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Optimal Ordering And Delivery Policies Under Carbon Emission And Advance Payment

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ABSTRACT
The retailer makes payment after receiving the ordered commodities in the traditional economic order quantity (EOQ) models. It is impractical in cases in which the supplier has the power to control the risk of the cash flow. This paper therefore constructs an inventory model with a partial advance payment with added considerations for order quantities and carbon emissions. Five practical orientations are incorporated into the proposed model: (1) the opportunity costs due to making advance payments; (2) the interest charge caused by the bank loans; (3) the carbon emission costs resulting from the deliveries; (4) the quantity discounts offered by suppliers to induce the retailer to order greater quantities; and (5) employing the lot-splitting delivery policy to help prevent there being insufficient retailer storage space or reduce storage costs. The objective of this study is to determine the number of shipment and the inventory lot size corresponding to unit-purchasing cost minimizing the annual total cost. We then develop a two-stage solution procedure and construct an efficient algorithm to help managers making quick and accurate decisions. Numerical examples are given to verify the validities of the proposed model and algorithm. Managerial implications are also explored.
Keywords: *Inventory, Carbon emission, Advance payment, Quantity discounts, lot-splitting shipment.*

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I. INTRODUCTION

Facing the increasingly volatile business environments, many companies employ different strategies to achieve competitive edges. One of the key strategies in recent is the timing of the payment for ordering cost. There are three different basic policies for paying the ordering cost: (i) payment at the time of delivery, (ii) postponed payment or credit payment, and (iii) advance payment (Taleizadeh, 2014). Although the influences of delayed payment on inventory policies have attracted great attention, the advance payment and its influences on inventory decisions are rarely addressed (Zhang *et al*., 2014). Thus, in this paper, we focus on advance payment (AP) policy in which a seller is powerful and wants to control the risk of the cash flow and he would like the buyer to pay in a fixed period before the date of delivery. Especially, partial advance payment is used to control the risk of buyer's canceling order or to finance the procurement of material or parts used in production of the ordered product. One of the pioneers focusing on the AP issue is Zhang (1996) developing a model for determining the optimal cash deposit amount when there is a fixed per-payment cost. He used a renewal theory approach to obtain the long-term expected total cost per period when the bill amounts in a future period are deterministic, exponentially distributed, normally distributed, or Poisson occurrence based with a constant per occurrence charge. Maiti *et al*. (2009) studied the effect of advanced payment for price-dependent demand in a stochastic environment in which they considered the holding, ordering, purchasing and advertising costs to be constant in the proposed model. Although they consider the payment scheme as an additional factor to make the model more realistic, they did not treat it as the core topic of the paper. Gupta *et al*. (2009) developed an inventory model with the incorporation of the effect of AP by the retailer to the wholesaler and considering the inventory costs as interval valued numbers in which they employed genetic algorithm to solve an inventory model with advance payment and interval valued inventory costs. Taleizadeh *et al*. (2011) used uncertain programming to develop a multiproduct, multi-constraint inventory control problem importing raw material from another country for which a fraction of the purchasing cost is paid as prepayment. Thangam (2012) incorporated the advance payment scheme and two-echelon trade credits into a supply chain for perishable items in which the advance payment was used between the final customer and the retailer, and the vendor still offered full delayed payment to the retailer. Taleizadeh *et al*. (2013a) developed an economic order quantity (EOQ) model for a deteriorating product with and without shortage under consecutive prepayments in which they assumed that the supplier asks purchasers to pay a fraction of the order's cost in advance and may allow them to divide the prepayment into multiple equal-sized parts to be paid during a fixed lead time. Taleizadeh *et al*. (2013b) constructed a fuzzy rough EOQ problem with quantity discount and prepayment for a deteriorating product as a mixed integer nonlinear programming model in which meta-heuristic algorithms were employed to find the optimal solution. In the mean time, Taleizadeh (2014) developed an economic order quantity model for a deteriorating product with partial backordering and partial consecutive prepayments with a real case study of a gasoline station. Zhang *et al*. (2014) investigates the buyer's inventory policy under advance payment, including all payment in advance and partial-advanced–partial-delayed payment in which the buyer's ordering policy is derived by minimizing his total inventory costs including inventory holding cost, ordering cost, and interest cost caused by advance payment or delayed payment. Recently, Pourmohammad-Zia and Taleizadeh (2015) developed an EOQ model with backordering under a hybrid payment scheme which is also linked to order quantity, involves multiple advance payments as well as delay payment. Based on a real-life situation in Iran, Lashgari1 *et al*. (2016) developed an EOQ model with down-stream partial delayed payment and up-stream partial prepayment under three different scenarios were developed in various situations with (1) no shortage, (2) full backordering, and (3) partial backordering. Zhang *et al*. (2016) proposed a two-stage optimization model to characterize a retailer's ordering policy in a supply chain with demand and supply uncertainties sequentially realized, where the advance payment could be conducted before the selling season to stable the supplier's capacity. In contrast to those previous EOQ models with advance payments such as in Taleizadeh *et al*. (2014) and Pourmohammad-Zia and Taleizadeh (2015), without taking expiration dates into consideration, Teng *et al*. (2016) have developed an EOQ model for the retailer with partial backordering and lost sales when the supplier requests a partial prepayment before delivery, and the product gradually deteriorates to 100% as its expiration date approaches. Tao and Xu (2019) focused on inventory management under two common carbon-regulation policies: carbon-tax regulation and cap-and-trade regulation in which they found that the allocated cap does not affect the optimal order size under cap-and-trade regulation. Mashud et al. (2021) aimed at reducing the carbon emissions of a retailer's inventory system in which a greenhouse product retailer is investing in advanced technology for lower carbon emission logistics activity. Li et al. (2022) integrated economic production quantity with economic order quantity models and designs multiple carbon policies in which abatement rate and collection rate are the key variables used to explore the mechanism of the interaction between carbon tax and cap-and-trade, inventory dynamics of closed-loop supply chain and impact of multiple carbon policies on closed-loop supply chain. Other related works on carbon emission and advance payment include research undertaken by Singh and Chaudhary (2023), Nia et al. (2023), Dey et al. (2023), among others found in their references.

Based on the above arguments, we know that the previous studies dealt with EOQ models with advance payment built upon the following assumptions: (1) unit purchasing cost is irrelevant with regard to ordering quantity; (2) a single order delivery; (3) no additional charges for carbon emissions. However, in order to achieve their economics of scale, the supplier usually offers quantity discounts as an incentive for the retailer to order larger quantities. To benefit from the quantity discounts, the retailer orders greater amounts of goods, and also requires the supplier to send deliveries in multiple shipments. Moreover, with there being greater environmental awareness, governments have begun levying carbon emission taxes or making attempts to impose limits on carbon emissions caused by business activities. These practical dimensions should be included in models examining inventory systems that entail making advance payments. This paper therefore constructs an inventory model with a partial advance payment with added considerations for order quantities and carbon emissions. Five practical orientations are incorporated into this proposed model: (1) the opportunity costs due to making advance payments; (2) the interest charge caused by the bank loans; (3) the carbon emission costs resulting from the deliveries; (4) the quantity discounts offered by suppliers to induce the retailer to order greater quantities; and (5) employing the lot-splitting delivery policy to help prevent there being insufficient retailer storage space or reduce storage costs. To the best of our knowledge, this research is the first to incorporate the PAP scheme, carbon emission, lot-splitting deliveries, and quantity discounts into an EOQ model. This paper not only to find the optimal ordering and delivery policies but also the quantity discounts policy by minimizing the retailer's total cost under partial advance payment and carbon emission. An example incorporating with some managerial insights are also explored.

II. NOTATION AND ASSUMPTIONS

The following notation and assumptions are introduced to define the EOQ model with advance payment and carbon emission.

Notation

The following parameters and variables are used to develop the problem.

D demand rate
S cost of placing one order
Q order size
N number of shipments per cycle (integer value)

I_h holding cost rate for a unit of item per period, expressed as a fraction of dollar value
α percent of purchase cost paid in advance, $0 \le \alpha \le 1$
I_d opportunity cost per \$ investment in other market per year
t length of advance payment
I_c interest charges per \$ investment in stocks per year
T planning horizon
R receiving cost per shipment
A_0 fixed transport cost of a shipment
c_d variable transport cost per unit
A_1 fixed GHG emission cost of a shipment
Ω fuel price at the time of the order
d distance traveled in kilometers
l fuel consumption in liters per km
c_k unit-purchasing cost of kth level
p unit sell price in \$

Assumptions
The following assumptions are made to develop the mathematical model.
(1) Demand rate for item is known and constant
(2)Shortages are not allowed
(3) Time horizon is infinite
(4) Replenishments are instantaneous
(5) Ordering quantity is manufactured at one setup and the shipment is a fixed quantity and delivered at regular intervals
(6) All-unit quantity discounts scheme is employed where c_k be the unit-purchasing cost of kth level and Q_{k-1} be the lowest order quantity of the kth level. This means when the order quantity is between $[Q_{k-1}, Q_k]$, then the unit purchase cost is c_k , where $c_k > c_{k+1}$
(7) the retailer is requested by the supplier to pay part of the payment in a fixed period before the date of delivery and the remaining unpaid balance must be paid when the moment of the first shipment.
(8) the retailer employs cash in hand to pay the partial advance payment and thus sustains a loss of opportunity cost; Alternately, he gets loan from a bank to pay the remaining unpaid balance and thus incurs a loan interest, I_c.
(9) the sales revenue for the end of each sub-batch (*i.e.* pQ/N .) does not immediately use to pay the loan but use to invest another market to obtain the capital gains, I_d, where $I_d > I_c$. At the end of the whole cycle, $(T+t_0)$, all of the loans are paid.

III. Mathematical model

To control the risk of buyer's canceling order or to finance the procurement of material used in production of the ordered product, the seller usually requests that the buyer pays a certain percentage of the total purchase cost per cycle as a partial advance payment (PAP). The decision regarding the amount of PAP to be made has a crucial impact in the total cost and inventory decisions because PAP is a real life phenomenon. In this section, we construct a mathematical model with the total cost minimization objective for retailer's optimal ordering and delivery policies under advance payment and carbon emission. The time-weighted inventory behavior when partial payment is paid in advance is illustrated in Fig. 1 in which a single order and lot-splitting shipments are considered (*say N* shipments) during the planning horizon, T. For each shipment, the delivery interval and amount are the same. At time 0, the retailer pays the supplier partial advance payment and incurs an opportunity cost with $\alpha tDTc_kI_d$, which is constant until time t. At time t, the retailer gets loans from bank to pay the remaining unpaid balance and thus incurs an interest. From time t, the sale begins and the revenue from sale is continuously used to pay the loan, and the amount of outstanding loan is decreasing. At time $t+t_1$, all of the loans are paid and then the revenue begins to earn interest until time $t+T$. Thus, we have $t_1 = \frac{(1-\alpha)Qc_k}{pD}$.

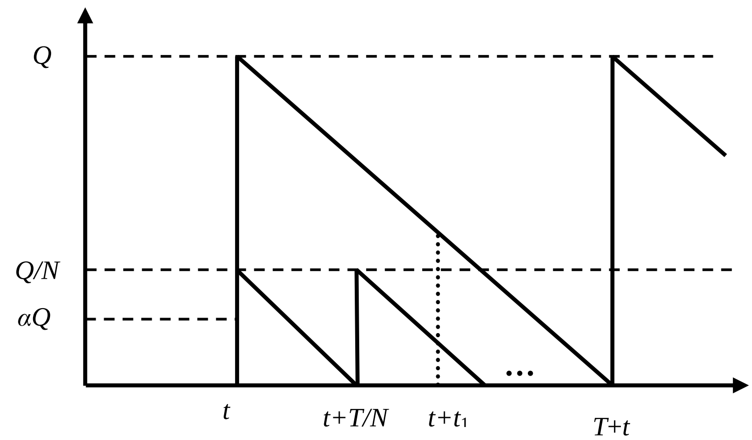

Fig. 1. Time-weighted inventory when partial payment is paid in advance.

For the retailer, the total variable costs under lot-splitting shipments consist of the following elements:
(1) Cost of placing order, S.
(2) Cost of stock holding (excluding interest charge), $c_k I_h Q^2/(2DN)$
(3) Opportunity cost, $\alpha Q c_k I_d t$
(4) Cost of interest charge, $(1-\alpha)Qc_k I_c t_1/2$
(5) Interest earning, $pDI_d(T-t_1)^2/2$
(6) Cost of purchasing items, $c_k Q$
(7) Receiving cost, NR
(8) Cost of delivery items (including fixed and variable delivery cost), $N(A_0 + c_d\, Q/N)$.
(9) Cost of carbon emission (including fixed and variable carbon emission cost), $N(A_1 + dl\Omega\, Q/N)$.
Then, the total cost is

$$TC_k(Q,N) = S + \frac{c_k I_h Q^2}{2DN} + \alpha Q c_k I_d t + \frac{(1-\alpha)Qc_k I_c t_1}{2} - \frac{pDI_d(T-t_1)^2}{2}$$
$$+ c_k Q + NR + N\left(A_0 + c_d \frac{Q}{N}\right) + N\left(A_1 + dl\Omega \frac{Q}{N}\right),\ k = 1,2,\ldots,r \qquad (1)$$

Because $T = (Q/D)$, the total cost per unit time is

$$ATC_k(Q,N) = \frac{D[S + N(R + A_0 + A_1)]}{Q} + \left[\frac{c_k I_h}{2N} + \frac{(1\text{-}\alpha)^2 c_k{}^2 I_c}{2p} - \frac{pI_d}{2}\left(1 - \frac{(1-\alpha)c_k}{p}\right)^2\right]Q$$
$$+ \alpha D c_k I_d t + c_k D + c_d D + dl\Omega D,\ k = 1,2,\ldots,r \qquad (2)$$

Since the objective of this paper is to minimize the total cost per unit time by simultaneously determining the optimal order lot size, number of shipments and unit-purchasing cost, the convexity of the function needs to be proved to find the unique solution, which requires to show that the Hessian matrix of Eq. (2) is positively definite for finding the solution using differential calculus. However, it is not easy to determine the concavity of the Hessian matrix in Eq. (2). Therefore, in the following section we develop an alternative procedure to identify the feasible solutions and then develop an algorithm to determine the overall optimal solution.

IV. SOLUTION PROCEDURES AND ALGORITHM

For the derivation of the optional solution in the total cost per unit $ATC_k(Q,N)$, We need the following property.
Property 1: For a fixed value of N, $ATC_k(Q,N)$ is convex in Q.
Proof: For a fixed value of N, $ATC_k(Q,N)$ in Eq. (2) is a function of single variable Q. Therefore, the sign of $\partial^2 ATC_k(Q,N)/\partial Q^2$ characterizes its convexity. Then, we have

$$\frac{\partial^2 ATC_k(Q,N)}{\partial Q^2}=\frac{2SD+ND(R+A_0+A_1)}{Q^3}>0,\ k=1,2,\ldots,r,$$

which shows that, for a fixed value of N, $ATC_k(Q,N)$ is convex in Q for all k.

Property 1 indicates there is a unique solution which minimizes Eq. (2) if the number of deliveries is given. Because the unit-purchasing cost depends on quantity ordered, we cannot obtain the optimal lot size form Eq. (2) immediately under the different cost curve corresponding to each cost level. This means the values for each Q_k in this stage are not optimal solutions but feasible solutions by letting $\left(\partial ATC_k(Q,N)/\partial Q\right)=0$. Setting $\left(\partial ATC_k(Q,N)/\partial Q\right)=0$ and letting it be $\tilde{Q}_k$ corresponding to each unit-purchasing cost, we have

$$\tilde{Q}_k(N)=\sqrt{\frac{D\left[s+N(R+A_0+A_1)\right]}{\left[\frac{c_kI_h}{2N}+\frac{(1\text{-}\alpha)^2c_k{}^2I_c}{2p}-\frac{pI_d}{2}\left(1-\frac{(1-\alpha)c_k}{p}\right)^2\right]}} \tag{3}$$

We here assume the supplier requires the rate of the partial advance payment is not significant (*e.g.* $\alpha<0.2$) and thus $\left\{\frac{(1\text{-}\alpha)^2c_k{}^2I_c}{2p}-\frac{pI_d}{2}\left(1-\frac{(1-\alpha)c_k}{p}\right)^2\right\}>0$. This implies the denominator in Eq. (3) is positive. Thus, Substituting eq. (3) into Eq. (2), we have

$$ATC_k(N)=2\sqrt{D\left[s+N(R+A_0+A_1)\right]\left[\frac{c_kI_h}{2N}+\frac{(1\text{-}\alpha)^2c_k{}^2I_c}{2p}-\frac{pI_d}{2}\left(1-\frac{(1-\alpha)c_k}{p}\right)^2\right]}$$
$$+\alpha Dc_kI_dt+c_kD+c_dD+dl\Omega D,\ k=1,2,\ldots,r \tag{4}$$

Ignoring the terms independent of N and taking square of $ATC_k(N)$, we know that minimizing $ATC_k(N)$ is equivalent to minimizing the following:

$$Z_k(N)=\frac{1}{2N}\left[DSc_kI_h\right]+\frac{DS(1-\alpha)^2c_k^2I_c}{2p}+\frac{D\left[(R+A_0+A_1)c_kI_h-pSI_d\left(1-\frac{(1-\alpha)c_k}{p}\right)^2\right]}{2}$$
$$+ND(R+A_0+A_1)\left[\frac{(1\text{-}\alpha)^2c_k{}^2I_c}{2p}-\frac{pI_d}{2}\left(1-\frac{(1-\alpha)c_k}{p}\right)^2\right],\ k=1,2,\ldots,r \tag{5}$$

Treating N as a continuous variable and taking the first and the second derivative of $Z_k(N)$with respect to N, one has

$$\frac{dZ_k(N)}{dN}=\frac{-DSc_kI_h}{2N^2}+D(R+A_0+A_1)\left[\frac{(1\text{-}\alpha)^2c_k{}^2I_c}{2p}-\frac{pI_d}{2}\left(1-\frac{(1-\alpha)c_k}{p}\right)^2\right],$$
$$k=1,2,\ldots,r$$

$$\frac{dZ_k^2(N)}{dN^2}=\frac{DSc_kI_h}{N^3}>0,\ k=1,2,\ldots,r$$

Therefore, we know $Z_k(N)$ is convex in N. This implies $ATC_k(N)$ is also convex in N. Letting $\left(dZ_k(N)/dN\right)=0$, we have

$$N_k=\sqrt{\frac{Sc_kI_h}{2(R+A_0+A_1)\left\{\frac{(1\text{-}\alpha)^2c_k{}^2I_c}{2p}-\frac{pI_d}{2}\left(1-\frac{(1-\alpha)c_k}{p}\right)^2\right\}}},\ k=1,2,\ldots,r \tag{6}$$

Because the denominator in Eq. (6) is positive, we have N_k is also positive. However, N_k is an integer value of the discrete variable. Eq. (6) does not guarantee an integer could be provided. Thus, employing the following relationships,

$$Z_k(N-1)\geq Z_k(N)\leq Z_k(N+1).$$

One can obtain the following condition to determine the candidate optional number of shipments:

$$\hat{N}(\hat{N}-1)\leq \frac{Sc_k I_h}{2(R+A_0+A_1)\left\{\frac{(1\text{-}\alpha)^2 c_k{}^2 I_c}{2p}-\frac{pI_d}{2}\left(1-\frac{(1-\alpha)c_k}{p}\right)^2\right\}}\leq \hat{N}(\hat{N}+1),$$

$$k=1,2,\ldots,r \tag{7}$$

Plugging $\hat{N}$ obtained from Eq. (7) into Eq. (3), we therefore obtain $\hat{Q}_k(\hat{N})$ corresponding to each unit-purchasing cost (*i.e.* c_k). We note that $\hat{Q}_k(\hat{N})$ is valid when the condition $Q_{k-1}\leq \hat{Q}_k(\hat{N})<Q_k$ holds. This indicates that if $\hat{Q}_k(\hat{N})$ is invalid corresponding to c_k, two cases occur:

Case I : $\hat{Q}_k(\hat{N})>Q_k$, where Q_k is an upper bound of ordering quantity corresponding to c_k.

In this case, the vendor may adopt the lower unit-purchasing cost (*e.g.* c_x for $c_x<c_k$) to obtain the lot size $\hat{Q}_k(\hat{N})$. This implies $ATC_x(\hat{N})<ATC_k(\hat{N})$ from Eq. (4). This result indicates the optical solution may not occur at this case.

Case II : $\hat{Q}_k(\hat{N})<Q_{k-1}$, where Q_{k-1} is a lower bound of ordering quantity corresponding to c_k.

In this case, we can fix *Q* and *N* individually when c_k is given. Two possible candidates under c_k may exist.

Scenario 1 : The perspective of break point Q_{k-1} corresponding to c_k

In this scenario, the candidate optimal lot size may occur at the breakpoint Q_{k-1} with its corresponding candidate number of deliveries. Therefore, for given Q_{k-1}, corresponding to c_k, one may determine the candidate integer deliveries for $\tilde{N}_k$ is given by

$$\tilde{N}_k(\tilde{N}_k-1)\leq \frac{c_k I_h}{2D(R+A_0+A_1)}\leq \tilde{N}_k(\tilde{N}_k+1),\ k=1,2,\ldots,r \tag{8}$$

Scenario 2 : The perspective of fixed *N*

Given a reasonable *N*, the candidate optimal solution may occur at $(Q_k^{\Delta}(N),N)$ in which the condition of $Q_{k-1}\leq \hat{Q}_k(\hat{N})<Q_k$ holds. Plugging Eq. (3) into $Q_{k-1}\leq Q_k^{\Delta}(N)$, we have

$$\eta N^2+\beta_k N-\omega_k>0,\ k=1,2,\ldots,r \tag{9}$$

where $\eta=2D(R+A_0+A_1)$

$$\beta_k=2DS-2(Q_{k-1})^2\left\{\frac{(1\text{-}\alpha)^2 c_k{}^2 I_c}{2p}-\frac{pI_d}{2}\left(1-\frac{(1-\alpha)c_k}{p}\right)^2\right\}$$

$$\omega_k=(Q_{k-1})^2 c_k I_h$$

Solving Eq. (9) for *N*, we have

$$N_k>\frac{-\beta_k+\sqrt{(\beta_k)^2+4\eta\omega_k}}{2\eta}\equiv \overline{N}_k,\ k=1,2,\ldots,r \tag{10}$$

Taking $N_k^{\Delta}>\lfloor \overline{N}_k \rfloor$ in which $\lfloor \overline{N}_k \rfloor$ is the smallest integer larger than $\overline{N}_k$, We then have $Q_k^{\Delta}(N_k^{\Delta})$ from Eq. (3). Alternatively, if $Q_k^{\Delta}(N_k^{\Delta})>Q_k$, no feasible solution occurs in this scenario.

We now have obtained some potential sets of optimal lot size and shipments from above discussions. However, the optimal solution until now could not be obtained. We therefore develop an algorithm to find the optimal ordering quantities and shipments as follows:

Algorithm:

Step 1: Obtain $\hat{N}_k$ from Eq. (7)

Step 2: Plugging $\hat{N}_k$ obtained in Step1 into Eq. (3) and then obtain $\hat{Q}_k\left(\hat{N}_k\right)$, where $k = 1,2,...,r$.

Step 3: FOR k=1 to r

IF $\hat{Q}_k\left(\hat{N}_k\right) > Q_k$, NEXT k

ELSE {

IF $Q_{k-1} \le \hat{Q}_k\left(\hat{N}_k\right) < Q_k$, DO {

{Compute $ATC_k\left(\hat{Q}_k, \hat{N}_k\right)$ from Eq. (2) and record them}

{NEXT k}

}

}

ELSE {

IF $\hat{Q}_k\left(\hat{N}_k\right) < Q_{k\text{-}1}$, DO {

{Obtain $\tilde{N}_k$ from Eq. (8) and compute $ATC_k\left(Q_{k-1}, \tilde{N}_k\right)$ from Eq. (2) and record them}

{Obtain $\overline{N}_k$ from Eq. (10) and take $N_k{}^{\Delta} = \left\lfloor \overline{N}_k \right\rfloor$; Compute $Q_k{}^{\Delta}\left(N_k{}^{\Delta}\right)$ from Eq. (3).

If $Q_{k-1} \le Q_k{}^{\Delta}\left(N_k{}^{\Delta}\right) < Q_k$, {

Compute $ATC_k\left(Q_k{}^{\Delta}, N_k{}^{\Delta}\right)$ from Eq. (2) and record them

}

ELSE {

$ATC_k\left(Q_k{}^{\Delta}, N_k{}^{\Delta}\right) = \infty$ and record them

}

}

{ $ATC_k\left(Q_k^{@}, N_k^{@}\right) = Min\left[ATC_k\left(Q_{k-1}, \tilde{N}_k\right), ATC_k\left(Q_k{}^{\Delta}, N_k{}^{\Delta}\right)\right]$ and then record them}.

{NEXT j}

}

}

Step 4: $ATC\left(Q^*, N^*\right) = Min\left[ATC_k\left(\hat{Q}_k, \hat{N}_k\right), ATC_k\left(Q_k^{@}, N_k^{@}\right)\right]$

Step 5: The optional order lot size is Q^* corresponding to its unit-purchasing cost; the optimal number of shipments is N^*; and the minimum cost is $ATC(Q^*,N^*)$.

V. NUMERICAL EXAMPLE

To illustrate the behaviors of the optimal order lot size Q^*, the optimal number of shipments N^*, and the minimum cost $ATC(Q^*,N^*)$, let us consider the following example. All necessary variable costs and fixed costs are estimated from existing data and listed in Table 1.

Table 1. All necessary variable costs and fixed costs

Description and parameters	Value	Unit
Demand rate (D)	96000	units/year
Ordering cost (S)	1200	\$/cycle
Receiving cost (R)	10	\$/shipment
Holding cost rate for a unit of item (a fraction of dollar value) (I_h)	0.20	\$/unit/year
Unit sell price of items (p)	50	\$/unit
Fixed transport cost of a shipment (A_0)	153.2	\$/shipment
Fixed GHG emission cost of a shipment (A_1)	40.46	\$/shipment
Interest charges(I_c)	0.15	\$/dollar/year
Length of advance payment(t)	0.1	year
Opportunity cost (I_d)	0.1	\$/dollar/year

Variable transport (c_d)	0.1	\$/unit
Distance traveled(d)	500	km
Fuel consumption(l)	0.3	litters/km
Fuel price (Ω)	0.5107	\$/(litters˙unit)/shipment
Percent of purchase cost paid in advance(α)	0.16	

In addition, the supplier provides a price discount schedule as the following intervals: $[1,3000)$ corresponding to $c_1 = 20.15$, $[3000,6000)$ corresponding to $c_2 = 20.10$, $[6000,10000)$ corresponding to $c_3 = 20.05$, $[10000,15000)$ corresponding to $c_4 = 20.02$, and $[15000,\infty)$ corresponding to $c_5 = 20$.

Using the algorithm developed in Section 4, one has the optimal ordering quantity, number of shipments and minimum annual total cost as follows:

Step 1: From Eq. (7), we have

$$\hat{N}_k = 3, k = 1,2,\ldots,5.$$

Step 2: Plugging $\hat{N}_k = 3(k = 1,2,\ldots,5)$ into Eq. (3) and then obtain

$$\hat{Q}_1(3) = 14570.7,\ \hat{Q}_2(3) = 14632.3,\ \hat{Q}_3(3) = 14694.7,$$

$$\hat{Q}_4(3) = 14732.5,\ \hat{Q}_5(3) = 14757.8$$

Step 3: (1) Because $\hat{Q}_1(3) > 3000$, $\hat{Q}_2(3) > 6000$ $\hat{Q}_3(3) > 10000$, This implies the optimal solution may not occur at these results. We therefore do not further compute them.

(2) Because $10000 \le \hat{Q}_4 < 15000$, we substitute $\hat{Q}_4 = 14732.5$ and $\hat{N}_4 = 3$ into Eq. (2) and thus obtain $ATC_4(14732.5,3) = 9312277.0$

(3) Because $\hat{Q}_5 < 15000$, from Eq. (8), we then have

$\tilde{N}_5 = 5$ and $ATC_5(15000,5) = 9308923.4$

(4) Obtaining $\overline{N}_5$ from Eq. (10) and take $N_5{}^{\Delta} = \lfloor \overline{N}_5 \rfloor$, we have $N_5{}^{\Delta} = 4$

Computing $Q_5{}^{\Delta}(4)$ from Eq. (3), one has $Q_5{}^{\Delta}(4) = 17499.1$

Because $Q_5{}^{\Delta}(4) > 15000$, we have $ATC_5(17499.1,4) = 9350652.0$

(5) Taking $ATC_5(Q_5^{@}, N_5^{@}) = Min\left[ATC_5(Q_4, \tilde{N}_5), ATC_5(Q_5{}^{\Delta}, N_5{}^{\Delta})\right]$, we have

$$ATC_5(Q_5^{@}, N_5^{@}) = ATC_5(15000,5) = 9308923.4$$

Step 4: Taking $ATC(Q^*, N^*) = Min\left[ATC_4(\hat{Q}_4, \hat{N}_4), ATC_5(Q_5^{@}, N_5^{@})\right]$, one has

$$ATC(Q^*, N^*) = ATC_5(15000,5) = 9308923.4$$

Step 5: The optional order lot size is 15000 units corresponding to its unit-purchasing cost \$20; the optimal number of shipments is 5 times; and the minimum cost is \$9308923.4.

Based on the above discussion, we know that the optimal solution for the given parameter set is Q^*=15000 units and N^*=5 times, and the annual total cost is \$9308923.4., which meets the results obtained in our developed Algorithm. Note that if a single delivery policy is adopted, the optimal order quantity obtained from Eq. (3) is then 7931.9 units. The annual total cost is $ATC_3(7931.9,1) = 9325536.9$. Thus, the effects of multiple deliveries and quantity discounts, in general, for the powerful retailer is larger than the effect of carbon emission under single delivery.

VI. CONCLUSION

In contrast to those previous EOQ models with advance payment, we have taken quantity discount, lot-splitting deliveries, and carbon emission into consideration. As a result, this paper has developed the retailer's optimal policy when the supplier requests a partial advance payment before delivery and provides quantity discounts. We then demonstrate how the constructed model is a convex function and also derive property to help develop a two-stage solution procedure. An efficient algorithm has been developed to help managers make quick and accurate decisions. Numerical results in general showed that: (1) the lowest unit purchasing cost in the cost discount schedule may not guarantee that the retailer could obtain minimum annual total cost because the extra quantity purchased may add additional holding cost and thus add the annual total cost; (2) the length of the advance payment, the quantity discounts, and the cost of carbon emissions do impact the retailer's inventory policy; and (3) in general, the more carbon emission cost is, the larger ordering quantity is and the less number of shipment is.

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