in section 4. Finally, section 5 is allocated to the conclusions and recommendations for future research.

II. Review of new papers in the various field of Supply Chain by Game Theory Approach

This paper [1] examine the issues of pricing strategies in relation to a manufacturer in a two-echelon supply chain including a manufacturer and two rivals retailer by taking warranty dependent on demand. Manufacturer as a leader in the Stackelberg game specifies wholesale prices for two competing retailers. Manufacturer chooses wholesale prices in the three following:

- Set the same wholesale price for both the retailer without taking into account the different retail price of each of them.
- Set the different wholesale price according to their retail price.
- Set the same wholesale price for both with respect to their average retail price in the market.

In this paper the optimum periods of retailers warranty and optimum benefit of each of them as well as the wholesale price of optimum profits of manufacturers with respect to the various pricing strategies with the use of the concepts of the Game theories is obtained.

The analysis showed that the results for retailers are the same, according to the first and third strategy. In addition reviews and compare the effects of different pricing strategies on production decisions and supply chain profit opportunities. It can be concluded that in the presence of symmetrical information of sale price; manufacturer adopt second strategy and otherwise, will adopt third strategy.

Durable products after sales service play an important role in customer purchase decisions. In this paper [2] manufacturer offer warranty to all customers while the retailer offer warranty only to customers who have paid the cost of facilities. Interactions between these two service programs are reviewed in the form of two types of customer. The first customer only uses the manufacturer's warranty by default and without charge and the second customer, in addition to the manufacturers' warranty, also uses extra after sales optional service of retailer which is provided by paying charge. In fact, the main objective of creating a balance between maximum profits obtained through competition between the retailer and manufacturer services program taking into account the increase in the level of customer satisfaction. Analytical modelling is provided in five modes:

- Nash equilibrium game model for both main parties regardless of any interactive chain between them,
- Determine the overall optimization when consider both chain members as an integrated.
- A Stackelberg game between a manufacturer and retailer based on the type of their services,
- The sensitivity of the price when there is power of choice over the services offered.
- Considering the interactions between two types of service programs offered.

Finally it has been concluded that warranty that is just offered to maximize profits would not lead to the optimum service levels to highest level of customer satisfaction.

In the paper, [3] a supply chain consists of a manufacturer and a retailer which manufacturer is intended to produce a single product and sells it through an exclusive retailer distributors. The framework of the supply chain in the form of a game theory model is presented for two common methods of selling and presentation. Manufacturer warranties product directly, but retailer warranties product indirectly. This paper is a comparison between the two decentralized models with a centralized system where the manufacturer as a unit is responsible for sale and providing warranty. It also examines the various causes of inefficiency in both decentralized modeland offers coordinating mechanisms to eliminate inefficiencies, for example, regulating contracts for the wholesale price paid to achieve harmony. Two decentralized system has been shown that when a retailer take longer warranty period, the system obtains more profit than the Warranty period of manufacturer decentralized system.

In this paper, [4] a model with a competitive price and service factors between the two decisions in order to obtain the optimal supply chain is provided under the potential demand. Each of these includes a supplier chain and a risk-averse retailer that offer the competition between the retailers to determine the retail price and optimal service and the competition between suppliers to determine the wholesale price under demand. This paper analyses the sensitivity influences of retailers on the players ' strategies, wholesale suppliers, the price performance of service and retail price. With evaluating the results show that the risk sensitivity of a retailer, lower service level and its less retail price occurs when the effects of the allergy risk competing on its decision is influenced by the interchangeability of the two products together. If the interchangeability is low optimum wholesale price of a supplier first increased and then reduced by two retailer risk sensitivity otherwise, it reduced by risk sensitivity. Higher service level performance for a retailer lead to lower the optimal retail price and the service level offered by his rival. Overall, retail profit is an increasing function of its expected profit, but it is a decreasing function of the uncertainty and risk sensitivity. Retailer is trying to maximize their profits than expected profit. In the end, the effect of retailers risk on the expected order number of his competitive and bargaining power.

In the paper [5] a distributed system consisting of one manufacturer and two independent retailers, with regard to competitive factors, service and retail prices are studied the results of the total market share, sales and profitability is examined with different strategies. Manufacturer will calculate wholesale price and retailers calculate retail price and optimal service level with regard to the concepts of cooperation and non-cooperative game theory of system. Results are compared.

In the paper [6] a manufacturer-retailer supply chain of a multi-product are investigated in an Stackelberg game under Two different scenarios where non-linear demand affect the price of each product and advertising cost. In the first scenario, manufacturer normally regarded as the leader of the game and in the second scenario retailer played a role as a dominant member of the chain. To get the optimum quantities of wholesale and retail prices, the cost of marketing and production policies bi-level mathematic programming approach have been used Due to the NP-Hard, several processes have been proposed to solve these issues, for example Imperialist Competitive Algorithm (ICA), and modifications, and evolution strategy. The results are compared with each other and finally numerical examples are presented to assess the effectiveness and efficiency of the model and solving method. In the first scenario the manufacturer as the leader controls production distance and wholesale price and retailer as a follower control the cost of marketing and retail price. Second scenario is like first scenario with the difference that retailer has the main power. In addition to the limitation of the budget available for the value and production and marketing of investment costs are considered. In this paper, [7] a supply chain consists of a manufacturer and a retailer with a taking back of the consumer purchase is examined and consumer return policy buying return policy of manufacturer in the form of a model, as well as a positive impact on the demand and potential negative impact of keeping product by the consumer is merged .The impact of repayment rates to customers are examined in the various contract models based on game theory.

III. Reverse supply chain and closed-loop chain

In recent years, attention to environmental issues, legal and economic benefits from recycling activities, raised attention to reverse logistics activities in a closed -loop supply chain - both in industry and in scientific research [8]. Chain which uses a combination of production and reproduction system is a closed loop supply chain [9]. A closed-loop supply chain consists of a forward supply chain and one reverse supply chain. Reverse supply chain can be defined as logistics activities to collect used goods from the customer to remanufacture make them usable in market [10]. Created costs of reverse logistics activities in the United States of America is more than $ 35 billion, and the cost of reproduction fee is $ 53 billion in the United States of America [11]. Undoubtedly the closed-loop supply chain has become a strategic issue. Actually it is an element that companies must consider in decisions making about the design and development process of their supply chain [12]. Nowadays concepts of deterministic and probabilistic models with different parameters in the closed-loop supply chain have been used in many research areas [13-17]. In this study, the input parameters are described as absolute values or specify probability distributions. Although these models show some of the general concepts of closed-loop supply chain behavior under different assumptions, but they were unable to describe the full range of conditions of real life.

Reconstruction is a process in which some components of the products used to separate again, clean, process, inspection, and then reassembled being used. Consumer awareness, views of non-governmental organizations, and regulatory requirements encourage manufacturers to produce green products and eco-friendly, and thus, more manufacturers are now creating reverse channels for recycled products used for remanufacturing. However, not only in terms of environmental concerns, but because of economic interests pursuant to reproduce, it is the main focus for manufacturers. Reduce costs associated with raw materials, in a production system not only an opportunity to reduce the environmental burden, but also provides lower production costs. According to a recent report by Global Industry Analysts (2010), global automotive production is growing, and by 2015, it is forecast to reach 104.8 billion dollars in America. High growth in regeneration also present in other industries, such as toner and inkjet cartridges, electrical equipment, consumer electronics, and furniture also exists (Hauser and Lund, 2008).

In practice, for manufacturers adjusting their sales strategies in response to the introduction of the reconstruction is important. For example, one of the largest PC manufacturer is HP now, that have considered the restructuring plan called "renewal program HP" for recycling and sales of products remanufactured or refurbished. The reconstruction program has certified that the Reproduced products have good performance and can be replaced by new products with lower prices, as well as warranty and service for these products is provided. Consumers should return the reproduced products or new products based on the information related to prices and service offerings to choose. As part of this business model, a comparative study is between price and service for new products versus reproduced products is useful for manufacturers and retailers that it must be provide from the valuable perspective of the interactions between price and service, decisions of sales and performance.

IV. Literature Review of closed-loop and reverse supply chain

All Previous studies have examined the remanufacture models that just consider remanufactured product without competitive environment, or competitive models just on factor. The paper [19] has been presented a combination of competitive prices and services in the nature of a supply chain model composed of one common retailer and manufacturer. A manufacturer engaged in the production of the new product, and then the second manufacturer produces the remanufactured product in a competitive environment. The model, analyzes the effects of competition and so we consider the following four competitive interactions: The presence of both price and service factors, only competitive factors, only competitive price factor and service competitive factors, competitive factors, as well as the absence of both price and service. Note that the interaction between both competitive prices and service factor can be used as general model that includes demand functions, the members of the chain, profit functions, and balance decisions for other interaction can be achieved from this competitive model.

Our analysis shows that when the reconstruction is leading to further cost savings, manufacturer provides to customers a higher level of services than traditional manufacturer. However, the cost of manufacturing always has a negative effect on the decisions of the chain. However, the effects of recovery costs and investment of services in the decision-making equilibrium strongly depends on price competitive factors and service, especially in determining the equilibrium's decisions of the manufacturer of new product. By comparing the different interactions we find that in the presence of competition, manufacturers generate an incentive for remanufacturer in order to product during the competition. In addition, for retailers, competitive factors of price generally increase profits from reproduced product sales that this to remanufacturer advantage as well. Meanwhile, competitive factor service is profitable for retailers, while detrimental to both the manufacturers. However, remanufacturer competes for service when the cost savings of restructuring is significant or recovery cost is low. In addition, the reconstruction of an effective strategy is very sensitive to market prices, even in circumstances where there are price competitive factor.

The paper [18] ,provides a fuzzy model of closed-loop supply chain with a manufacturer, a retailer and a third sector provider and consumer demand, restructuring costs, and costs of product collection has been used as a fuzzy have been considered .the retailer sells them to consumers. Manufacturer In the forward supply chain, produce new products, using the original components or rebuilding the products that is collected from consumers in the end, and new products are to be sold to a major retailer that then their retailer agreement sells them to consumers. There are three different modes for the collection of used products For the reverse supply chain in the paper, namely, manufacturer collects used products directly from the customer (MC), manufacturer by a contract will allow a retailer to collect used products (RC), and the manufacturer let a contract to a third party (TPC). Using the concepts of game theory in the form of a Stackelberg game in which manufacturer plays role as leader and the retailer or third sector as follower and integrating it with the theory of fuzzy, the results obtained from the above three methods have been used in the case of product collected and analyze the decisions of the manufacturer, the retailer and the third-party liability, as well as check and calculate the expected benefit of them. In addition, some studies also is given using management perspectives number.

Variable cost of production per unit of product: $\mathrm{C_m}$
Fuzzy variable reworking cost per unit of product: $\widetilde{\mathrm{C_r}}$
fuzzy cost of collecting every unit of product: $\widetilde{\mathrm{P_c}}$
Handling costs per unit collected product to the manufacturer: $\mathrm{P_b}$ ($\mathrm{P_b} \geq \widetilde{\mathrm{P_c}}$)
Wholesale price of manufacturer: W
Retail Price of retailer: P
The amount of returned products that are capable of working again: τ ($0 \leq \tau \leq 1$)
Linear demand function: $D(P) = \tilde{\alpha} - \tilde{\beta} \times P$
The total cost of the collection: $\mathrm{C}(\tau) = \mathrm{K} \times \tau^2 + \widetilde{\mathrm{P_c}} \times \tau \times \mathrm{D(P)}$

Decision by the MC:

In this model, the manufacturer as leader identifies the optimal wholesale price and optimal value τ and then retailer as the follower identifies the retail price in the form of a Stackelberg game.

Retailer profit function:

$$\mathrm{E}[\Pi_{\mathrm{R}}^{\mathrm{MC}}(\mathrm{P})] = (\mathrm{P} - \mathrm{W}) \times (\mathrm{E}(\tilde{\alpha}) - \mathrm{E}(\tilde{\beta}) \times \mathrm{P})$$

Manufacturer profit function:

$$\mathrm{E}[\Pi_{\mathrm{M}}^{\mathrm{MC}}(\mathrm{W}, \tau)] = \mathrm{E}\left[\left(\mathrm{W} + \left(\mathrm{C_m} - \widetilde{\mathrm{C_r}}\right)\tau - \mathrm{C_m} - \widetilde{\mathrm{P_c}}\tau\right)(\mathrm{D(P)})\right] - \mathrm{K}\tau^2$$

Decision by the RC:

In this model, the manufacturer as leader identifies the optimal wholesale price and then retailer as the follower identifies the retail price and optimal value τ in the form of a Stackelberg game.

Retailer profit function:

$$E[\Pi_R^{RC}(P,\tau)] = E[(P-W)\times D(P) - K\tau^2 - (\widetilde{P_c} - P_b)\times \tau \times D(P)]$$

Manufacturer profit function:

$$E[\Pi_M^{RC}(W)] = E\left[\left(W + \left(C_m - \widetilde{C_r}\right)\tau - C_m - P_b \times \tau\right)(D(P))\right] - K\tau^2$$

Decisions by the TPC:

First the manufacturer identifies wholesale price, then retailer the retail price, and the third part optimal value of τ.

Retailer profit function:

$$E[\Pi_R^{TPC}(P) = E[(P-W)\times D(P)]$$

Part of profit function of third part:

$$E[\Pi_T^{TPC}(\tau)] = E[(P_b - \widetilde{P_c})\times \tau \times D(P) - K\tau^2]$$

Manufacturer profit function:

$$E[\Pi_M^{TPC}(W)] = E\left[\left(W + \left(C_m - \widetilde{C_r}\right)\tau - C_m - P_b \times \tau\right)(D(P))\right]$$

In the paper [19] a supply chain includes two manufacturer's type and a common retailer was considered. The first manufacturer is a traditional one that produces new components of the product while the second manufacturer produce and remanufacture the goods consumed through a reverse channel. Both manufacturers provide their products with warranty and advertising services and sell via a common retailer. Manufacturers determine the optimal service level and retailers determine the optimal retail prices of products.

This paper identifies and reviews the characteristics of the balance and each profit of the members of the chain in the form of the theory of the games by taking the services competitive factors and pricing and according to the computational analysis, a vision and management and economic results is expressed for the members of the chain.

Competition between manufacturers is considered in four states;

Competition between the services provided by the manufacturers;

Competition between the prices offered by manufacturers;

Competition between prices and offered service by the manufacturers;

Lack of consideration of any competitive factor;

Competitive equilibrium price and service factors may influence decisions regarding the recovery of costs and service fees for the manufacturers. The amount of income -producing retail price competition and lower manufacturing cost increases due to the product being remanufactured. On the other hand whatever cost savings by remanufacturer increases causes a higher service level than the first manufacturer to be provided to the consumer.

$0 \le \tau \le 1$: part of the materials used that are capable of working again.
S_j: level of j th manufacturer service.
$j,k=\{M,RM\}, j \neq k$
W_j: Wholesale price of manufacturer j.
P_j : Retail price of product j.
D_j: market demand for product j.
α_j: potential size of market demand for product j.
$\gamma_s,\gamma_p,\beta_s,B_p \ge 0$: positive coefficients.
C: variable cost per unit product production.
C_r: variable cost per unit of rework production ($\delta = C - Cr \ge 0$).

Considering the demand function in terms of price and level of service as a linear function:

$$D_j = \alpha_j - \beta_p \times P_j + \gamma_p \times \left(P_k - P_j\right) + \beta_s \times S_j - \gamma_s(S_k - S_j)$$

$\frac{m \times S_j{}^2}{2}$: Service level cost
$C \times D_j$: Manufacturing cost

$\frac{b\times\tau^2}{2}$: remanufacturing cost
First manufacturer's profit function:

$$\Pi_M = (W_M - C)\times D_M - \frac{m\times {S_M}^2}{2}$$

Second manufacturer's profit function:

$$\Pi_{RM} = (W_{RM} - C + \delta\times\tau)\times D_{RM} - \frac{m\times {S_{RM}}^2}{2} - \frac{b\times\tau^2}{2}$$

Retailer's profit function:

The purpose of this paper [20] is pricing decisions about collection of products used in the reverse supply chain based on various scenarios and compares them with each other in terms of models based on game theory. This paper considers a recycling network includes collecting operation and processing stages of the product in which the manufacturer do both operations or a third part to the collecting and procedures do processing stages or the retailer to collect and procedures do processing stages or the third part both collect and processing operations. The cost of processing a used product will increase the collecting cost function which has great influence on the willingness of the final consumer. The manufacturer, by checking price decision with the different items prefers to its own do processing operations and the third part joins to reverse chain to have deeper cooperation means not only collect but also do processing steps.

The cost of processing a used good increases the cost of collecting function. This function has a significant impact on consumer sentiment final. By examining pricing decisions for various cases, manufacturer prefers to do processing operations and remanufacturing due to higher profits rather than concede to others. And the third part adds to reverse chain with the hope to have a deeper cooperation means that not only collected but also do processing. In this study [21], a closed-loop supply chain including a manufacturer, retailer, and a third party is considered that in its reverse chain used products in reverse supply chain are collected from end-customers during various scenarios. Hence the rate of return used products is influenced by consumer sentiment to consume and this sentiment is influenced by the cost of collecting. In the forward supply chain, wholesale optimal price and optimal retail price is affected by the costs of the collection. This paper focuses on the management costs of collecting, wholesale prices and retail prices for a closed loop supply chain. Under this assumption, the rate of products usable return of an ascending function by price per unit of product in the collection has been taken. Optimal cost of collecting, wholesale price and retail price is calculated according to the following three models: the model the manufacturer directly assembles products from consumer (CMRM), model that retailer collects the goods (CRMRC) and the model that a third part has the contractual obligation to collect it (CTMRC). These model forms will be analyzed and compared ina Stackelberg game that manufacturer is the leader. Comparing The optimum prices and overall system profits in each of the models it can be conclude that its manufacturer prefers to collect the product himself and it is better for him because of deleting collecting cost. Finally this paper presented a numerical example for accuracy and optimum results.

The paper [22] examines the interaction of a closed-loop supply chain including a manufacturer and retailer. Reverse channel of manufacturers is to collect and send the goods and strategic decisions in the product pricing in the forward channel is adopted when retailers are competing with each other .two models has been offering in one, the products consumed is directly collected by the manufacturer and in another indirectly by other retailers. Either of the two models in both centralized (direct centralized (CD) and centralized indirect (CI)) and decentralized (decentralized direct (DD) are examined. Also a model in which the gathering consumer products is not the case (NR), is also presented and discussed at the end of the model results are compared with each other . In the case of indirect acquisitions, competition profits will be divided between retailers. As a result, the impact of a reverse chain network on wholesale prices and retailers' competitive behavior is characterized.

Formulation of the models is presented in the following format:
The entire chain in focus: C
Manufacturer: M
First and second retailer: R1, R2
i={C, M, R1,R2}&k={NR,CD,CI,DD,DI}&j={1,2}
The variable cost of producing one unit of the new product: C_m
Variable cost per unit of product rework: C_r ‘ $(C_m - C_r - \Delta \geq 0)$
Profit function for the member ith in kth model : Π_i^k
The percentage of returned products that have the ability to rework: τ
Wholesale price Manufacturer: W
Retail price of retailer j th: P_{R_j}

Back of the purchase cost per unit: b
Total cost of the collection: $C(\tau)=B\times\tau^2$
The potential size of market demand: ϕ
Interactive tensile modulus of the price: β

Linear function of demand:

$$D_j\left(P_{R_j}, P_{R_{3-j}}\right) = \phi_j - P_{R_j} + \beta \times P_{R_{3-j}} \quad , \quad \text{s.t. } 0 \le \beta \le 1 \,, \quad j = 1,2$$

ModelCD:

$$\max_{P_{R_1},P_{R_2},\tau} \Pi_C^{CD} = (P_{R_1} - C_m + \Delta\tau)\times(\phi_1 - P_{R_1} + \beta \times P_{R_2}) + (P_{R_2} - C_m + \Delta\tau)\times(\phi_2 - P_{R_2} + \beta \times P_{R_1}) - B\tau 2$$

ModelCI:

$$\max_{P_{R_1},P_{R_2},\tau_{R_1},\tau_{R_2}} \Pi_C^{CI} = (P_{R_1} - C_m + \Delta\tau_{R_1})\times(\phi_1 - P_{R_1} + \beta \times P_{R_2}) + (P_{R_2} - C_m + \Delta\tau_{R_2})\times(\phi_2 - P_{R_2} + \beta \times P_{R_1}) - B\tau_{R_1}2 - B\tau_{R_2}2$$

ModelDD:

$$\max_{P_{R_j}} \Pi_{R_j}^{DD} = (P_{R_j} - W)\times(\phi_j - P_{R_j} + \beta \times P_{3-j}) \qquad j=1,2$$

$$\max_{W,\tau} \Pi_M^{DD} = [D_1(W) + D_2(W)]\times(W - C_m + \Delta\tau) - B\tau 2$$

ModelDI:

$$\max_{P_{R_j},\tau_{R_j}} \Pi_{R_j}^{DI} = (P_{R_j} - W + b\tau_{R_j})\times(\phi_j - P_{R_j} + \beta \times P_{3-j}) - B(\tau_{R_j})2 \qquad j=1,2$$

$$\max_{W} \Pi_M^{DI} = D_1(W,b)[W - C_m + (\Delta - b)\tau_{R_1}(W,b)] + D_2(W,b)[W - C_m + (\Delta - b)\tau_{R_2}(W,b)$$

In terms of retailers, direct Collection system, is preferred due to lack of fees collected by the manufacturer of the product as well as lower wholesale prices. In indirect Collection system manufacturer has not Collection costs and competition between retailers results in lower profit margins, retailers, and retail price reduction. That is why this system due to the increase in the total benefit of the manufacturer, is preferred by manufacturer. In the paper [23], the optimal price decision in a fuzzy closed-loop supply chain with retail competition is considered. Customer demands, costs and fees collected for a Fuzzy is reconsidered. Using game theory and fuzzy theory, the optimal decision on wholesale price, retail price and rework rates are taken under scenarios of centralized and decentralized. The closed-loop supply chain consisting of one manufacturer and two competing retailers which manufacturer will collect used goods. Manufacturer will determine optimal wholesale price and optimal return rates of used product and retailers determined the optimal retail price under the model by the theory-based games.

Description Model:
Variable cost of producing one unit of the new product: C_m
Fuzzy variable reworking cost per unit of product: $\widetilde{C_r}$
fuzzy cost of collecting per unit of product: $\widetilde{P_c}$
Wholesale price of manufacturer: W
Retail price of retailer ith: P_i
Percentage of returned products that have the ability to rework: τ
Total cost of collecting: $C(\tau) = K\times\tau^2$

Linear function of demand:

$$D_i(P_i, P_j) = \widetilde{\alpha_i} - \widetilde{\beta_1} \times P_i + \widetilde{\beta_2} \times P_j \qquad \text{s.t. } i = 1,2 \,, \; j = 3 - i$$

Decentralized model:

$$\max_{W,\tau} E[\Pi_m(W,\tau)] = \max_{W,\tau} E\left[\left(W + \left(C_m - \widetilde{C_r}\right)\tau - C_m - \widetilde{P_c}\tau\right)(D_1(P_1,P_2) + D_2(P_2,P_1))\right] - K\tau^2$$

$$\max_{P_i} E\left[\Pi_{R_i}(P_i)\right] = \max_{P_i} E\left[(P_i - W)D_i\left(P_i,P_j\right)\right]$$

Centralized model:

$$\max_{P_1,P_2,\tau} E = \max_{P_1,P_2,\tau} E\left[P_1 \times D_1(P_1,P_2) + P_2 \times D_2 + \left((C_m - \widetilde{C_r})\tau - C_m - \widetilde{P_c}\tau\right)(D_1(P_1,P_2) + D_2(P_2,P_1))\right] - K\tau^2$$

After computing the optimal values of both models we know that the total expected profit in the centralized system is more than decentralized mode, the rate of collecting used goods in concentrated mode is more than decentralized optimize form , and the price petty sales under the centralized state is less than under decentralized mode. Table 2-4 will discuss for an overview of several papers in the field of closed-loop supply chain by game theory and next section with regard based on the vacuum in the field will provide a mathematical modeling. According to the literature review have been presented in this chapter, the majority of this research focuses on competition between members of closed-loop supply chain and ignore the impact of competition of other chains. It seems that there is the gap in the use of game theory in closed-loop supply chain.

V. Defining the problem

In this paper, two closed-loop three-level supply chain consists of a manufacturer and a retailer and a third section compete to collect the products used under two different Scenarios over the wholesale price and the retail value of the product used for optimal return coefficient of collector. Inthe competition, manufacturer in addition to the product attempted to remanufacture the product as well. Manufacturers sell their product, with the wholesale price to the retailer, and the retailer also sells the same product with the customer's retail price. Reversely, the third sectors also collect used products and send it to the manufacturer to remanufacture. To consider the competition between the two chains, two closed-loop chains treated under different scenarios. In the first scenario, each of the links has a separate poster collection and in the second one there is a in a common collector to collect the two chains. In the form of a Stackelberg game in which manufacturer is leader and retailer and collector have a follower role, modeling and balance points are obtained and compared with each other.

5.1 The assumptions

- Both chains produce the same product with the same quality.
- Product Quality is the same as remanufactured one.
- The product is considered a single-period, ie game is single- stage and is not intended to duplicate.
- Chain members have symmetric information.
- Chain members have reasonable and common risk and their behaviors are wise and informed.
- Linear demand function is considered linear and definitive.
- Reproduction cost of used product is less than the cost of producing each unit of product. (Profit for reproduction)
- The cost of collecting each used product unit is less than the cost of delivering a product to manufacturer. (Profit for collectors)

5.2 Definition of Symbols, Variables and Parameters

To facilitate index M, R are used to represent the manufacturer, retailer and t, T collectors, respectively.

5.3 Variables

D_i Product demand for i th chain i, i, j = 1,2 ($i \neq j$)

Π_{R_i} Retailer profit for i th chain

Π_{t_i} Collector profits for the i th chain

Π_{M_i} Manufacturer profit for the ith chain

5.4 Parameters

α_i : Market potential size of product in I th chain

β : stretching factor of product prices ($0 \leq \beta \leq 1$)

θ : Price competition coefficient ($0 \leq \theta \leq 1, \beta \geq \theta$)

P_i : Retail price of product for ith chain

W_i : Wholesale price of product for ith chain

P_{b_i} : Delivering unit product cost from collector to the manufacturer for ith chain,

P_{c_i}: Collecting unit product costs by a collector for ith chain ($P_{b_i} \geq P_{c_i}$)

τ_i: Capability Rate of used product by collectors for ith chain, $(0 \leq \tau_i \leq 1)$
b : fixed Parameter of collection fee
C_{m_i} : Production cost per unit of product for the ith chain
C_{r_i} : Reproduction cost per unit of product for the ith chain $(C_{m_i} \geq C_{r_i})$

VI. Mathematical modeling of the problem and how to calculate the equilibrium points

Demand function used in this study, has linear and deterministic property and increase its price lower than the competitive price. The more price difference between the two products, the less the product demand of a chain are affected by price level of other chains. Demand function used in this study is the same as demand function in (Wei & Zhao, 2011) and is as follows:

$$D_i = \alpha_i - \beta . P_i + \theta . P_j \ ; \quad i, j = 1,2 \ \ (i \neq j)(1)$$

According to the paper [18] the total collection cost of a product C (τ) is shown $C(\tau) = b.\tau^2 + P_c.\tau.D_i$ where $b.\tau^2$ represents the fixed fee collection used product by the customer and $P_c.\tau.D_i$ represents variable costs of collection and $\tau.D_i$ also represents the total number of used collected products. According to the material presented, the model will be discussed under the following two scenarios: First scenario: Each of the two chains has separated collection, as shown schematically in Fig. 1.

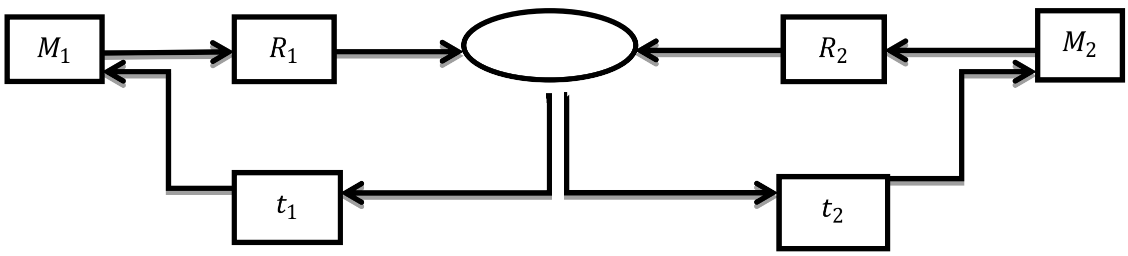

Fig. 1: Schematic of the two chains compete under the first scenario

According to the above model and calculate the total cost of harvesting used in accordance with $C(\tau_i) = b.\tau_i{}^2 + P_{c_i}.\tau_i.D_i$ each member profits functions are defined as follows :

$$\Pi_{R_i} = (P_i - W_i).D_i ; \quad i, j = 1,2 \ \ (i \neq j) \ \ (2)$$

$$\Pi_{t_i} = (P_{b_i} - P_{c_i}).\tau_i.D_i - b.\tau_i{}^2 \ ; \quad i, j = 1,2 \ \ (i \neq j) \quad (3)$$

$$\Pi_{M_i} = [W_i - (1-\tau_i).C_{m_i} - (C_{r_i} + P_{b_i}).\tau_i].D_i \quad ; \ i, j = 1,2 \ \ (i \neq j) \quad (4)$$

For ease of calculation and further analysis of the problem, if get $\Omega_i = P_{b_i} - P_{c_i}$ as the amount of revenue collected for the chain ith have been collected from manufacturer due to each unit of used production and $\Delta_i = C_{m_i} - C_{r_i}$ get as the amount of profit that the manufacturer gain for chain i, for every used recurring single product reproduction, Then, the profit function of collector and manufacturers, will be as follows:

$$\Pi_{t_i} = \Omega_i.\tau_i.D_i - b.\tau_i{}^2 \ ; \quad i, j = 1,2 \ \ (i \neq j) \quad (5)$$

$$\Pi_{M_i} = [W_i - C_{m_i} + (\Delta_i - P_{b_i}).\tau_i].D_i \ ; \ i, j = 1,2 \ \ (i \neq j) \ \ (6)$$

It is clear that manufacturer profits for reproduction per unit of used product returns must be higher than the cost per unit of product collected, in order to practice of reproduction be profitable for manufacturers$(\Delta_i \geq P_{b_i})$. In the model, first, manufacturers as leaders in Stackelberg game identify wholesale price W_i^* that maximizes its profit function and then according to the wholesale price of each; retailers and collectors as followers assign the optimal retail price P_i^* and return the product used τ_i^*. To this end, considering the optimal wholesale price and retail sales and collectors clear response functions, backward method is used. How to get the optimal point considering out of the chain competition of chain members is important and is the main innovation of this thesis.

Theorem) given the best response function of the retailer, the optimal retail price is as follows:

$$P_i^* = \frac{2\beta(\beta W_i+\alpha_i)+\theta(\beta W_j+\alpha_j)}{4\beta^2-\theta^2} \quad ; \qquad i,j=1,2 \ \ (i \neq j) \quad (7)$$

proof) by taking the first derivative of the profit function of the retailer than Pi and putting it equal to zero, the optimal retail price and retailer 's best response function is obtained as follows:

$$\frac{\partial \Pi_{R_i}}{\partial P_i} = 0(8)$$

$$\alpha_i - \beta P_i + \theta P_j - (P_i - W_i)\beta = 0 \ \Rightarrow \ P_i^* = \frac{2\beta(\beta W_i+\alpha_i)+\theta(\beta W_j+\alpha_j)}{4\beta^2-\theta^2}; \qquad i,j=1,2 \ \ (i \neq j)(9)$$

In order to assess the condition of optimality and maximize the profit of the retailer, the second order derivatives of the retailer's profit function will be obtain:

$$\frac{\partial^2 \Pi_{R_i}}{\partial {P_i}^2} = -2\beta \leq 0 \qquad (10)$$

Assuming $0 \leq \beta$ the second derivative is negative, and this indicates the optimality of retail price.

Theorem) given the best response function of the collector, the optimum rate of return used product for collector is as follows:

$$\tau_i^* = -\frac{\beta\Omega_i}{2} \times \frac{(-2\beta\alpha_i-\theta\alpha_j+2\beta^2W_i-\theta\beta W_j-W_i\theta^2)}{b.(4\beta^2-\theta^2)}; \qquad i,j=1,2 \ \ (i \neq j) \quad (11)$$

Proof) by assign P_i^*in the profit function equal to zero the first order derivative of the profit function with respect to τ_i^*best response function of collector and τ_i^* is obtained as follows.

$$\frac{\partial \Pi_{t_i}}{\partial \tau_i} = 0 \Rightarrow \Omega_i\left(\alpha_i - \beta P_i^* + \theta P_j^*\right) - 2b\tau_i = 0; i,j=1,2 \ \ (i \neq j)(12)$$

$$\Omega_i\left(\alpha_i - \beta\frac{2\beta(\beta W_i+\alpha_i)+\theta(\beta W_j+\alpha_j)}{4\beta^2-\theta^2} + \theta\frac{2\beta\left(\beta W_j+\alpha_j\right)+\theta(\beta W_i+\alpha_i)}{4\beta^2-\theta^2}\right) - 2b\tau_i = 0(13)$$

$$\tau_i^* = -\frac{\beta\Omega_i}{2} \times \frac{(-2\beta\alpha_i-\theta\alpha_j+2\beta^2W_i-\theta\beta W_j-W_i\theta^2)}{b.(4\beta^2-\theta^2)} \quad ; \quad i,j=1,2 \ \ (i \neq j) \quad (14)$$

In order to assess the condition of optimality and collects profit maximization, the second order derivative of the profit function of the collector obtains:

$$\frac{\partial^2 \Pi_{t_i}}{\partial {\tau_i}^2} = -2b \leq 0; \qquad i,j=1,2 \ \ (i \neq j) \ \ (15)$$

Assuming $0 \leq b$ the second derivative is negative, and this indicates the τ_i^* optimality. Finally, to obtain the optimal wholesale manufacturers price W_i^*, with replacement values P_i^* and τ_i^* In the interest of manufacturers and make equal to zero the first order derivative of manufacturer profit function towards W_i , W_i^* can be achieved.

$$\left(1 - \frac{1}{2}\frac{\left(\Delta_i - P_{b_i}\right)\beta\Omega_i(2\beta^2-\theta^2)}{b(4\beta^2-\theta^2)}\right)\left(\alpha_i - \beta P_i^* + \theta P_j^*\right) + \left(W_i - C_{m_i} - \left(\Delta_i - P_{b_i}\right).\tau_i^*\right)\left(-\frac{2\beta^3}{4\beta^2-\theta^2} + \frac{\theta^2\beta}{4\beta^2-\theta^2}\right) = 0 \quad ;$$
$$i,j=1,2 \ \ (i \neq j)(17)$$

Indeed W_i^* values calculated from the model solution and the answers are provided in Appendix at the end due to the high volume.

$$\mathrm{Max}_{W_i}\,\Pi_{M_i} = \max_{W_i}\left[W_i - C_{m_i} + \left(\Delta_i - P_{b_i}\right).\tau_i^*\right].(\alpha_i - \beta P_i^* + \theta P_j^*)(18)$$

s. t:

$$P_i^* = \frac{2\beta(\beta W_i+\alpha_i)+\theta(\beta W_j+\alpha_j)}{4\beta^2-\theta^2} \quad ; \qquad i,j = 1,2 \ \ (i \neq j) \qquad (19)$$

$$\tau_i^* = -\frac{\beta\Omega_i}{2} \times \frac{(-2\beta\alpha_i-\theta\alpha_j+2\beta^2 W_i-\theta\beta W_j-W_i\theta^2)}{b.(4\beta^2-\theta^2)} \quad ; \qquad i,j = 1,2 \ \ (i \neq j) \qquad (20)$$

$$\frac{\partial^2 \Pi_{M_i}}{\partial {W_i}^2} = 2(1-\frac{1}{2}\frac{\left(\Delta_i-P_{b_i}\right)\beta\Omega_i(2\beta^2-\theta^2)}{b(4\beta^2-\theta^2)})(-\frac{2\beta^3}{4\beta^2-\theta^2}+\frac{\theta^2\beta}{4\beta^2-\theta^2}) \leq 0 \ \ (21)$$

In order to assess optimality and the second condition of manufacturer profit maximization, the second order derivative of the manufacturer profit function obtains:

$$\frac{\partial^2 \Pi_{M_i}}{\partial {W_i}^2} = 2(1-\frac{1}{2}\frac{\left(\Delta_i-P_{b_i}\right)\beta\Omega_i(2\beta^2-\theta^2)}{b(4\beta^2-\theta^2)})(-\frac{2\beta^3}{4\beta^2-\theta^2}+\frac{\theta^2\beta}{4\beta^2-\theta^2}) \leq 0 \ (22)$$

Given $\beta \geq \theta$, the value of $\left(-\frac{2\beta^3}{4\beta^2-\theta^2}+\frac{\theta^2\beta}{4\beta^2-\theta^2}\right) \leq 0$ ؛ (22);always are negative,since($0 \leq \beta, \theta \leq 1 \ \ ; \Delta_i \geq P_{b_i}$)and great fixed parameter value of collecting cost (b) we have:

$$0 \leq \frac{1}{2}\frac{\left(\Delta_i-P_{b_i}\right)\beta\Omega_i(2\beta^2-\theta^2)}{b(4\beta^2-\theta^2)} \leq 1 \rightarrow 2\left(1-\frac{1}{2}\frac{\left(\Delta_i-P_{b_i}\right)\beta\Omega_i(2\beta^2-\theta^2)}{b(4\beta^2-\theta^2)}\right) \geq 0 \ \ (23)$$

As the multiplication of positive value (23) in the negative value (22) is always negative, thus second order derivative of manufacturer profit function towards the W_i is always negative and guarantees the optimality of W_i^* .

The second scenario: Both chains have a common collector, as it is shown schematically in Fig. 2.

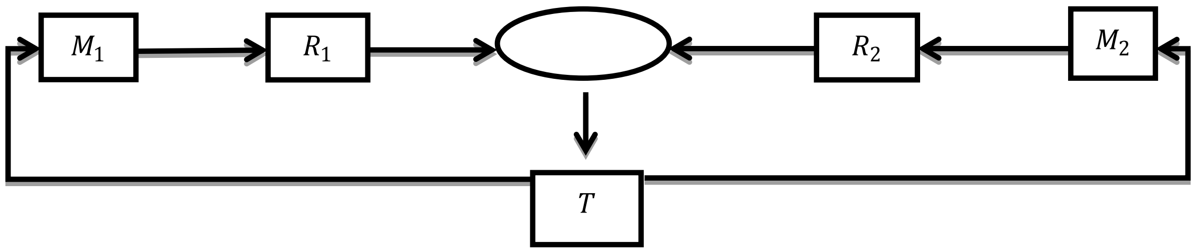

Fig. 2: Schematic chain competition under the second scenario

According to the model and calculate the amount of the total cost of the collection of used products according to the $C(\tau) = b.\tau^2 + P_c.\tau.D_i$ retailer's profit functions , common collector and manufacturer respectively defined as follows:

$$\Pi_{R_i} = (P_i - W_i).D_i; \quad i,j = 1,2 \ \ (i \neq j) \ (24)$$
$$\Pi_T = (P_b - P_c).\left(D_i + D_j\right).\tau - b.\tau^2 \ ; \ \ i,j = 1,2 \ \ (i \neq j) \ \ (25)$$
$$\Pi_{M_i} = \left[W_i - (1-\tau).C_{m_i} - \left(C_{r_i} + P_b\right).\tau\right].D_i \quad ; \ \ i,j = 1,2 \ \ (i \neq j) \ \ (26)$$

For easer calculating and further analysis of the issue if $\Omega = P_b - P_c$ is as the collector's income that takes for each unit of used production from manufacturer and $\Delta_i = C_{m_i} - C_{r_i}$ is as the manufacturer's income that takes for the ith chain for the reproduction of each unit used production that return, then the collector and manufacturer profit function will be as follows:

$$\Pi_T = \Omega \times (D_i + D_j) \times \tau - b \times \tau^2; \quad i,j = 1,2 \ \ (i \neq j) \qquad (27)$$
$$\Pi_{M_i} = [W_i - C_{m_i} + (\Delta_i - P_b) \times \tau] \times D_i; \ i,j = 1,2 \ \ (i \neq j) \quad (28)$$

In this model first manufacturers as Stackelberg game leaders, specify the wholesale price W_i^* that maximizes the profit function of each one. And then according to the wholesale price set by each, retailers and collectors as followers of the game specify the optimal value of the retail price P_i^* and the rate of return used products τ^*. For this purpose, according to wholesale optimal price and getting determined the response functions retailer and collector use the backward method. Importantly; how to obtain optimal points according to the chain members' external competition is taking as the main innovation of this article.

Given the identical functions of the retail profit, way to calculate the optimal amount P_i^* is the same as first scenario.

Theorem) with respect to the function responsecollector, the optimal rate of return used product for collectors is as follows:

$$\tau^* = -\frac{1}{2}\frac{[(W_i+W_j)(\beta-\theta)-(\alpha_i+\alpha_j)]\beta\Omega}{(2\beta-\theta)b}; \qquad i,j = 1,2 \ \ (i \neq j) \quad (29)$$

proof) by taking the value obtained P_i^* in collector's profit function and by using zero the first derivative of the profit function towards collector than τ, collector's best response function and τ^* is obtained as follows:

$$\frac{\partial \Pi_T}{\partial \tau} = 0 \quad (30)$$

$$\Omega[(\alpha_i - \beta P_i^* + \theta P_j^*) + (\alpha_j - \beta P_j^* + \theta P_i^*)] - 2b\tau = 0 \ (31)$$

$$\tau^* = -\frac{1}{2}\frac{[(W_i+W_j)(\beta-\theta)-(\alpha_i+\alpha_j)]\beta\Omega}{(2\beta-\theta)b} i,j = 1,2 \ \ (i \neq j) \quad (32)$$

To evaluate the optimality and the second maximization condition of the profit collector is and, the second order derivative of the profit collector function obtains:

$$\frac{\partial^2 \Pi_T}{\partial \tau^2} = -2b \leq 0 \quad (33)$$

Assuming that $0 \leq b$ the second order derivative is negative and this gives an indication of τ^*optimality. Finally, to obtain the optimum amount of wholesale prices manufacturer W_i^* with replacement values P_i^* and τ^* in manufacturer profit function and putting equal to zero the first order derivative of the manufacturer profit function towards W_i values W_i^* can be achieved.

$$\frac{\partial \Pi_{M_i}}{\partial W_i} = 0 \qquad (34)$$

$$(1 - \frac{1}{2}\frac{(\Delta_i - P_b)\beta\Omega(\beta-\theta)}{b(2\beta-\theta)})(\alpha_i - \beta P_i^* + \theta P_j^*) + (W_i - C_{m_i} - (\Delta_i - P_b).\tau^*)\left(-\frac{2\beta^3}{4\beta^2-\theta^2} + \frac{\theta^2\beta}{4\beta^2-\theta^2}\right) = 0; \qquad i,j = 1,2 \ \ (i \neq j)(35)$$

Indeed W_i^* values calculated from the above model solution and its answers are provided in Appendix at the end due to the high volume.

$$\max_{W_i} \Pi_{M_i} = \max_{W_i} \ [W_i - C_{m_i} + (\Delta_i - P_b).\tau^*] \times (\alpha_i - \beta P_i^* + \theta P_j^*) \ (36)$$

s.t:

$$P_i^* = \frac{2\beta(\beta W_i + \alpha_i) + \theta(\beta W_j + \alpha_j)}{4\beta^2 - \theta^2} \quad ; \qquad i,j = 1,2 \ \ (i \neq j) \qquad (37)$$

$$\tau^* = -\frac{1}{2}\frac{[(W_i+W_j)(\beta-\theta)-(\alpha_i+\alpha_j)]\beta\Omega}{(2\beta-\theta)b} \qquad i,j = 1,2 \ \ (i \neq j) \quad (38)$$

In order to assess optimality and the second condition of manufacturer profit maximization, the second order derivative of the manufacturer profit function obtains:

$$\frac{\partial^2 \Pi_{M_i}}{\partial {W_i}^2} = 2(1 - \frac{1}{2}\frac{(\Delta_i - P_b)\beta\Omega(\beta-\theta)}{b(2\beta-\theta)})(-\frac{2\beta^3}{4\beta^2-\theta^2} + \frac{\theta^2\beta}{4\beta^2-\theta^2}) \leq 0 \qquad (39)$$

Given $\beta \geq \theta$, the value of $(-\frac{2\beta^3}{4\beta^2-\theta^2} + \frac{\theta^2\beta}{4\beta^2-\theta^2}) \leq 0$ (40) is always a negative value and since ($0 \leq \beta, \theta \leq 1 \ ; \ \Delta_i \geq P_b$) and great fixed parameter value of collecting cost (b) we have:

$$0 \le \frac{1}{2}\frac{(\Delta_i - P_b)\beta\Omega(\beta-\theta)}{b(2\beta-\theta)} \le 1 \rightarrow 2\left(1 - \frac{1}{2}\frac{\left(\Delta_i - P_{b_i}\right)\beta\Omega_i(2\beta^2-\theta^2)}{b(4\beta^2-\theta^2)}\right) \ge 0 \qquad (41)$$

As the multiplication of positive value (41) in the negative value (40) is always negative, thus second order derivative of manufacturer profit function towards the W_i is always negative and it guarantees the optimality of W_i^* .

VII. Presentation and analysis of two numerical examples

This section presents two numerical examples to illustrate and compare the resulting relations. The example model presented in this study can be solved with the help of MAPLE software. For this purpose, the following parameters to compare the effect of the amount of revenue collected($\Omega_i = P_{b_i} - P_{c_i}$) the interest of each member in the chain of tables and diagrams presented have been used:

$$\alpha_1 = \alpha_2 = 200\,, \beta = 0.9\,, \theta = 0.6\,, C_{m_1} = 40\,, C_{m_2} = 38\,, C_{r_1} = 14\,, C_{r_2} = 16\,, P_{c_1} = \quad P_{c_2} = P_c = 4\,, b = 1000$$

To this end, the two chains with a common product type with the same quality and size of the potential market, is intended. To analyze the process changes, in production costin the first chain is greater than the second, but remanufactures cost in the first chain is less than the second one. In fact, by changing the parameter Ω, which on one hand is considered as the benefit arising from each collected product unit for a collector and the other hand as the cost arising from the receipt of each collected product unit for a remanufacturer its influence on the chain members profit functions can be seen. With equate assuming of cost amounts collected per unit earning rate of collector profit; in fact, how the impact of the delivery cost of each product unit as a variable on profit functions of each member.

In Table 1 different amounts of different members profit function of two chains based on two raised scenario for Ω values are provided.

Table1. Display the profit chain members function based on Ω changes

	chain	scenario	Ω1=Ω2=Ω			
			6	7	8	9
manufacturer profit function	1	1	20627.35561	20651.63687	20672.25286	20689.17348
	2	1	20706.99177	20708.65009	20706.58058	20700.80112
	1	2	20976.78320	21035.61518	21083.61682	21120.67035
	2	2	20946.55669	20964.27280	20971.12310	20967.06794
retailer profit function	1	1	9198.316266	9225.805834	9248.02683	9264.925152
	2	1	9189.571080	9199.493991	9204.093469	9203.365396
	1	2	9277.540082	9313.092551	9341.788898	9363.540893
	2	2	9238.954084	9252.014925	9258.190072	9257.450750
collector profit function	1	1	74.50636165	101.7145093	133.1716273	168.8532608
	2	1	74.43552571	101.4244102	132.5389459	167.7313342
	1+2	1	148.9418874	203.1389191	265.7105732	336.5845956
	-	2	299.9668797	409.3595120	535.6766887	678.7296369

As shown in Table 1 is in all cases in second scenario, members gain more profits than the first scenario and even in the case of a collector, the total profit of second scenario is more than both chain profits in the first scenario.

In the case of the manufacturer and retailer profit, the second scenario also acquires more values, that benefit change process of two chains members of two under different scenarios is important. For example, in the case of manufacturers, in the first scenario, the second chain profits (chain that has a lower cost of production) and in the second scenario, the first chain profits is more, but in the case of suppliers and retailers profit functions it can be said chain that has less cost of reproducing (the first chain) can give more profit to them. Even in some cases in spite of an increase in Ω, profit loss occurs and increasing Ω to some extent enhances profits. Like the second scenario, the second chain, with increasing Ω (the value of 8 to 9), reduces the profits of manufacturers. However generally as Ω increase the amount of profits increased, and the values is more for the second scenario. The benefit changes process of manufacturers, retailers and collectors are presented in diagram 1 and 2 and 3.

In the diagram 1, the changes process of retail profits is displayed by Ω change. As observed in the event that the cost of sending each used product unit cost be more than collect cost ,the chain with less remanufacture cost give more profit to retailers.

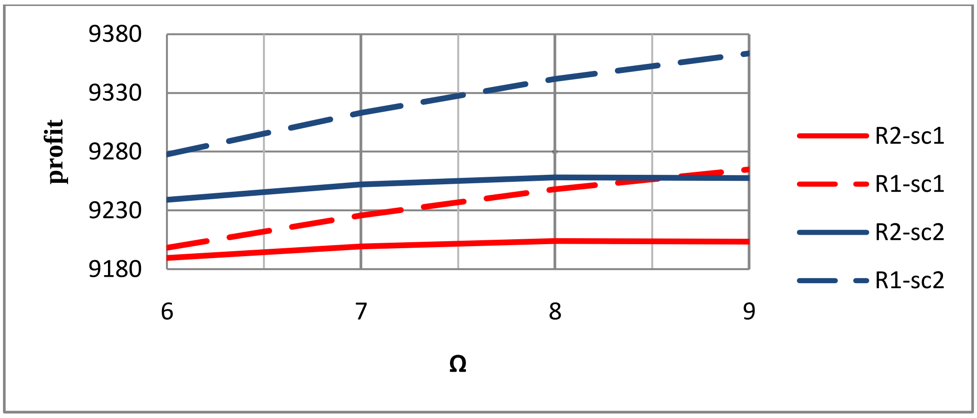

Diagram1: the impact of Ω increasing on retailer profit function

In the diagram 2, profit-producing function of the process changes is displayed according to changes in the value of Ω. As shown in the diagram from the viewpoint of the manufacturer when both chains have a common collector; less cost of reproducing give more profit to him, but at a time when every chain has a distinct collector; less production cost can increase her profits.

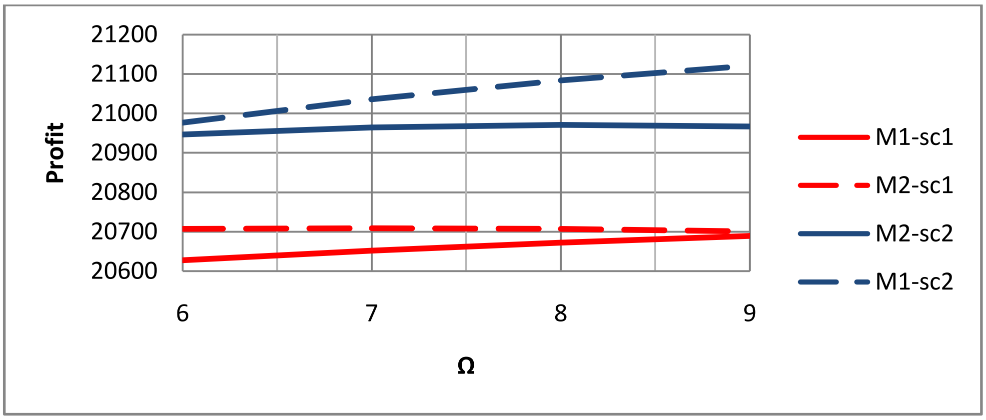

Diagram 2: the impact of Ω increasing on manufacturer profit function

In the diagram 3 trend of the profit function changes according to changes in the amount of visitors collected Ω is displayed. As shown in the diagram view according to the values of the function sum profit participants; the existence of a common picker is far better than the existence of two separate collectors in the system and it is also the first profit under the scenario by gathering suppliers almost equal.

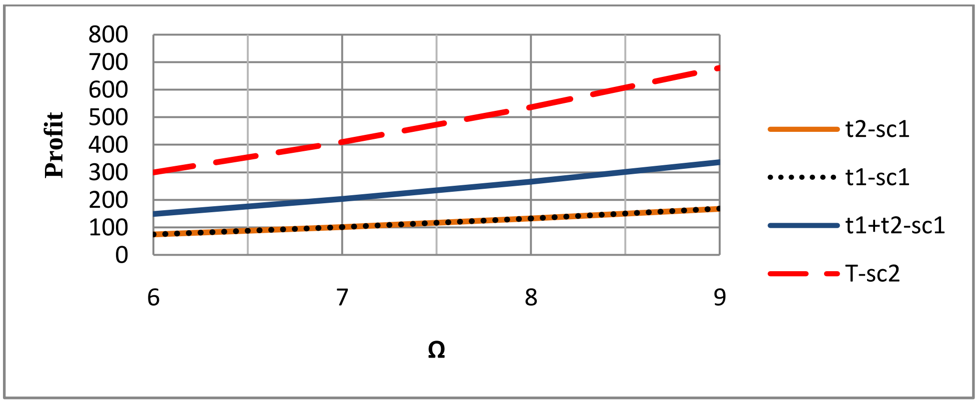

Diagram 3: the impact of Ω increasing on collector profit function

To compare the effect of rate and $\Gamma = \Delta_i - \Delta_j$ (the difference is earning money to pay for the process of chain remanufactures) on the following parameters of the chain member benefit, and the results of the calculations presented in the following diagram.

$$\alpha_1 = \alpha_2 = 200\,, \beta = 0.9\,, \theta = 0.6\,, C_{m_1} = 40\,, C_{m_2} = 38\,, P_{b_1} = P_{b_2} = P_b = 12, \Delta_1 = 26, \Omega_1 = \Omega_2 = \Omega = 8\,, b = 1000$$

To this end, two chains which produce a common product with the same quality and potential market size, are considered in which the production costs in the first chain is more than the second one. With fixed reproduction cost in first chain and variable reproduction cost on the second chain remanufactures the process chain of members profit functions will be examined. In fact, we want to show the effect of the parameter Γ on the interest rate of chain members. In Table 2 the profit function of two different values of both chain different members is provided based on two raised scenario for values of Γ.

Table 2- Display the profit chain members function based on Γ changes

	chain	scenario	$\Gamma = \Delta_i - \Delta_j$				
			1	2	3	4	6
manufacturer profit function	1	1	20626.67813	20642.91056	20657.10209	20672.25286	20687.36306
	2	1	20796.89106	20766.70039	20736.59704	20706.58058	20676.65064
	1	2	21027.16883	21046.01467	21064.83065	21083.61682	21102.37329
	2	2	21159.82115	21096.74440	21033.84526	210971.12310	20908.57738
retailer profit function	1	1	9227.64174	2934.455618	9241.251768	9248.029683	9254.789452
	2	1	9288.828607	9260.454007	9232.209228	9204.093469	9176.105965
	1	2	9316.777770	9325.128039	9333.465074	9341.788898	9350.099566
	2	2	9367.015845	9330.596118	9294.321070	9258.190072	9222.202450
collector profit function	1	1	132.8780328	132.9761609	133.0740253	133.1716273	133.2689679
	2	1	133.7591319	133.3505375	132.9481127	132.5389549	132.1359257
	1+2	1	266.6371647	266.326984	266.0178380	365.7105732	265.4048936
	-	2	538.0922834	537.2848439	536.4796481	535.6766887	534.8759567

As shown in table in all cases in second scenario, members gain more profits than the first scenario and even in the case of a collector; the total profit of second scenario is more than both chain profits in the first scenario. In the case of the manufacturer and retailer profit, the second scenario also acquires more values, that benefit change process of two chains members of two under different scenarios is important. The benefit changes process of manufacturers, retailers and collectors are presented in diagram 4 and 5 and 6. In the diagram 4, the changes process of retailer profits is displayed as Γ. As can be seen the balance and replacement point of retailers profit between two chains under scenario is expressed in $2 < \Gamma < 3$.

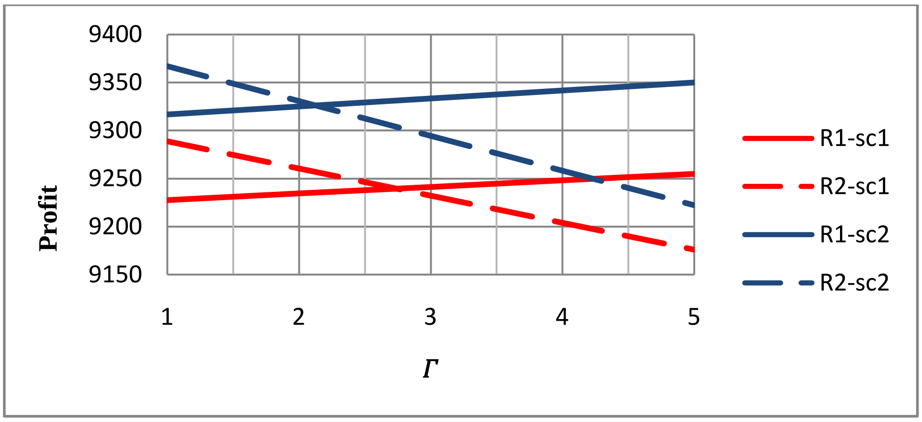

Diagram 4: increasing impact of Γon the retailer's profit

In the diagram 5 process changes the function of profit value changes displayed according to the providers' collection is Γ. As shown in the diagram, balance view according to the values of the function sum profit participants; the existence of a common picker is far better than the existence of two separate picker in the system and it is also the first profit under the scenario by gathering suppliers .in fact the amount of difference between earning a significant impact from the two chain remanufactures the interest rate total participants.

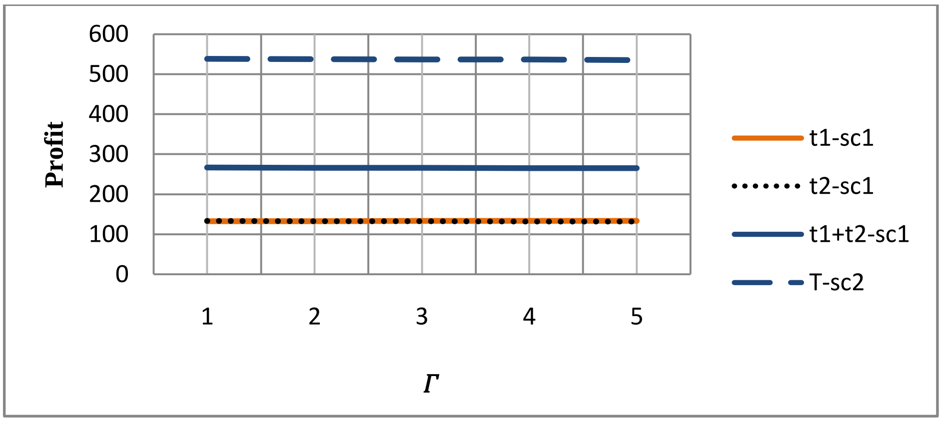

Diagram 5: increasing impact of Γon the collector's profit

In the diagram 6, the changes process of retailer profits is displayed as Γ. As can be seen the balance and replacement point of retailers profit between two chains under second scenario is expressed in $2 < \Gamma < 3$But in second scenario is in $4 < \Gamma < 5$ when second chain reproduction cost is two units more than the cost of reproducing the first chain.

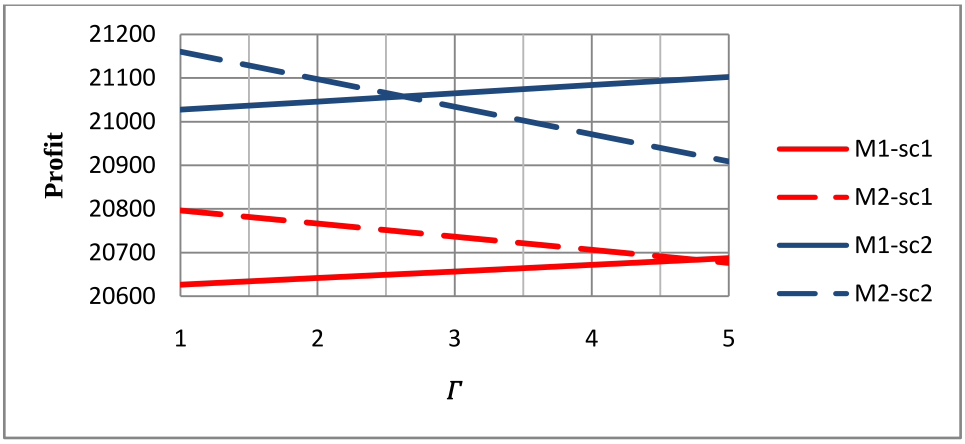

Diagram6: increasing impact of Γon the manufacturer's profit

VIII. Conclusion

The paper, examines competitive modeling and comparisons between the two closed-loop three-level supply chains with game theory concepts considering two different scenarios in order to collect the used product to the customer. Using concepts and definitions of game theory for modeling this closed-loop chains in the form of a Stackelberg game to get optimum amounts of wholesale and retail prices and the optimal values of the of the product backward coefficient to the optimum benefit amounts and collector as well as each of the members is taking by out chain competition. As observed, the optimal retail price levelsP_i^* is equal in both scenarios but optimal values of the collecting coefficients for collectors τ_i^* and the optimal values of the wholesale price W_i^* and all earned profits by members of the chain is more in the second scenario. This represents the advantage of cooperation between members of the chain. The cost of reproducing in this chain has a great role in the more profitable for all members and competitive chain should have less remanufacture cost. Definitely the chain members profit is influenced by manufacturers profit as the leader of the game. In an outside the chain competition, the chain in which manufacturer gain more profit, its members gain more profit as well.

Finally, a development of research is deleting every research default issue. If any of these assumptions are not true in research, the conditions will change and modeling and its response will be different as well. Including the use cases of inventory costs, maintenance of order, or a lack of research on the process modeling is that each model structure and the answers will change. In the meantime the demand being linear dependent on commodity prices is a very simplistic approach. With other factors such as being dependent on the demand for quality, complementary goods are goods successor, or even the cost of advertising and marketing, model output and greater conformity between the fact will be established. Resolve modeling has become more difficult in such situations but in practice has more applications.

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