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# Car Dynamics using Quarter Model and Passive Suspension, Part VI: Sprung-mass Step Response

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***Abstract***: *The objective of the paper is to investigate the step response of a 2 DOF quarter-car model with passive suspension. The mathematical models of the sprung-mass displacement and acceleration as response to the step road disturbance are derived. The step response of the system is plotted using MATLAB for a 100 mm step amplitude. The effect of the suspension parameters on the step response of the sprung-mass is investigated in terms of motion displacement and acceleration. The maximum percentage overshoot and settling time of the step response are evaluated as time-based characteristics of the dynamic system for a specific range of the suspension parameters. The maximum acceleration of the sprung-mass due to the step road disturbance did not conform with the ride comfort requirements according to ISO 2631-1 standard .*

***Keywords:*** *Car dynamics; quarter car model; passive suspension; step response, sprung-mass acceleration*

## I. Introduction

Car dynamics can be studied in various ways including time response to speed humps, frequency response and step response. The latest is borrowed from automatic control engineering to investigate the dynamic system in the time domain using the most known input type. This type of input is used by a large number of researchers dealing with car dynamics.

Masi (2001) used computer simulation to examine the effect of control methods on the performance of semiactive dampers in controlling the dynamics of a quarter-car model. One of the inputs he used was a step input of 12.7 mm amplitude [1]. Askerda et. al. (2003) developed a methodology for analyzing the impact of data errors on control system dependability. Their applied their technique on a car slip-control brake-system. They used a step input command signal in terms of the normalized car-slip [2]. Toshio and Atsushi (2004) presented the construction of a pneumatic active suspension system for a one-wheel car model using fuzzy reasoning and disturbance observer. They used a step input of 1 V amplitude to the actuator [3]. Sakman, Guclu and Yagiz (2005) investigated the performance of fuzzy logic controlled active suspension on a nonlinear model with 4 DOF. They demonstrated the effectiveness of the fuzzy logic controller for active suspension systems. They used a half-car model with 4 DOF. They used a step road surface input of 35 mm amplitude resulting in chassis maximum acceleration of about 3.5 m/s$^2$ [4].

Faheem, Alam and Thomson (2006) investigated the response of quarter and half-car models to road excitation. They studied the time response to a step road input and the frequency response of the sprung and unsprung masses [5]. Maila and Priyandoko (2007) presented the design of a control technique for active suspension system of a quarter-car model using adaptive fuzzy logic and active force control. They tested their control strategy using a 50 mm step road input resulting in a 75 mm maximum sprung-mass displacement and 20 m/s2 maximum acceleration using passive elements. The active force control techniques reduced the maximum acceleration to about 10 m/s$^2$ [6]. Hanafi (2009) designed a fuzzy logic control system for a quarter-car model. He used a step road disturbance of 100 mm amplitude [7]. Alexandru and Alexandru (2010) presented a virtual prototype of an automobile suspension system with force generator actuator. They used a quarter-car model and a step input of 100 mm amplitude. The time response of the sprung-mass exhibited an overshoots and undershoots [8].

Unaune, Pawar and Mohite (2011) simulated the time response of a quarter-car model to a step input with passive elements suspension. They investigated the effect of mass ratio, stiffness ratio and damping coefficient ratio on the sprung-mass dynamics. They didn't show the amplitude of the step input used in the simulation [9]. Fayyad (2012) presented a control system for active suspension system to improve ride comfort even at resonance. He used a step road input of 80 mm amplitude to a quarter-car model. The passive elements resulted in a sprung-mass overshoot of 59 % while with the active control it was 22.5 %. The maximum acceleration to the step input used was 17 m/s2 with passive elements and 1.3 m/s$^2$ with active suspension [10]. Chikhale and Deshmukh (2013) performed a vibration analysis of a quarter-car model to a step input using MATLAB and MSc. ADAMS. They used the system state space representation in deriving the mathematical model of the system. They used a step input of 25 mm amplitude. The sprung-mass had 62 % overshoot in its displacement response to the step input [11]. Faruk, Bature, Babani and Dankadai (2014) used PID conventional controller and integrated fuzzy logic control schemes for good driving comfort and effective isolation of road disturbance. They simulated the quarter-car step input response for a 1 m step amplitude !!. The time response without control had a 75 % overshoot reduced to zero with PID and fuzzy controllers [12]. Patole and Swant (2015) studied the efforts of researchers about nonlinearity of passive suspension parameters. They referred to ride comfort through vibration amplitude and ride comfort level for model inputs including a step type [13].

## II. The Quarter-Car Model

Researchers use quarter car model in studying the vehicle dynamics. This model depending on the degree of simplification used by the researcher may be considered as a single (1DOF), two (2DOF) or three degree of freedom (3DOF) [14,15,16]. A quarter-car model with 2 DOF will be considered in this work because of the complexity of its mathematical model if 3 DOF one is considered. The line diagram of the 2 DOF quarter-car model is shown in Fig.1 [17].

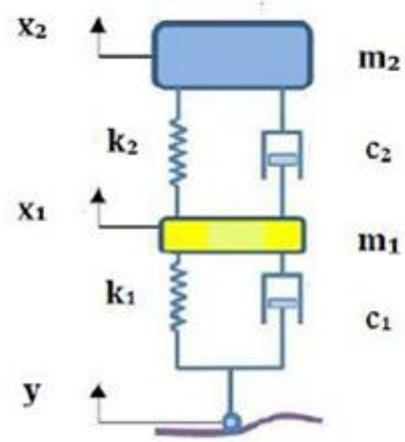


**Fig. 1 2** DOF quarter-car model [17].

The model of Fig.1 consists of:

- One tire and wheel and its attachments having the parameters: mass ($m_1$), damping coefficient ($c_1$) and stiffness ($k_1$).
- Sprung mass which is one quarter of the car chassis having the parameters: mass ($m_2$), damping coefficient ($c_2$) and stiffness ($k_2$).
- $c_2$ and $k_2$ are the suspension parameters.

## III. Model Parameters

The 2 DOF quarter-car model parameters used in this analysis work are given in Table 1. The suspension parameters $k_2$ and $c_2$ are set in the range given in the table to investigate their effect on the step response of the vehicle. The tire and wheel parameters ($m_1$, $k_1$ and $c_1$) and sprung mass ($m_2$) are borrowed from reference [17].

**Table 1 2** DOF model parameters

| Parameters | Description | Value |
|---|---|---|
| **$k_1$ (N/m)** | Tire stiffness | 135000 |
| **$c_1$ (Ns/m)** | Tire damping coefficient | 1400 |
| **$m_1$ (kg)** | Un-sprung mass | 49.8 |
| **$k_2$ (N/m)** | Suspension stiffness | 1000 – 3000 |
| **$c_2$ (Ns/m)** | Suspension damping coefficient | 1000 – 4000 |
| **$m_2$ (kg)** | Sprung mass | 466.5 |

## IV. Sprung-Mass Displacement

The mathematical model of the 3 DOF quarter-car model is defined by three differential equations obtained by applying Newton's third law to the free body diagram of the three lumped masses of Fig.1. The dynamic motions of the system are: $x_1$, $x_2$ and $x_3$. The irregularities of the road is defined by the dynamic motion y which is assumed sinusoidal. Of course, the irregularities are random. The frequency contents of this irregularity can be defined using FFT (fast Fourier transform). Thus, the sinusoidal input motion y represents one component in this transform. All the parameters of the system are passive and assumed having the constant values presented in section 3. The differential equations are written in standard form as follows:

For the tire and wheel mass, $m_1$:

$$m_1x_1'' + (c_1+c_2)x_1' - c_2x_2' + (k_1+k_2)x_1 - k_2x_2 = c_1y' + k_1y \quad (1)$$

For the sprung mass, $m_2$:

$m_2x_2'' - c_2x_1' + c_2x_2' - k_2x_1 + k_2x_2 = 0$ (2)

The Laplace transform of Eqs.1 and 2 gives:

$[m_1s^2 + (c_1+c_2)s + k_1+k_2]X_1(s) – (c_2s+k_2)X_2(s) = (c_1s+k_1)Y(s)$ (3)

And $(m_2s^2+c_2s+k_2)X_2(s) – (c_2s+k_2)X_1(s) = 0$ (4)

Where $X_1(s)$ and $X_2(s)$ are the Laplace transform of the motions $x_1$ and $x_2$ respectively, and s is the Laplace operator.

Solving Eqs.3 and 4 for X2(s)/Y(s) gives:

$X_2(s)/Y(s) = (c_1s+k_1)(c_2s+k_2) / \{[m_1s^2 + (c_1+c_2)s + k_1+k_2] (m_2s^2+c_2s+k_2) – (c_2s+k_2)^2\}$ (5)

Eq.5 is the transfer function of the dynamic system. Writing Eq.5 in a standard form gives:

$X_2(s)/Y(s) = N_1(s) / D_1(s)$ (6)

Where: $N_1(s) = c_1c_2s^2 + (c_1k_2+c_2k_1)s + k_1k_2$

and $D_1(s) = m_1m_2s^4 + [m_1c_2+m_2(c_1+c_2)]s^3 + [m_1k_2+c_2(c_1+c_2)+m_2(k_1+k_2)-c_2^2]s^2 + [k_2(c_1+c_2)+c_2(k_1+k_2)-2c_2k_2]s + [k_2(k_1+k_2)-k_2^2]$

## V. Sprung-Mass Acceleration

- In the derivation of the sprung mass displacement, the variables x'', x' , x and y are used.
- For sake of the mathematical model of the sprung mass acceleration, the acceleration variables $a_1$, $a_2$, velocity variables $v_1$, $v_2$ and displacement variables $x_1$, $x_2$ will be used.
- The differential equations of the dynamic system (Eqs.1 and 2) in terms of the acceleration, velocity and displacement variables will take the form:

$m_1a_1 + (c_1+c_2)v_1 – c_2v_2 + (k_1+k_2)x_1 – k_2x_2 = c_1y' + k_1y$ (7)

And $m_2a_2 - c_2v_1 + c_2v_2 – k_2x_1 + k_2x_2 = 0$ (8)

- The acceleration, velocity and displacement variables are related through:

$a_1 = v_1'$ (9)

$a_2 = v_2'$ (10)

$v_1 = x_1'$ (11)

$v_2 = x_2'$ (12)

- Taking the Laplace transform of Eqs.7 through 12 gives the Laplace transform ratio A2(s)/Y(s) as:

$A_2(s)/Y(s) = N_2(s) / D_2(s)$ (13)

Where: $N_2(s) = c_1c_2s^4 + (k_1c_2 + k_2c_1) s^3 + k_1k_2s^2$ (14)

$D_2(s) = m_1m_2s^4 + [m_2(c_1+c_2) + m_1c_2] s^3 + [m_2(k_1+k_2) + c_2(c_1 + c_2) + m_1k_2 – c_2^2] s^2 + [c_2(k_1+k_2) + k_2(c_1+c_2) – 2k_2c_2] s + k_2(k_1+k_2) – k_2^2$ (15)

## VI. Sprung-Mass Step Displacement Response

The sprung mass displacement is evaluated using the transfer function of the spring-mass displacement relative to the ground motion (Eq.6). MATLAB command 'step' is used for this purpose for an input step of 100 mm amplitude [18]. The dynamics of the quarter-car model is evaluated in response to a step road input for suspension parameters in the range:

$1000 \leq c_2 \leq 4000$ Ns/m , $1000 \leq k_2 \leq 3000$ N/m (16)

The step response of the sprung mass for the suspension parameters in the range of Eq.16 is shown in Figs.2 through 6.

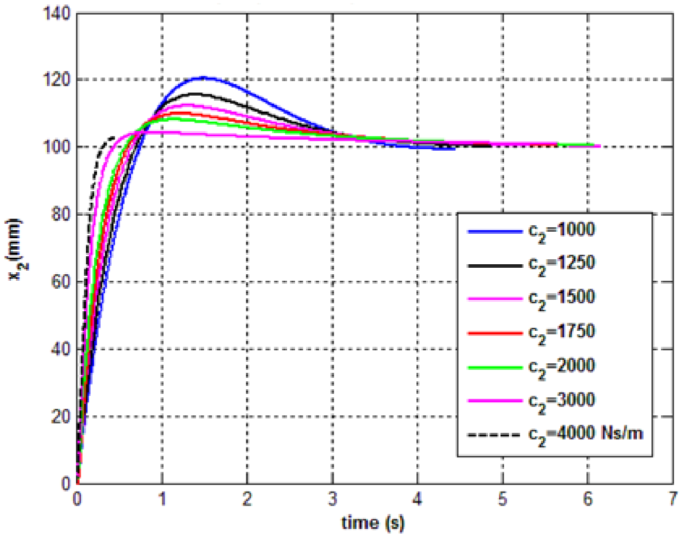


**Fig.2** Sprung-mass step response for $k_2$ = 1000 N/m.

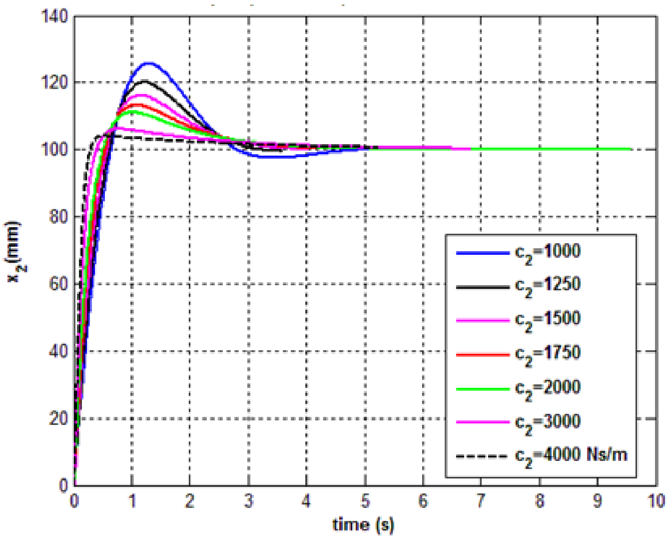


**Fig.3** Sprung-mass step response for $k_2$ = 1500 N/m.

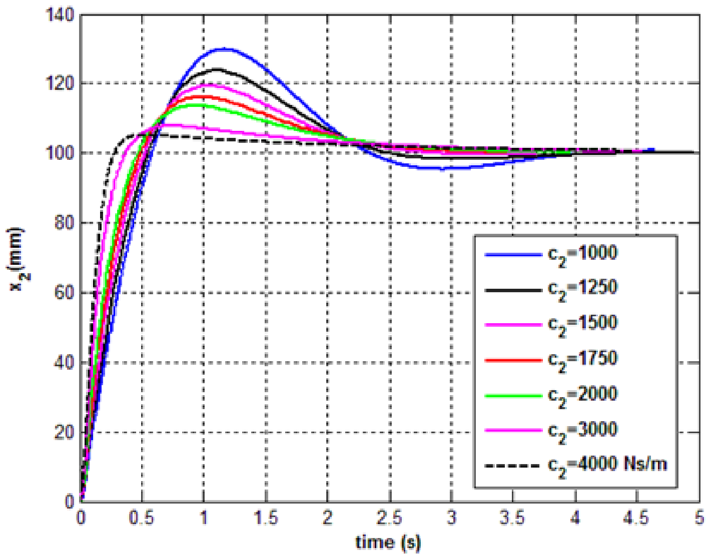


**Fig.4** Sprung-mass step response for $k_2$ = 2000 N/m.

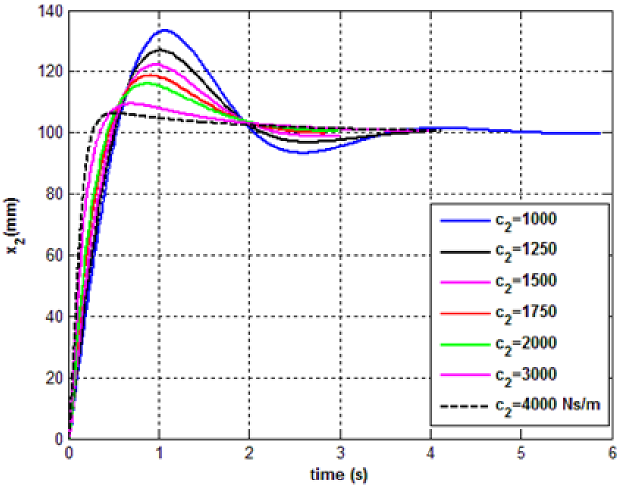


**Fig.5** Sprung-mass step response for $k_2$ = 2500 N/m.

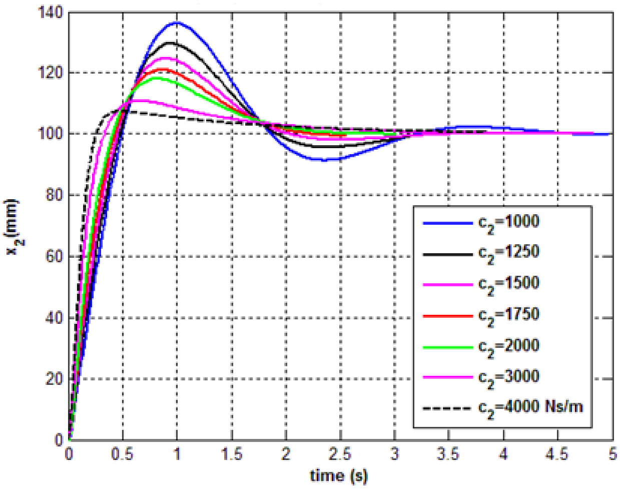


**Fig.6** Sprung-mass step response for $k_2$ = 3000 N/m.

- The step response of the sprung-mass exhibits an overshoot and undershoot depending of the level of the suspension damping coefficient.
- The maximum overshoot and maximum undershoot decreases as the damping coefficient increases.
- All the step response curves of the sprung-mass displacement intersect in more than one point.
- The effect of the suspension parameters $c_2$ and $k_2$ on the maximum percentage overshoot of the sprung-mass step response is shown graphically in Fig.7.

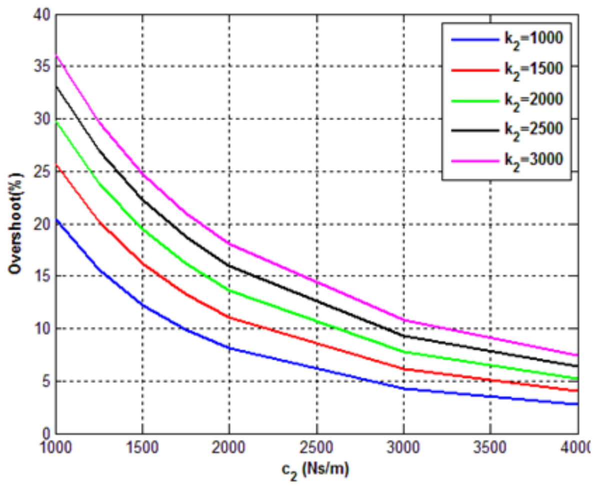


**Fig.7** Sprung-mass maximum displacement overshoot.

- The maximum percentage overshoot decreases as the suspension damping coefficient increases and the suspension stiffness decreases.
- The settling time of the sprung-mass displacement is shown in Fig.8.

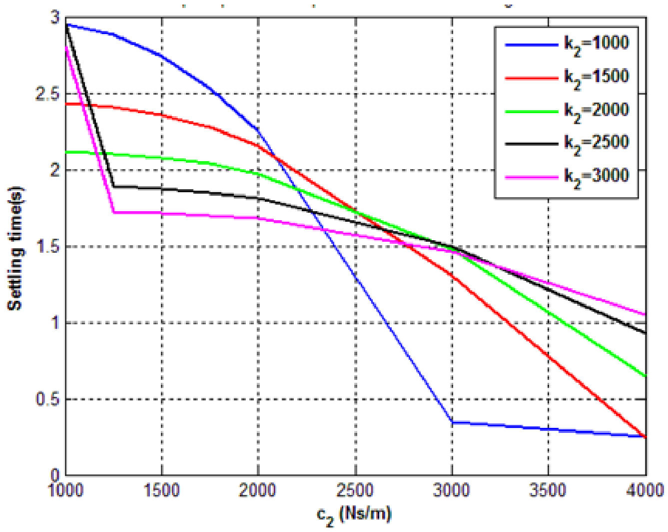


**Fig.8** Sprung-mass displacement settling time .

- The settling time decreases as the suspension damping coefficient increases.
- The effect of the suspension stiffness is not uniform.
- versed between the first and second intersections.

## VII. Sprung-Mass Step Acceleration Response

- The step response of the sprung-mass acceleration is simulated using Eq.13 and the 'step' command of MATLAB.
- The step response in terms of the sprung-mass acceleration is in Figs.9-13 for suspension stiffness of 1000, 1500, 2000, 2500 and 3000 N/m respectively.

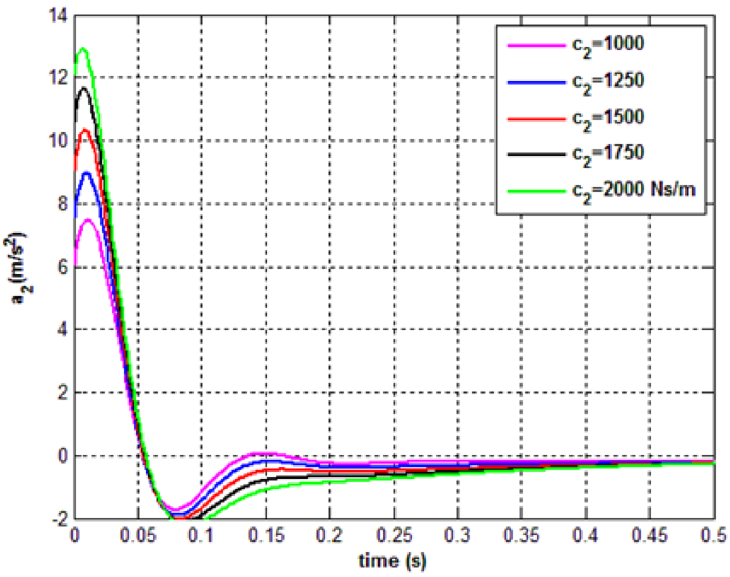


**Fig.9** Sprung-mass step response (acceleration) for $k_2$ = 1000 N/m.

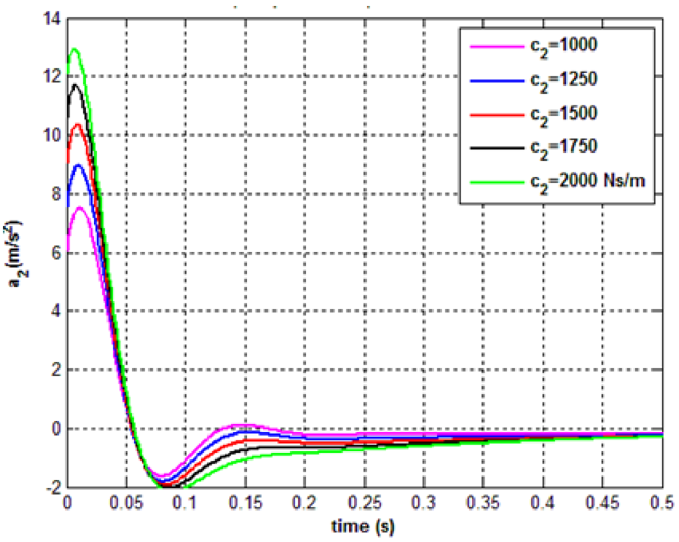


**Fig.10** Sprung-mass step response (acceleration) for $k_2$ = 1500 N/m.

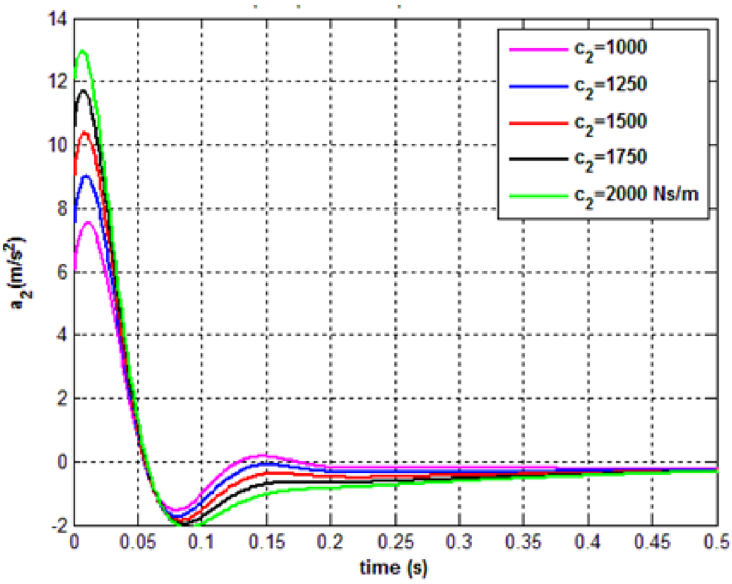


**Fig.11** Sprung-mass step response (acceleration) for $k_2$ = 2000 N/m.

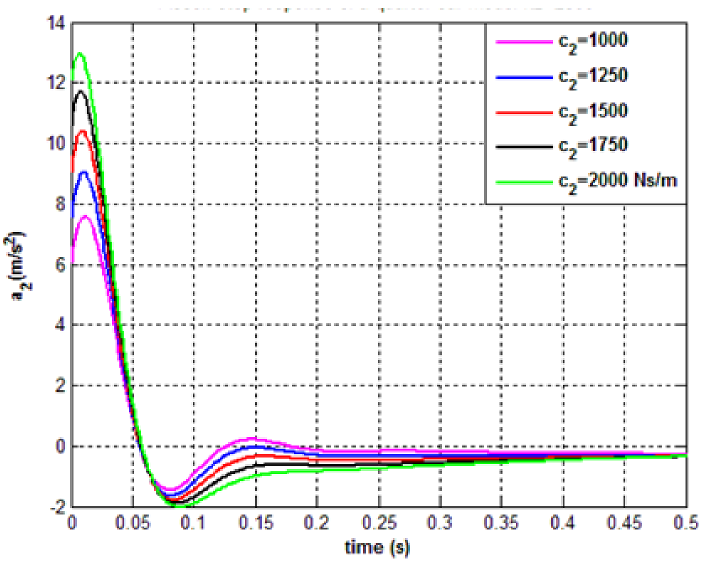


**Fig.12** Sprung-mass step response (acceleration) for $k_2$ = 2500 N/m.

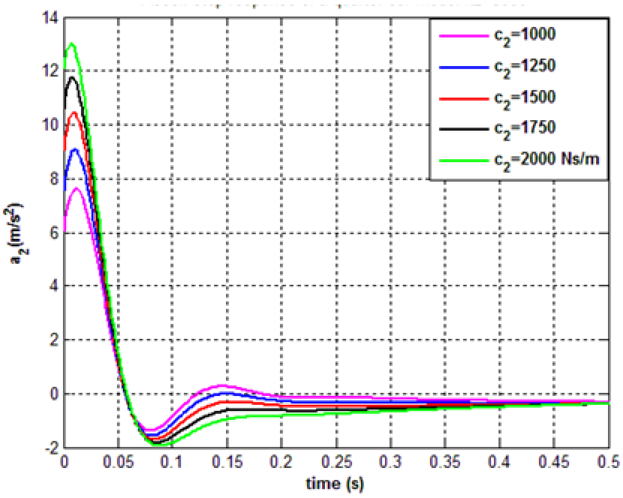


**Fig.13** Sprung-mass step response (acceleration) for $k_2$ = 3000 N/m.

- The sprung-mass acceleration exhibits overshoot and undershoot characteristics relative to the zero acceleration line.
- The effect of the suspension parameters on the maximum acceleration of the sprung-mass is shown in Fig.14.

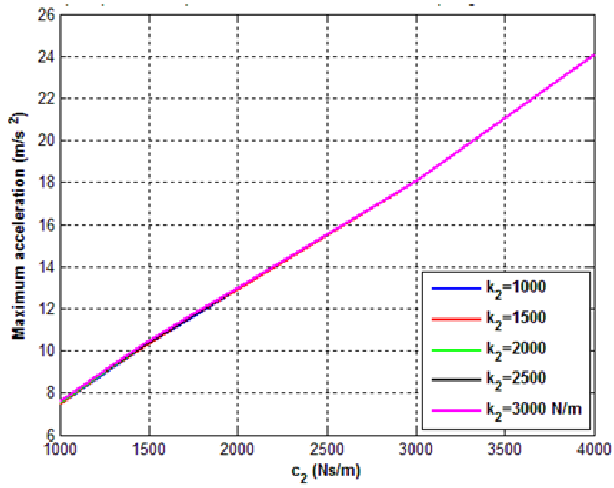


**Fig.14** Maximum acceleration of the sprung-mass due to the step input.

- The maximum acceleration of the sprung mass increases with increased damping.
- The maximum acceleration increases almost linearly with the suspension damping coefficient.
- The suspension stiffness has almost no effect on the maximum acceleration of the sprung-mass.
- Fig.15 shows the effect of the suspension parameters on the acceleration response settling time as evaluated by MATLAB.

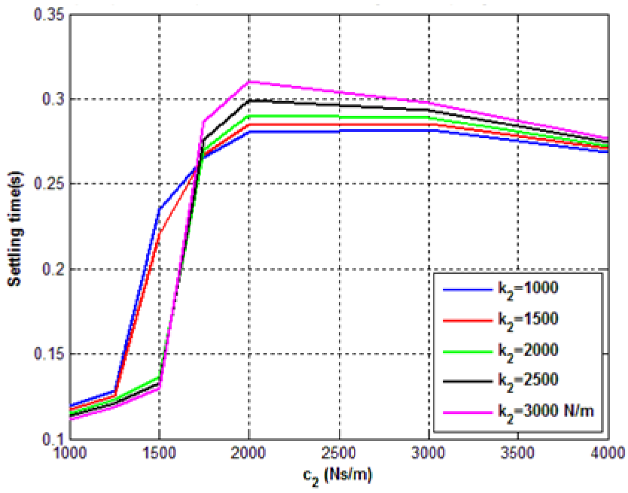


**Fig.15** Settling time of the sprung-mass due to the step input.

- The settling time increases to a maximum and then decreases again.
- For all values of the suspension stiffness, the settling time is maximum at a damping coefficient of about 2000 Ns/m.
- All the characteristic curves of the settling time intersect at a damping coefficient of about 1700 Ns/m.

## VIII. Conclusion

- The dynamic model of the 2 DOF quarter-car dynamic system was derived assuming passive parameters for all dampers and springs in the system.
- The step response of the system was presented in terms of the suspension parameters (damping coefficient and stiffness).
- Using MATLAB, the step response of the sprung-mass was presented for suspension parameters of damping coefficient in the range 1000 to 4000 Ns/m and stiffness in the range 1000 to 3000 N/m.
- The maximum overshoot of the sprung-mass displacement has decreased from 20.30 to 2.73 % when the damping coefficient of the suspension increased from 1000 to 4000 Ns/m at 1000 N/m suspension stiffness.
- The settling time of the sprung-mass displacement has decreased from 2.95 to 0.248 s when the damping coefficient of the suspension increased from 1000 to 4000 Ns/m at 1000 N/m suspension stiffness.
- The suspension stiffness did not affect the maximum acceleration of the sprung-mass during the step response.
- With suspension damping coefficient of $\leq$ 1700 Ns/m, increasing the suspension stiffness decreased the settling time of the sprung-mass acceleration. With damping coefficient > 1700 Ns/m, this action was reversed.
- The sprung-mass settling time of its acceleration step response was increased with increased suspension damping coefficient up to 2000 Ns/m after which it started decreasing with less rate.
- The maximum acceleration due to the 100 mm step road disturbance was completely extremely uncomfortable according to ISO 2631-1 [19].
- Such sudden road disturbance has to be avoided because of its harmful effect on passengers and vehicles.
- The alternative is using more smooth speed control humps such as simple harmonic motion hump [15] or polynomial hump [20].

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**Biography**

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