

# Investigation of Task Bottleneck Generalized Assignment Problems in Supply Chain Optimization Using Heuristic Techniques

Dr. V.Kalyana Chakravarthy<sup>\*</sup>, Dr. V.V.Venkata Ramana<sup>\*\*</sup>,  
Dr. C.Umashankar<sup>\*\*\*</sup>

Professor, Department of Aeronautical Engineering Surya group of institutions, Villupuram  
Tamilnadu, India

Sr.Technical Director, National Informatics Centre, Vijayawada  
Andhra Pradesh, India

Registrar(Rtd), Rashtriya Sanskrit Vidyapeetha, Tirupathi, Andhra Pradesh, India

Corresponding Author: Dr. V.Kalyana Chakravarthy

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**Abstract:** This paper aims to look at the notion of optimization in a supply chain to avoid bottlenecks that exists frequently in manufacturing area. Supply chain consists of raw material supplier, manufacturer, wholesaler, retailer and customer. Here attention is given to a specific chain link called process operations in which the inspection line process operations of Liquefied Petroleum Gas (LPG) cylinder manufacturing unit is studied. Task Bottleneck Generalized Assignment Problem (TBGAP) aims at assigning the suitable quality supervisors to each process inspection stages and at the same time avoid the bottleneck in all these process inspection stages. Here optimizing the inspection time of the individual process inspection stage and the effective functioning of quality supervisors assigned, treated as one of the chain level optimization in the outsourcing of supply chain. The inspection time of the process operations analyzed using heuristic techniques namely Lexi Search methodology and Genetic algorithms. The outcome would be treated as the optimal solution in terms of inspection time reduction and saving manufacturing cost, whichever shows effect on the minimizing the maximum manufacturing cost, and time bound assignment.

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## I. Introduction

Bottleneck occurs in manufacturing or fabrication process in Liquefied Petroleum Gas (LPG) cylinder manufacturing very frequently because of its more number of process inspection stages, though there are many process inspection stages available to avoid the bottlenecks because of the continuous working of machinery and quality supervisors by mistake or by accident. In LPG gas cylinder manufacturing unit, there are 21 stages of process inspection. The major inspection stages are mentioned here as strip circumferential welding stage, foot ring welding stage, blanking operation stage, hydraulic press of providing non defective circular halves inspection stage, Physical material inspection of the incoming material stages, testing the sample materials for shear strength and compressive strength stage, quality control stage, coat thickness inspection stage, torque test, pneumatic inspection stage. Assigning the appropriate supervisor to a specific inspection stage plays a major role because of manufacturing cost and delivery time saving of the product. Here a study conducted and tested numerically in the assignment of supervisors to appropriate process inspection stages based on the importance of process operations.

The Generalized Assignment Problem (GAP) is comprised of a set of agents and with a set of tasks. The costs of each agent to perform each of the jobs, the amount of resources consumed by each agent performing them and the resource capacity available with each of the agents are also known. An assignment of jobs to agents is to be made in such a way that all the jobs are to be performed and the total resource needs on any agent does not exceed its capacity; such that the sum of the costs corresponding to the assignments is minimum[24].

VenkataRamana [12] discussed in his doctoral thesis about the various combinatorial programming problems in detail. The GAP with a mini-max (bottleneck) objective function instead of the usual minimum sum objective is studied. Many times, the appropriate objective is not the sum of the costs of the active assignments, but the maximum of the costs of these assignments. Problems of this type are called as the Task Bottleneck Generalized Assignment Problems (TBGAP) [11]. It can be defined as follows: there are a number of agents and number of tasks. Any agent can be assigned to perform any task, incurring some cost that may vary

depending on the agent-task assignment. It is required to perform all tasks by assigning exactly one agent to each task in such a way that the maximum cost among the individual assignments is minimized. A common type of application of the problem explains the term “bottleneck”, where the cost is the duration of the task performed by an agent. In this setting the "maximum cost" is "maximum duration", which is the bottleneck to schedule the overall job, to be minimized. The objective of TBGAP is to maximize the rate of production of the entire assembly line and the same may be achieved with the concept.

Supply Chain Management (SCM), which evolved to the rescue of many manufacturers with its outstanding solutions at every link level in the supply chain as shown below figure [17].

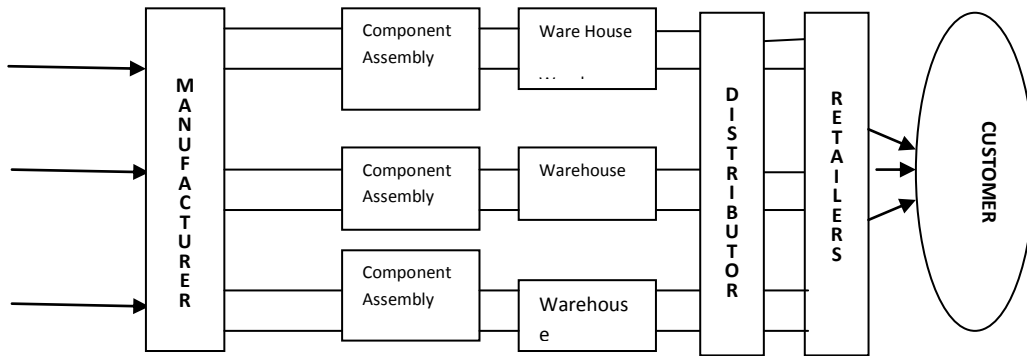


Figure 1: A Simple Supply Chain with chain link concept

**1.1 TBGAP in relation to SCM:** The Task Bottleneck Assignment Problem [15, 16] is the Combinatorial Optimization Problem in the branch of optimization. Maximum cost" is "maximum duration", which is the bottleneck to schedule the overall job, to be minimized. When there are a number of suppliers and a number of orders, any supplier would be assigned to perform any order in the supplier manufacturer module of SCM, incurring some cost that may vary depending on the supplier order assignment. It is required to perform all orders by assigning exactly one supplier to each order in such a way that the total cost of the assignment is minimum which very fact is the objective of the TBGAP. To minimize the total cost with minimum assignment, TBGAP problem is consider to study at supplier – manufacturer chain link of SCM using Lexi Search Algorithm and Genetic Algorithm processes.

## II. Work Of To The Project

### 2. 1 PROBLEM DESCRIPTION:

Let  $I=\{1,2,\dots,m\}$  and  $J=\{1,2,\dots,n\}$  be the index sets of quality supervisors and process inspection stages respectively. For  $i \in I, j \in J$ , let  $C_{ij}$  be the cost of quality supervisor  $i$  to perform process inspection stages  $j$ ,  $R_{ij}$  be the corresponding process stage requirement and let  $B_i$  be the budget capacity of supervisor  $i$ . The TBGAP is to assign each job to one supervisor so that total process inspection stages requirement for any process inspection stages does not exceed its availability and the maximum cost incurred is minimized. Thus, we have, mathematically

$$\text{Minimize (Max } \{C_{ij} X_{ij}\}) \tag{1}$$

$$\text{Subject to } \sum_{i=1}^{I_j} R_{ij} X_{ij} \leq B_i, i \in I \tag{2}$$

$$\sum_{i=1} X_{ij} = 1, j \in J \tag{3}$$

$$X_{ij} = 1, \text{ if job } j \text{ is assigned to agent } i \tag{4}$$

$$= 0, \text{ otherwise.}$$

It is to be noted that equations (2) - (4) define the usual constraint set of the general minimum GAP, whose objective function is  $\text{Min } \sum \sum C_{ij} X_{ij}$   $i j$  **(5)**

It is well known that GAP is NP-Hard in the strong sense, since even its feasibility question is so [20]. Hence the same results apply to TBGAP.

## III. Lexi Search Algorithm Process:

The name Lexicographic-Search or Lexi-Search method implies that the search is made for an optimal solution in a systematic way, just as one searches for meaning of a word in a dictionary[2]. When the process of feasibility checking of a partial word becomes difficult, though lower bound computation is

easy, Pattern Recognition Technique can be used. Lexi-Search algorithms, in general, require less memory, due to the existence of a Lexicographic order of partial words. If Pattern Recognition Technique is used, the dimension requirement of the problem can be reduced, since it reduces the two-dimensional cost array into a linear array and the problem can be reduced to a linear form of finding an optimal word of length  $n$  [20], and hence reduces computational work in getting an optimal solution.

Looking into the simplicity of the structure of the TBGAP, the Classical Lexi Search approach is made use of an algorithm is developed to solve the TBGAP.

Step 1: Read the Cost, Requirement and the Resource Limit data. Arrange the alphabet Table, by arranging the costs column wise, in the increasing order of various agents for each job. Store the corresponding row indices. BTCA gives the bottleneck cost of assignment at any point of time. BND is the bound value on the cost; VT is the trial value for bound (999999) at the start. Array LW will contain the optimal assignment.  $J=0, K=1$ .

Step 2: Given cost matrix is converted into reduced form.

Step 3: Sorting the values of the matrix with index position in terms of rows of the given cost Matrix.

Step 4: Rearrange the values of each column in the ascending order of the modified cost matrix

Step 5: Lexi search calculations to carry out for bound calculations, value of the word (VLK) and also the length of the word (LK) to be checked for sensitivity

Step 6:  $LB < VT$  go to feasible checking otherwise comeback

Step 7: As a part of feasible check, column repetition to check and the requirement matrix should not violate the maximum capacity.

At the end, if  $VT = 999999$  there is no feasible solution, otherwise solution with least bottleneck value is optimal.

Consider the following example of 3 quality supervisors and 4 process inspection stages, as given in the below table. The Alphabet Table of the search is as given in the table, where in each cell, the first letter denotes the actual row identity while the next entry is the corresponding cost, after arranging the original cost matrix column wise in ascending order. Thus, we can see the striking difference in the Alphabet Table preparation. In this case, one should not subtract the column minima from the respective columns, as the problem is to find out the bottleneck objective value. The Search Table for the same is given in the below table.

	Cost Matrix- C				Process Inspection Stage - R				Budget Limit- B
Quality Supervisors	44	55	47	22	34	16	30	22	195
	24	46	38	59	19	33	26	32	170
	53	26	39	53	16	27	27	20	190

**ALPHABET TABLE**

Modified Cost Matrix C			
2-24	3-26	2-38	1-22
1-44	2-46	3-39	3-53
3-53	1-55	1-47	2-59

**SEARCH TABLE**

S.no	1	2	3	4	Value	Remarks
1.	2-24				24	Accept
2.		3-26			26	Accept
3.			2-38		38	Accept
4.				1-22	22	Accept, VT=22
5.				3-53	53	Reject, > VT

<b>6.</b>		<b>3-39</b>	<b>39</b>	<b>Reject, &gt; VT</b>
<b>7.</b>	<b>2-46</b>		<b>46</b>	<b>Reject, &gt; VT</b>
<b>8.</b>	<b>1-44</b>		<b>44</b>	<b>Reject, &gt; VT</b>

A complete solution word for the above problem is thus (2, 3, 2, 1) implying the assignment that process inspection stages 1,2,3,4 are respectively to be assigned to the quality supervisors 2,3,2,1 and the optimal bottleneck objective value is 22.

#### **IV. Algorithm**

##### **4. GENETIC ALGORITHM (GA) PROCESS:**

**History and Genesis of Genetic Algorithm (GA):** The father of the original GA is John Holland [22], who invented in the early 1970s. Genetic Algorithms are Adaptive Heuristic Search Algorithms, based on the evolutionary ideas of natural selection and genetics. The basic techniques of the Genetic Algorithms are designed to simulate processes in natural systems necessary for evolution; especially those follow the principles first laid down by Charles Darwin of “Survival of the Fittest”. Since, in nature, competition among individuals for scanty resources results in the fittest individuals dominating over the weaker ones. A GA allows a population composed of many individuals to evolve under specified selection rules to a state that maximizes the “fitness” (i.e., minimizes the cost function).

##### **4.1 ALGORITHM**

To develop the optimal solutions in the search space the following are the steps required:

Step 1: Generate the initial population in random with predefined size using the population Scaling law, which represents the feasible solution of the problem.

Step 2: The populated chromosomes are initialized with a defined matrix structure and will be Evaluated thereafter.

Step 3: Assigning the fitness value to all the chromosomes of the population and selections are made Using Rank Space Selection method.

Step 4: Parents are selected in a Rank Order to create the new population using linear inversion Crossover. A new mutation method called Alternate Column Exchange mutation is applied to improve the quality of solution with a low Probability.

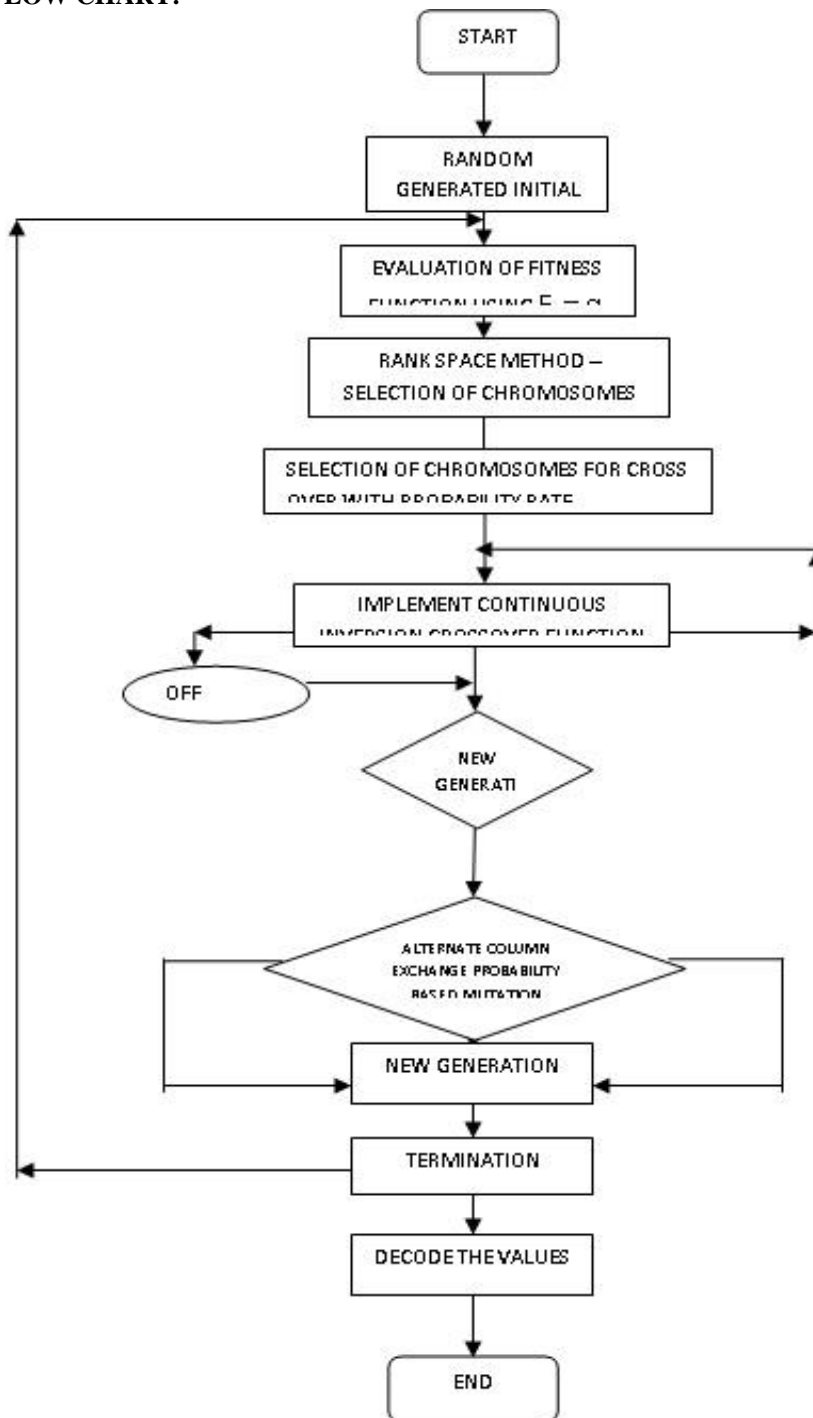
Step 5: Quality solution will be selected during the above steps whereas the poor ones be rejected.

Step 6: The selected quality solution called offspring having the properties of their parents and may be Chosen as next generation parent.

Step 7: The process may be repeated for several generations from step 3 to step 6 until the optimal Solution is reached.

Step 8: Terminate the process at the required stage.

**4.2 GA FLOW CHART:**



**V. Conclusion**

Average inspection time of the individual process inspection stage and the effective functioning of quality supervisors in terms of optimal cost are mentioned in below tables T1,T2, treated here as the chain level optimization in the outsourcing of supply chain. The Lexi search method results computed in rapid but as of the problem size increases the computational time increases because of the exhaustive search, whereas the genetic algorithm gave few of the optimistic results in case of the higher sized problems because of the larger solution search space. Though the marginal time is higher between these two methodologies, in view of computations genetic algorithms can adopt in any programming languages easily.

**TABLES: T1. GENETIC ALGORITHM SAMPLE TABLE OF VALUES:**

S.no	Problem instance	Population size 'Ps'	Number of generations 'Ng'	Type of Cross over 'Lxc'	Type of Mutation with probability 'Acm'	Optimal Cost
1	4 x 6	20	20	0.65	0.01	62
	5 x 5		30	0.65	0.01	89
	5 x 10		40	0.65	0.01	188
2	5 x 15	40	30	0.65	0.01	234
	5 x 20		40	0.65	0.01	283
	10 x 15		50	0.65	0.01	212
3	10 x 25	60	40	0.65	0.01	190
	15 x 20		50	0.65	0.01	171
	15 x 25		60	0.65	0.01	173

**T2.EXPERIMENTAL RESULTS:**

S.NO	PROBLEM INSTANCES	LEXISEARCH METHOD	GENETIC ALGORITHM APPROACH(best values)	OPTIMAL SOLUTION	
				GA	LSA
1	4 x 6	63	62	0.003	0.0275
2	5 x 5	98	89	0.28	0.1375
3	5 x 10	179	188	0.1836	0.165
4	5 x 15	225	234	0.2015	0.188
5	5 x 20	305	283	0.324	0.38
6	10 x 15	212	212	0.748	0.55
7	10 x 25	223	190	1.993	0.0825
8	15 x 20	181	171	2.852	0.1325
9	15 x 25	173	173	3.685	0.597
10	25 x 50	266	190	8.203	1.19

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