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Inventory Management for Post-Epidemic Era

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Abstract *This wave of COVID-19 Pandemic is highlighting the crisis of overstocking and understocking, and how to balance just-in-time and just-in-case has become a difficult problem for many companies. The COVID-19 epidemic spread disturbed the whole world to generate a huge impact on traveling, business operations, and civilian activities. Owing to the vaccination rate being over eighty percent and the buster shots being popular among many countries, people are planning to live with business operations following the epidemic era. To handle this new challenge, we discuss the non-convexity phenomenon of the replenishment run time problem without the use second derivative from calculus. We provide two approaches: (a) Our first result is based on the geometric skill of asymptote and a tangent line, and (b) Our second approach is a direct numerical computation, to help practitioners with an alternative approach other than analytical methods. Our results can be helpful for researchers and practitioners facing business operations and marketing management after the COVID-19 epidemics to develop the best management models for warehouse management systems in the post-pandemic era.*

Keywords: *COVID-19 epidemics; Replenishment run time; Machine breakdown; Convexity; Failure in rework*

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I. Introduction

When the epidemic first broke out, demand for many commodities fell sharply, leaving companies with excessive inventories. But the subsequent border closures, tightening of the supply chain, and gradual rebound in demand proved that it was wise to stock up. Traditionally, researchers have used calculus to examine the first and second partial derivatives of objective functions to solve for the optimal solution of inventory models. However, due to the strong mathematical background needed for this approach, the development of a non-calculus approach will greatly increase the comprehension of inventory models for researchers without a mathematical background. Grubbstrom [10] pioneered this approach by deriving the standard EOQ formula from an algebraic approach. Grubbstrom and Erdem [11] generalized the algebraic method for inventory models with shortages. Moreover, Cardenas- Barron [1] studied an EPQ model with shortages by an algebraic method, and then Chang [5] applied this approach for inventory models with a variable lead-time. Ronald et al. [19] studied the inventory models in Grubbstrom and Erdem [6] and Cardenas-Barron [1] and revealed that some questionable results implicitly referring to findings of calculus, which led to the derivation of a complex method to obtain the optimal solution presented in Ronald et al. [19]. This method, which lends its origins to Montgomery et al. [18], searches for the minimum solution along each beam in the first octant, and then connects the minimum solutions to yield a new objective function, thereby locating its minimum. This method however is too tedious to be absorbed by ordinary researchers. Chang et al. [6] amended Ronald et al. [19] to present a more intuitive algebraic approach in finding the optimal solution. Similarly, Sphicas [20] applied an intuitive algebraic method to locate the optimal solution for EOQ and EPQ systems with linear and fixed backorder costs. Minner [17] examined the algebraic derivation for a finite planning horizon inventory system and then took its limit to yield the optimal solution for infinite planning horizon inventory systems. Cardenas-Barron [2] applied the perfect square method to construct a supply chain inventory system with equally divided cycle time. Lan et al. [14], taking a page from Grubbstrom and Erdem [11], Cardenas-Barron [1], Chang [4], and Ronald et al. [19], presented a clearer algebraic method to solve inventory systems with stochastic lead time, in which the algebraic approach is applied to inventory systems under a more general setting. Teunter and Dekker [21] used the marginal cost to obtain the optimal order level without the tedious derivation of the average cost. Hsieh et al. [12] applied the Cauchy–Schwarz inequality and the arithmetic-geometric mean inequality to find the optimal solution for an integrated single-vendor single-buyer inventory problem without referring to the traditional algebraic method of completing the square. Cardenas-Barron [3] used algebraic methods to solve the optimal solution for EOQ and EPQ models. However, he did not check the sign before he took the square root. Hence, he overlooked the degenerate cases, where the optimal solutions occurred on the boundary. Cardenas-

Barron et al. [3] offered a simpler method for deciding the integer solutions for the lot sizing replenishment policy for a two-stage inventory model. They also amended some questionable results in Drake et al. [9]. Chung [7] examined the second result of Lin and Chiu [16] to show the convex property proposed by Lin and Chiu [16] containing questionable findings, and then he obtained a revision to find the optimal solution. This article is motivated by Hung et al. [13] that provided a new solution method to solve an old inventory problem and Lin et al. [15] that provided a neat approach to dramatically simplify the previous complicated solution of Deng et al. [8] and Wu et al. [22]. The purpose of this paper is to provide an intuitive explanation for the examination of the non-convexity property so that practitioners who are not familiar with calculus can fully understand the significant results of Chung [7].

II. Notation and assumptions

To be compatible with Lin and Chiu [16] and Chung [7], we adopt the same notation and assumptions as theirs.
Notation

C is the unit production cost.

C_k is the unit rework cost.

C_s is the disposal cost for one unit.

g is the constant reinstatement time for a breakdown machine

h is the holding cost per item per unit time of producer.

h_1 is the holding cost for reworked items.

K is the setup cost per production run.

M is the cost to restore a breakdown machine.

P is the annual production rate.

P_1 is the work rate to repair defective items

t_1 is the production uptime, the decision variable.

x is the portion of defective items in the production rate P .

β is the number of breakdowns per unit time (annual), a random variable that follows a Poisson distribution.

λ is the annual demand rate, with $P > \lambda$.

θ_1 is the portion of disposal after defective items are reworked.

π is an auxiliary function to simplify the expression.

$E[x]$ is the expectation of the portion of defective items.

$TCU\ (t_1)$ is the total production-inventory costs per unit time.

$E[TCU\ (t_1)]$ is the expected total production-inventory costs per unit time.

$f(y) = \dfrac{a - by}{e^y - 1}$ with $a > 0$ and $b > 0$ is an auxiliary function.

Assumptions

The manufacturing process is developed with the following features:

(1) It will randomly produce x portion of defective items that can be reworked.

(2) During the reworked process, there is a θ_1 portion that cannot be reworked to repair at rework cost, C_k per item, and then they will be discarded with disposal cost, C_s per item.

(3) The annual production rate P is greater than the annual demand rate λ and the reworked items are denoted as xP.

(4) Based on the mean time between failure data, only one machine breakdown happens with random occurrence times.

(5) The resume inventory control policy is followed after a breakdown occurs the maintenance procedure will be operated immediately to correct with a constant repair time. The disturbed lot will be restarted right after the reinstatement of the breakdown machine.

III. Materials and methods

First, we review previous results, Lin and Chiu [16] discussed the expected total inventory cost per unit time as follows

$$E[TCU(t_1)] = \frac{1}{1-\theta_1 E[x]}\left[\frac{K\lambda\beta}{P(1-e^{-\beta t_1})} + \frac{M\lambda\beta}{P} + C\lambda + C_k E[x]\lambda + C_s\theta_1 E[x]\lambda\right]$$
$$+\pi\left[\frac{-t_1 e^{-\beta t_1} - \dfrac{e^{-\beta t_1}}{\beta} + \dfrac{1}{\beta}}{(1-\theta_1 E[x])(1-e^{-\beta t_1})}\right], \tag{1}$$

with

$$\pi = hP\left(1 - 2\theta_1 E[x] + \theta_1^2 E[x^2]\right) - h\lambda + 2h\lambda\theta_1 E[x]$$
$$+\frac{P\lambda E[x^2](h_1 - h(1-\theta_1))}{P_1} + h\theta_1 E[x] g\lambda\beta \ . \tag{2}$$

For the detailed notation and expressions, please refer to Lin and Chiu [16]. They claimed that $E[TCU(t_1)]$ is a convex function.

Next, we recall their solution approach. Chung [7] studied Equations (1) and (2) to derive the second derivative of $E[TCU(t_1)]$ as

$$\frac{d^2}{dt_1^2}E[TCU(t_1)] = \frac{\beta e^{-\beta t_1}(1+e^{-\beta t_1})}{(1-\theta_1 E[x])(1-e^{-\beta t_1})^3}\left[\frac{K\lambda\beta^2}{P} + \frac{2\pi(1-e^{-\beta t_1})}{1+e^{-\beta t_1}} - \beta\pi t_1\right]. \tag{3}$$

Chung [7] used the following inequality,

$$\frac{1-e^{-\beta t_1}}{1+e^{-\beta t_1}} < 1 , \tag{4}$$

to imply that if

$$t_1 > \frac{1}{\beta\pi}\left(\frac{K\lambda\beta^2}{P} + 2\pi\right), \tag{5}$$

then $\frac{d^2}{dt_1^2}E[TCU(t_1)] < 0$, invalidating the claim of Lin and Chiu [16] as that $E[TCU(t_1)]$ is convex when t_1 is big enough. Chung [7] provided an excellent solution procedure to analyze the first derivative of $E[TCU(t_1)]$, proving the uniqueness of the minimum solution.

In this paper, we will provide a different approach to show that $E[TCU(t_1)]$ is not convex without the use of the second derivative.

IV. Our solution method

We will apply a different approach without referring to analytical skills as calculus. Owing to

$$\frac{1}{1-e^{-\beta t_1}} = \frac{e^{\beta t_1}}{e^{\beta t_1}-1} = 1 + \frac{1}{e^{\beta t_1}-1}, \tag{6}$$

and

$$\frac{-t_1 e^{-\beta t_1} - \dfrac{e^{-\beta t_1}}{\beta} + \dfrac{1}{\beta}}{1-e^{-\beta t_1}} = \frac{-1}{\beta} + \frac{-t_1}{e^{\beta t_1}-1}, \tag{7}$$

the expression of equation (1) was simplified as follows

$$E[TCU(y)] = \frac{a-by}{e^y - 1} + c , \tag{8}$$

for $y > 0$ where the following new parameter and abbreviations are adopted,

$$y = \beta t_1, \tag{9}$$

$$a = \frac{K\lambda\beta}{P(1-\theta_1 E[x])}, \tag{10}$$

$$b = \frac{\pi}{\beta(1-\theta_1 E[x])}, \tag{11}$$

and

$$c = a - b + \frac{1}{1-\theta_1 E[x]}\left[\frac{M\lambda\beta}{P} + C\lambda + C_k E[x]\lambda + C_s \theta_1 E[x]\lambda\right]. \tag{12}$$

First, it is apparent to show that $a > 0$, since $0 < \theta_1 < 1$ the failure in rework rate and $E[x]$ is the expectation of the random defective rate with $E[x] < 1$.

Next, we need to show that proving that $b > 0$ is equivalent to proving that $\pi > 0$. We can rewrite π of Equation (2) as follows

$$\pi = h(P-\lambda)(1-2\theta_1 E[x]) + h\theta_1\left(P\theta_1 E[x^2] + E[x]g\lambda\beta\right)$$
$$\frac{P}{P_1}\lambda E[x^2](h_1 - h(1-\theta_1)), \tag{13}$$

where P is the annual production rate and λ is the annual demand rate, and so $P > \lambda$.

Next, we discuss the numerical examination from previous papers. In the numerical example of Chung [7] and Lin and Chiu [16], it was assumed that x follows a uniform distribution over $[0,0.2]$ such that $E[x] = 0.1$ and the failure in rework rate $\theta_1 = 0.1$ such that $1-2\theta_1 E[x] > 0$ is a reasonable assumption. Moreover, h_1 is the unit holding cost for reworked items with $h_1 = 0.8$, h being the holding cost per item per unit time, when $h = 0.6$, $h_1 - h(1-\theta_1) > 0$ is also logical based on the numerical examples of Chung [7] and Lin and Chiu [16]. Hence, we know that $\pi > 0$ and $b > 0$.

Our goal is to verify that for $y > 0$, the following auxiliary function

$$f(y) = \frac{a - by}{e^y - 1} \tag{14}$$

with $a > 0$ and $b > 0$, is not a convex function.

We will provide proof that is based on geometric properties: asymptote and tangent line for this problem.

We know that $a - by$ and $1/(e^y - 1)$ both are decreasing functions, owing to $a > 0$ and $b > 0$.

Moreover, $1/(e^y - 1)$ is always positive.

$a - by$ is positive for $0 < y < (a/b)$ and the sign of $a - by$ changes to negative for $(a/b) < y < \infty$.

Hence, the minimum of $f(y)$ should happen in the interval of $(a/b) < y < \infty$. Thus, our domain was restricted from original $0 < y < \infty$ to $(a/b) < y < \infty$.

We know that e^y goes to infinity faster than any polynomial so then $y \to \infty$, to imply that $f(y) = 0$, that is, $f(y)$ has a horizontal asymptote at $y \to \infty$ with $f(y) \to 0^-$, implying $f(y) < 0$ and $f(y) \to 0$ as $y \to \infty$.

By the way of contradiction, we assume that $f(y)$ is a convex function then $f''(y) > 0$ for $y > 0$. Because $f(y)$ is negative for $(a/b) < y$ and there is a horizontal asymptote when $y \to \infty$ so the minimum problem can be further restricted to a compact set so that the minimum problem has a solution in that specific compact set. Hence, we prove the existence of the minimum solution.

We may assume the absolute minimum point as y^*. We know that $f''(y) > 0$ and for $y > y^*$ so $f'(y) > 0$ for the restricted domain $y > y^*$.

We may consider the tangent line passing through $(y^* + 1, f(y^* + 1))$ and slope $f'(y^* + 1) > 0$ that will intersect the x-axis at a finite point, say $y^\#$, with

$$y^\# = y^* + 1 + \frac{(-1) f(y^* + 1)}{f'(y^* + 1)}. \tag{15}$$

If $f(y)$ is convex and $f(y)$ is above the tangent, which implies that $f(y)$ will intersect the x-axis before $y^\#$ that is contradicted that $f(y) < 0$ for $(a/b) < y < \infty$.

V. Our second approach: A numerical evaluation

Next, we will provide a more intuitive approach through numerical examination. There are two appealing approaches to check the convexity of a function:

(a) $f''(y) > 0$ in the entire domain,

and

(b) For any y_1, y_2, and $0 \le \varepsilon \le 1$,

$$f(\varepsilon y_1 + (1-\varepsilon) y_2) \le \varepsilon f(y_1) + (1-\varepsilon) f(y_2). \tag{16}$$

Chung [7] considered the first approach, showing that $f''(y) > 0$ is invalid when y is very large. Here we will use the second approach to find two points: y_1, and y_2, and a value of ε with $0 < \varepsilon < 1$,

$$f(\varepsilon y_1 + (1-\varepsilon) y_2) > \varepsilon f(y_1) + (1-\varepsilon) f(y_2). \tag{17}$$

We used the same data as Lin and Chiu [16] and Chung [7], to use the following values for parameters: $k = 450$, $\lambda = 4000$, $\beta = 0.5$, $P = 10000$, $\theta_1 = 0.1$, $E[x] = 0.1$ and $\pi = 3760.13$ to get the next table.

Table 1. The comparison between $f(y)$ and the middle point of a secant line

y	1	2	3	4
$f(y)$	-4367	-2363	-1189	-565
$\frac{f(y-1)+f(y+1)}{2}$		-2778	-1464	

From Table 1, we know that

$$f(3) = -1189.26 > \frac{f(2) + f(4)}{2} = -1464.43 \tag{18}$$

showing that $f(y)$ is not a convex function, under the domain, $1 \le y \le 3$. Therefore, we present a numerical examination to show that $f(y)$ is not convex.

VI. Applications of our findings

Many companies stressed that once trading conditions return to normal, they would still prefer to return to stocking levels before COVID-19 Pandemic. For companies, the huge inventory backlog means more money, space, management, insurance expenses, and a major problem, selling commodities before their expiration dates. Researchers and companies developed business models to cite data and relations among parameters and variables to describe commerce procedures. As time goes by, those business models involved more complicated and contained more parameters and variables and then their interrelationship becomes intractable. In this paper, we demonstrate two different approaches to studying business models.

The first approach is an analytic method to establish mathematical verifications to prove some assertions under rigorous definition and precious examinations which is independent of the values of parameters and variables. Our second approach is a numerical method that uses data of parameters directly to explain the phenomenon without tedious proof which provides an intuitive explanation for business models. We can claim that our two different solution approaches will help researchers and partitioners to deal with business operations and marketing management.

VII. Conclusion

Chung [7] used the second derivative to show the assertion of Lin and Chiu [16] to claim that their objective function is convex that is invalid. We offered two intuitive approaches: (a) a geometric approach with the use of tangent line and asymptote and (b) a numerical check to show the same result. Our work will help researchers who are not acquainted with the second derivation of calculus to reach a deeper understanding of the replenishment run time problem with breakdown and failure in rework for production systems.

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