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New Interval Linear Assignment Problems

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Abstract: *In this paper a ground reality the entries of the cost matrix are not always crisp. In many application this parameters are uncertain and this uncertain parameters are represented by interval. In this contribution we proposed a new Interval Hungarian Method to solve the mid values of each interval in the cost matrix with four different cases of the assignment problem and finally we conclude that same optimum solution.*

Keywords: *Interval Analysis, Hungarian Method, Assignment Problems.*
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I. Introduction

The basic assumption of assignment problem is that one person can perform one job at a time. An assignment plan is optimal if it minimizes the total cost (or) maximizes the profit. This type of linear assignment problem can be solved by very well-known Hungarian method, which was derived by the two Mathematician D.Konig and E.Egervary. Saragam Majumdar [1] was introduced an interval linear assignment problems.

G.Ramesh and K. Ganesan [2] studied an Assignment Problem with generalized interval arithmetic. Dr. S. Amutha, S. Lakshmi, S. Narmatha [4] developed by method of solving extension of interval in Assignment problem Cloud, Michael J. Moore, Raman E Kearfott. R. Baker [5], Introduction to interval analysis. The first section review of interval arithmetic, in next section, definition and details of new interval Hungarian method. In fourth, sixth sections are solved by balanced, unbalanced example problems and result is analysed. Finally conclusions are drawn in this paper discussed about Saragam Majumdar of its numerical example problems solved by new interval Hungarian method and finally we conclude that the same optimal solution.

II. Definition

2.1. Interval Equality

The convention of denoting intervals and their end points by capital letters. The left and right end points of an interval X will be denoted by $\underline{X}$ and $\overline{X}$ respectively. Thus

$$X = \left[\underline{X} \cdot \overline{X}\right]$$

Two intervals X and Y are said to be equal if they are the same sets if their corresponding end points are equal X = Y if $\underline{X} = \underline{Y}$ and $\overline{X} = \overline{Y}$.

2.2. Interval Arithmetic

The interval form of the parameters may be written as

$$\left[\underline{X}, \overline{X}\right] = \left\{x : x \in IR/\underline{x} \le x \le \overline{x}\right\}$$

where $\underline{x}$ is the left value and $\overline{x}$ is the right value of the interval respectively.

We define m = $\dfrac{\underline{x}+\overline{x}}{2}$ is the centre and w = $\underline{x} - \overline{x}$ is the width of the interval $\left[\underline{x}, \overline{x}\right]$.

Let $\left[\underline{x}, \overline{x}\right]$ and $\left[\underline{y}, \overline{y}\right]$ be two elements then the following arithmetic are well known [5].

(i) $\left[\underline{x}, \overline{x}\right] + \left[\underline{y}, \overline{y}\right] = \left[\underline{x} + \underline{y}, \overline{x} + \overline{y}\right]$

(ii) $\left[\underline{x}, \overline{x}\right] - \left[\underline{y}, \overline{y}\right] = \left[\underline{x} - \overline{y},\ \overline{x} - \underline{y}\right]$

(iii) $\left[\underline{x}, \overline{x}\right] \times \left[\underline{y}, \overline{y}\right] = \left\{\min\left\{\underline{x}\underline{y},\ \underline{x}\overline{y}, \overline{x}\underline{y},\ \overline{x}\overline{y}\right\},\ \max\left\{\underline{x}\underline{y},\ \underline{x}\overline{y}, \overline{x}\underline{y},\ \overline{x}\overline{y}\right\}\right\}$

III. New Interval Hungarian Method

In this section, we are proposed New interval Hungarian method and to solve the interval linear assignment problems.

Algorithm:

Step 1: Find out the mid values of each interval in the cost matrix with four different cases.

Case (i)

- **(i)** mid value of 1st column one difference with left and right value.
- **(ii)** mid value of 2nd column two difference and 3rd column one difference, 4th column two difference with left and right value etc.,

Case (ii)

(i) mid value of 1st column with three difference.

(ii) mid value of 2nd column four difference and 3rd column three difference, 4th column four differences with left and right value, etc.

Case (iii)

(i) mid value of 1st column one difference, mid value of 2nd column two difference, mid value of 3rd column three difference, mid value of 4th column four difference with left and right value, etc.

Case (iv)

(i) mid value of the element with left value ten difference and right value twelve difference, etc., Next mid value of the element with left value ten difference and right value twenty difference, etc.,

Step 2: Subtract the interval which have smallest mid value in each row from all the entries of its row. Suppose that left value is small and right value is highest in corresponding row, now we select the left smallest value from those rows.

Step 3: Subtract the interval which have smallest mid value from those columns, which have no intervals contain zero from all the entries of its column.

Step 4: Draw lines through appropriate rows and columns. So that all the intervals contain zero of the cost matrix are covered and minimum number of such lines is used.

Step 5: Test for optimality

(i) If the minimum number of covering lines is equal to the order of the cost matrix, then optimality is reached.

(ii) If the minimum number of covering lines is less than the order of the matrix, then go to step 6.

Step 6: Determine the smallest mid value of the intervals which are not covered by any lines. Subtract this entry from all uncrossed elements and add it to the crossing having an interval contain zero. Then go to step 4.

IV. Numerical Examples

Let us consider an assignment problem discussed by Sarangam Majumda [1]. The assignment cost of assigning any operator to any one machine is given in the following table [3].

Table 1 : Cost matrix with crisp entries

	I	II	II	IV
A	10	5	13	15
B	3	9	18	3
C	10	7	3	2
D	5	11	9	7

By applying Hungarian Method, Sarangam Majumdar got an optimal assignment as A, B, C, D machines are assigned II, IV, III, I operators respectively and minimum assignment cost is 16.

Now change the entry of the cost matrix by some interval form.

Case (i) The first column mid value with one difference of left value and right value, the second column mid value with two difference of left value and right value and so on. Then we get a new cost matrix as follow.

Table 2 : Cost matrix with interval entries

	I	II	II	IV
A	[9, 11]	[3, 7]	[12, 14]	[13, 17]
B	[2, 4]	[7, 11]	[17, 19]	[1, 5]
C	[9, 11]	[5, 9]	[2, 4]	[0, 4]
D	[4, 6]	[9, 13]	[8, 10]	[5, 9]

Applying their interval Hungarian method, Sarangam Majumdar obtained the optimal assignment as A, B, C, D machines are assigned to II, I, III, IV operators respectively and the optimum assignment cost as [14, 22]. After starting that the above assignment is optimal, they claim that the solution is not unique and another optimal assignment can be obtained as A, B, C, D are assigned to II, IV, III, I respectively and minimum assignment cost is [12, 20]. We apply the new Interval Hungarian Method and solve for the same numerical example problems. Finally we conclude that the optimum cost is [10, 22].

Hungarian Method	Interval Hungarian Method	New Interval Hungarian Method
16	[12, 20]	[10, 22]

(ie) Midvalue IHM = 16, NIHM = 16

Case (ii) the first column mid value with three difference of left and right value the second column mid value with four difference of left and right value and so on. Then we get a new cost matrix as follows

Table 3: Cost matrix with interval entries

	I	II	II	IV
A	[7, 13]	[1, 9]	[10, 16]	[11, 19]
B	[0, 6]	[5, 13]	[15, 21]	[-1, 7]
C	[7, 13]	[3, 11]	[0, 6]	[-2, 6]
D	[2, 8]	[7, 15]	[6, 12]	[3, 11]

Applying new Interval Hungarian method obtained the optimal assignment as A, B, C, D machines are assigned to II, IV, III, I operators respectively and minimum assignment cost is [2, 30].

Case (iii) The first column mid value with one difference of left and right value, second column mid value with two difference of left and right value. Third column three differences of left and right value and fourth column mid value with four difference of left and right value and so on. Then we get a cost matrix as follows.

Table 4 : Cost matrix with interval entries

	I	II	II	IV
A	[9, 11]	[3, 7]	[10, 16]	[11, 19]
B	[2, 4]	[7, 11]	[15, 21]	[-1, 7]
C	[9, 11]	[5, 9]	[0, 6]	[-2, 6]
D	[4, 6]	[9, 13]	[6, 12]	[3, 11]

We are apply new Interval Hungarian method obtained the optimal assignment as A, B, C, D machines are assigned to II, IV, III, I operators respectively and minimum assignment cost is [6, 26].

Case (iv) The mid value of the element with left value ten difference and right value 12 difference of the all cost entries. Then we get a new cost matrix as follows :

Table 5: Cost matrix with interval entries

	I	II	II	IV
A	[0, 22]	[-5, 17]	[3, 25]	[5, 27]
B	[-7, 15]	[-1, 21]	[8, 30]	[-7, 15]
C	[0, 22]	[-3, 19]	[-7, 15]	[-8, 14]
D	[-5, 17]	[1, 23]	[-1, 21]	[-3, 19]

We are apply new Interval Hungarian method obtained the optimal assignment as A, B, C, D machines are assigned to II, IV, III, I operators respectively and minimum assignment cost is [-24, 64].

Therefore, the various intervals of three cases are obtained by the same optimal assignment as A, B, C, D machines are assigned II, IV, III, I respectively and minimum assignment cost of mid value are same. (i.e.,) 16.

V. Unbalanced Interval Hungarian Method

The method of balanced assignment requires that the number of rows and columns is equal. When the given cost matrix is not a square matrix then the assignment is unbalanced. In such cases a dummy row(s) (or) column(s) are added in the matrix (with zeros as the cost elements) to make it a square matrix.

VI. Numerical Examples

We take a linear assignment problem as an example problem and solve this problem by Hungarian Method. The assignment cost of assigning any operator to any one machine is given in the following table [3].

Table 1: Cost matrix with crisp entries

	I	II	II	IV
A	10	5	13	15
B	3	9	18	3
C	10	7	3	2

Now we applying Hungarian method, then we get an optimal assignment as A, B, C, D machines are assigned to II, I, IV, III operators respectively and minimum assignment cost is 10.

Now change the entry of the cost matrix by some interval (i.e. new interval Hungarian method) then we get a new cost matrix as follows

	I	II	II	IV
A	[9, 11]	[3, 7]	[12, 14]	[13, 17]
B	[2, 4]	[7, 11]	[17, 19]	[1, 5]
C	[9, 11]	[5, 9]	[2, 4]	[0, 4]

The new cost matrix is not square matrix by adding a dummy row with cost is [0, 0].

	I	II	II	IV
A	[9, 11]	[3, 7]	[12, 14]	[13, 17]
B	[2, 4]	[7, 11]	[17, 19]	[1, 5]
C	[9, 11]	[5, 9]	[2, 4]	[0, 4]
D	[0, 0]	[0, 0]	[0, 0]	[0, 0]

We are applying the new interval Hungarian method and solve this problem. We get an minimum assignment cost is [5, 15] and optimal assignment as A, B, C, D machines are assigned to II, I, IV, III operators respectively. Finally we conclude that the solution is same for the both methods.

Hungarian Method	Interval Hungarian Method	New Interval Hungarian Method
10	[7, 13]	[5, 15]

VII. Conclusion

In this paper, the mid value difference of the column wise n, n+1, n+2,…and there is no difference between the optimal assignment and mid value is 16 for the both methods (i.e., Interval Hungarian Method, New Interval Hungarian Method). Otherwise, the mid value of element with left and right value difference are highest. Then the optimal assignments are same but mid value is different.

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