

A Clustered Based Novel Approach of Graph Coloring

Sapna Arya¹, Manish Dixit²

^{1,2}Department of Computer Science & Information Technology
Madhav Institute Of Technology & Science Gwalior-474002 (M.P.) INDIA

Abstract: An efficient graph coloring algorithm is proposed in this research paper that uses less number of colors for a graph coloring problem. The proposed algorithm is applicable for all types of graph. The algorithm works by dividing the vertices into two clusters i.e. non-visited type of clusters including the nodes that are not colored and visited type of clusters including the nodes that are already colored and finds maximum number of color that have been filled into visited nodes. An assumption is taken that k number of colors is already known and the colors are selected from the same k -colors. The proposed algorithm is implemented on random graphs along with well known ECG algorithm. In most of the cases the proposed algorithm provides better result and uses less number of colors as compare to ECG algorithm.

Key Words: Graph Coloring, Neighbors of a Vertex, Combinational Optimization.

I. Introduction

A graph is a symbolic representation of a network and of its connectivity. It implies an abstraction of the reality so it can be simplified as a set of linked nodes. A graph is an ordered pair $G = (V, E)$ where, V is the vertex set whose elements are the vertices, or nodes of the graph. This set is often denoted $V(G)$. E is the edge set whose elements are the edges, or connections between vertices, of the graph. This set is often denoted $E(G)$. If the graph is undirected, individual edges are unordered pairs $\{u, v\}$ where u and v are vertices in V . If the graph is directed, edges are ordered pairs (u, v) .

Graph coloring is an approach of coloring the vertices of a graph such that no two adjacent vertices share the same color; this is called a vertex coloring. Similarly, an edge coloring assigns a color to each edge so that no two adjacent edges share the same color, and a face coloring of a planar graph assigns a color to each face or region so that no two faces that share a boundary have the same color.

Graph Coloring Problem is labeling of vertices such that no two adjacent vertices have the same color. Coloring the graph with minimum number of colors is also of extreme importance as it influences how efficiently a problem can be solved. In this paper we introduce an algorithm that colors the graph with minimum number of colors.

In graph's theory, completeness theory and operational research, Graph coloring problem is one of the most studies NP-hard problems [1]. Graph coloring algorithms is applicable in the solution of many real time problems such as timetabling and resource assignment. In last decades many researchers has worked in Graph coloring algorithms.

In 1979, Daniel Brelaz [2] has describes new heuristic methods to color the vertices of a graph which relay upon the comparison of the degrees and structure of a graph. He developed a method that is exact for bipartite graphs and is important part of heuristic procedure to find maximal cliques in general graphs.

In 1987, A.Hertz and D. de Werra [3] have describe tabu search technique that are adapted to graph coloring problems. They exploit tabu method for recycling and being trpped in local minima. They provide almost optimal colorings of graphs having upto 1000 nodes and good heuristic for graph coloring.

In 1988, Jonathan S. Turner [4] have describe a simple and efficient heuristic algorithm for graph coloring problem and for all $k \geq 1$, it find an most advantageous coloring for almost all k -colorable graphs.

In 1999, Philippe Galinier and Jin-Kao Hao [5] have describe some hybrid algorithms for the graph coloring problem. These algorithms combine a new class of highly specialized crossover operators and a well-known tabu search algorithm.

In 2000, Anna Marino and Robert I. Damper [6] have describe a genetic algorithm that breaks the symmetry of the graph coloring problem by fixing the colors of the nodes in a large clique of a graph. They developed a approach that have been conducted on both standard and random graphs to demonstrate the effectiveness.

In 2006, Dr. Hussein Al-Omari and Khair Eddin Sabri [7] have describe two new heuristic graph-coloring algorithms, i.e. based on known heuristic algorithms. One is a modification of the largest degree ordering (LDO) algorithm, and the other one is a modification of saturated degree ordering (SDO) algorithm. They developed two new heuristic graph coloring algorithms were better than the original ones with respect to the number of used colors.

II. Proposed Methodology

This paper describes a new method to color vertices of a graph. Two clusters are formed by the algorithm that are visited node cluster and non visited node cluster. The architecture of the proposed methodology is shown in figure 1.

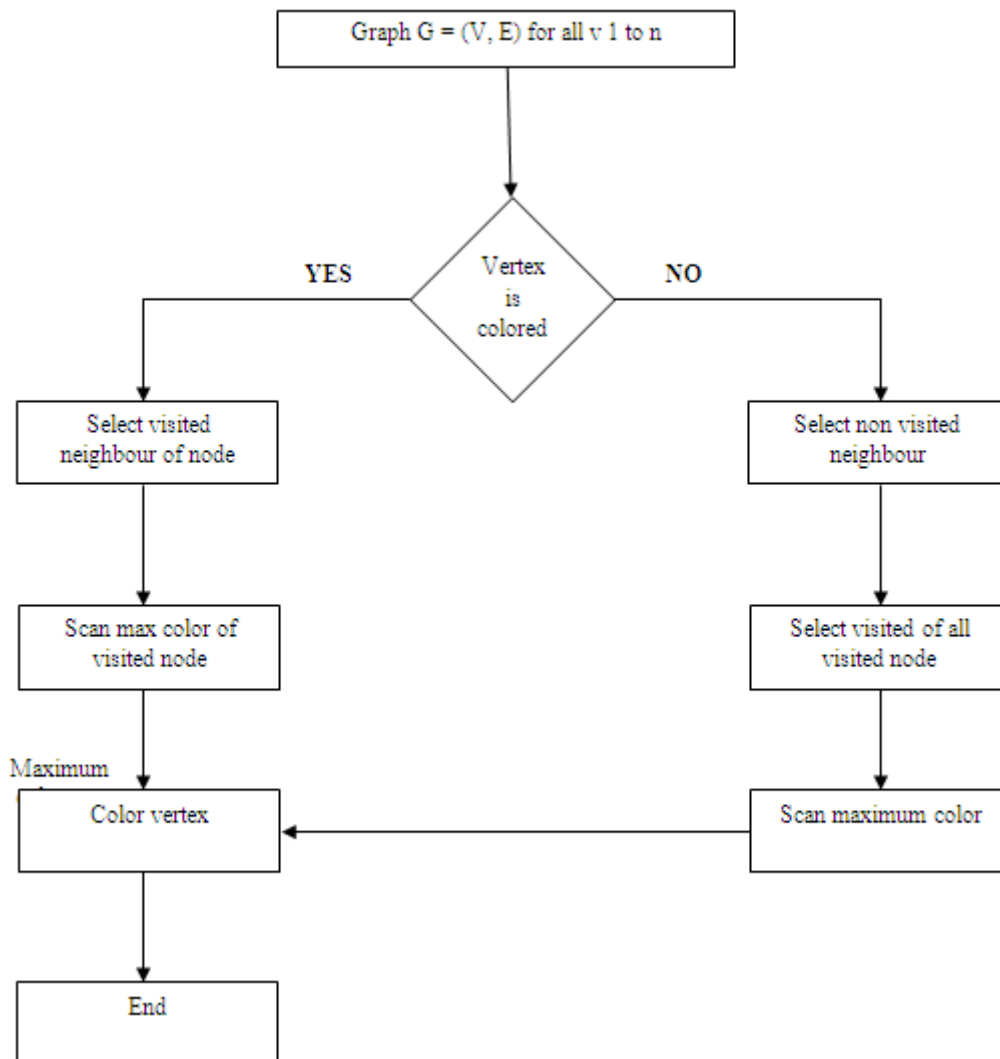


Fig 1 Architecture of proposed algorithm

Proposed Algorithm

Input: A Graph $G = (V, E)$ with number of vertices $V = N$; List of colors, K .

Output: The input graph $G = (V, E)$ with efficiently colored vertices.

While ($i \leq n$)

```

{
  if colored ( $V_i$ ) = true
  then
  {
     $N\_visited\_Neighbour\ i = select\_non\ visited\ (V_i)$ 
     $j = 1$ 
    While ( $j \leq N\_visited\_Neighbour\ i$ )
    {
       $visited\_Node\ j = select\_visited\_Node\ (V_j)$ 
       $C_k = Scan\_visited\ Node\_color\ (visited\_Node\ j)$ 
       $Color\ (V_j, C_{k+1})$ 
       $j++$ 
    }
  }
}
    
```

```

else
{
  V_Neighbour i = Select_visited Node ( Vi )
  Ck = Scan_visited Node ( visited_Neighbour i )
  Color ( Vi, Ck+1 )
}
i++
}
}

```

Implementation with Example

The proposed algorithm can be implemented globally for all graphs. The graph shown in fig 2 is used to evaluate the performance of the proposed algorithm.

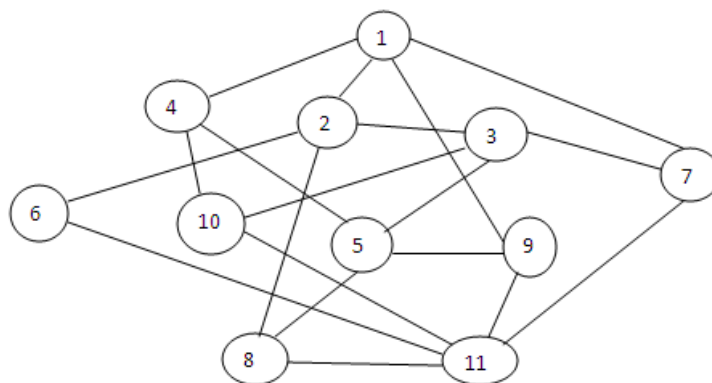


Fig 2. Random graph

No. of vertices in graph (n) = 11

Let the color vector C = [Blue, Red, Green, Yellow, Pink, Grey,...]

On implementing the proposed algorithm for above graph in a very first step the vertex 1 is colored using the color Blue and all the next steps are shown in table 1

| Steps | Current vertex | Color | Non visited neighbour | Visited neighbour |
|-------|----------------|--------|-----------------------|-------------------|
| 1 | 1 | Blue | 2, 4, 7, 9 | - |
| 2 | 2 | Red | - | 1 |
| 3 | 4 | Red | - | 1 |
| 4 | 7 | Red | - | 1 |
| 5 | 9 | Red | - | 1 |
| 6 | 3 | Blue | 5, 10 | 2, 7 |
| 7 | 5 | Green | - | 3, 4, 9 |
| 8 | 10 | Green | - | 3, 4 |
| 9 | 6 | Blue | 11 | 2 |
| 10 | 11 | Yellow | - | 6, 7, 9, 10 |
| 11 | 8 | Blue | - | 2, 5, 11 |

Table 1. Steps of implementation of proposed algorithm to Graph in fig 2

The algorithm uses 4 colors that are Blue, Red, Green and Yellow . Initially the vertices of the graph numbered from 1 to 11. The vertex 1 is colored with color Blue, now in step 2 the vertex 1 is already colored so it starts coloring its non visited neighbour. Vertex 2 has only 1 visited neighbour i.e. vertex 1 and on scanning the color of vertex 1 is Blue color therefore the algorithm colors the vertex 2 with Red color. Now it takes vertex 2 which is already colored therefore scanning is processed as a result vertex 6,4,7 and 9 are vertices found to be the non visited neighbour of vertex 3and the algorithm will continue until all the vertices of the graph are colored. As a result the graph is colored using 4 colors.

III. Result and Discussion

The proposed algorithm is implemented to random graphs that have different number of vertices and densities. The ECG algorithm is also implemented using the same graph the comparison among both the algorithm and their result is shown in table 2.

| Vertices | Edges | K _{ECG} | K _{NEW} |
|----------|-------|------------------|------------------|
| 450 | 16680 | 18 | 15 |
| 138 | 493 | 6 | 11 |
| 250 | 3218 | 10 | 8 |
| 64 | 728 | 12 | 9 |
| 95 | 755 | 9 | 7 |
| 191 | 2360 | 12 | 8 |
| 81 | 1056 | 12 | 10 |

Table 2. Experimental Results and Comparisons of ECG and our Proposed Algorithm

As it is clear that most of the cases the proposed algorithm of graph coloring provides better results than ECG algorithm.

References:

- [1]. M. Bessedic, R. Laib, A. Boulmerka, "Ant Colony System for Graph Colring Problem" CIMCA-IAWTIC'05
- [2]. Brelaz, D., "New methods to color vertices of a graph" Communications of the ACM 22, pp. 251 –256, 1979
- [3]. Hertz, A. and D. de Werra, "Using tabu search techniques for graph coloring" Computing 39 (4), 345–351, 1987
- [4]. Turner, J. S., "Almost all k-colorable graphs are easy to color" Journal of Algorithms 9 (1), pp. 63–82, 1988
- [5]. Galinier, P. and J-K. Hao, " Hybrid evolutionary algorithms for graph coloring" Journal of Combinatorics, 3(4), pp. 379 –397, 1999
- [6]. Marino, A. and Damper, R. I., "Breaking the symmetry of the graph colouring problem with genetic algorithms" Genetic and Evolutionary Computation Conference (GECCO-2000), 2000
- [7]. Al-Omari and Sabri, "New Graph Colouring Algorithms," American Journal of Mathematics and Statistics 2 (4), pp.739-741, 2006.
- [8]. Nikhil Chaturvedi, Saurabh Agarwal, Punit Kumar Johari, "A Novel approach of image retrieval based on Texture". IJERMCA 2014,pp 42-48.