IOSR Journal of Computer Engineering (IOSR-JCE)
e-ISSN: 2278-0661,p-ISSN: 2278-8727, Volume 18, Issue 4, Ver. II (Jul.-Aug. 2016), PP 56-62
www.iosrjournals.org

Lossless and Lossy Polynomial Image Compression

Ghadah Al-Khafaji and Maha A. Rajab
Department of Computers, College of Science, BaghdadUniversity, Baghdad, Iraq
Department of Computers, College ofIbn Al-Haytham, BaghdadUniversity, Baghdad, Iraq

Abstract: *This paper introduced a new hybrid image compression system of the lossless non-linear polynomial coding base and lossy linear polynomial base. The results are promising, since it would achieve high compression ratio with excellent medical image quality.*
Keywords: *lossless and lossy, linear and non-linear polynomial coding.*

I. Introduction

Nowadays, compression is become an essential requirement of transmission applications and storage. Image compression is the application of data compression on digital images [1], that aims to eliminate the data redundancies, which simply categorized into three essential redundancies: inter-pixel redundancy, coding redundancy and psycho-visual redundancy[2-3], for more details see [4-5].

In general, the image compression techniques classified into two types: lossless and lossy depending on the redundancies utilized, the first one also called information preserving or error free techniques, as their name indicates that no loss of information and the reconstructed image is identical to the original one and based on using the inter pixel redundancy and/or coding redundancy, that characterized by low compression ratio, with techniques such as Huffman coding, arithmetic coding and run length coding [6-7]. The second one, where some information may be lost through the processing but the distortion level of the reconstructed image must be acceptable and cannot be known by the human visual system. In other words, the original image can not be reconstructed exactly from the compressed data, where there is some degradation on image quality based on using of psycho-visual redundancy, either alone or combined with statistical redundancy with high compression ratio [8-9]. Review on various image compression techniques can be found in [6-10-11].

The linear polynomial coding is a modern effective image compression technique used by a number [12-13-14] based on using the image spatial domain that work either of linear base model or of nonlinear base model [15-16-17].

This paper is completely dedicated to the investigation of the hybrid compression system to compress the images effectively, using the lossy linear polynomial coding technique (first order Taylor series) and lossless non- linear polynomial coding technique (second order Taylor series).

This paper is organized as follows; section 2 discussed the proposed compression system. Section 3 explained experimental results and discussion. Conclusions are shown in Section 4.

II. The Proposed Compression System

The hybrid proposed compression system mixed the lossy and lossless types along with utilization of polynomial coding of linear (needthree coefficients a_0,a_1,a_2) and nonlinear base (i.e., require six coefficients a_0,a_1,a_2,a_3,a_4,a_5). The source coding and decoding of the proposed system are illustrated in figure 1and figure 2 respectively. Figure 3, shows an example of the proposed compression techniques.The following steps are illustrated the proposed hybrid image compression system:

1. Load the input uncompressed grayscale image *I* of size (*N*×*N*).

2.Partition the image (*I*) into nonoverlapped blocks of fixed size *n*×*n*, such as (8×8), then create an image called sampled image (*S*) of size quarter than the partitioned image *I*. in other words, the technique used two images one corresponding to the partitioned image *I* and the sampled image of small redundancy[7].

3. Apply non-linear polynomialcoding technique on the partitioned sampled image *S* of fixed size *n*×*n* to compute the estimated coefficients according to equations (1-17) below [18].

$$a_0=\frac{\begin{vmatrix} V_1 & W_2 & W_2 \\ V_2 & W_3 & W_4 \\ V_3 & W_4 & W_3 \end{vmatrix}}{\begin{vmatrix} W_1 & W_2 & W_2 \\ W_2 & W_3 & W_4 \\ W_2 & W_4 & W_3 \end{vmatrix}} \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots.(1)$$

$$a_1 = \frac{\sum_{x=0}^{n-1}\sum_{y=0}^{n-1} S(x,y)(x-xc)}{\sum_{x=0}^{n-1}\sum_{y=0}^{n-1}(x-xc)^2} \ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

$$a_2 = \frac{\sum_{x=0}^{n-1}\sum_{y=0}^{n-1} S(x,y)(y-yc)}{\sum_{x=0}^{n-1}\sum_{y=0}^{n-1}(y-yc)^2} \ldots\ldots\ldots\ldots\ldots\ldots\ldots (3)$$

$$a_3 = \frac{\begin{vmatrix} W_1 & V_1 & W_2 \\ W_2 & V_2 & W_4 \\ W_2 & V_3 & W_3 \end{vmatrix}}{\begin{vmatrix} W_1 & W_2 & W_2 \\ W_2 & W_3 & W_4 \\ W_2 & W_4 & W_3 \end{vmatrix}} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4)$$

$$a_5 = \frac{\sum_{x=0}^{n-1}\sum_{y=0}^{n-1} S(x,y)(x-xc)(y-yc)}{\sum_{x=0}^{n-1}\sum_{y=0}^{n-1}(x-xc)^2(y-yc)^2} \ldots\ldots\ldots (5)$$

$$a_4 = \frac{\begin{vmatrix} W_1 & W_2 & V_1 \\ W_2 & W_3 & V_2 \\ W_2 & W_4 & V_3 \end{vmatrix}}{\begin{vmatrix} W_1 & W_2 & W_2 \\ W_2 & W_3 & W_4 \\ W_2 & W_4 & W_3 \end{vmatrix}} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (6)$$

$$xc = yx = \frac{n-1}{2} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (7)$$

$$V_1 = a_0 W_1 + a_3 W_2 + a_4 W_2 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (8)$$

$$V_2 = a_0 W_2 + a_3 W_3 + a_4 W_4 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (9)$$

$$V_3 = a_0 W_2 + a_3 W_4 + a_4 W_3 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (10)$$

$$W_1 = n \times n \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (11)$$

$$W_2 = \sum_{x=0}^{n-1}(x-xc)^2 = \sum_{y=0}^{n-1}(y-yc)^2 \ldots\ldots\ldots\ldots\ldots (12)$$

$$W_3 = \sum_{x=0}^{n-1}(x-xc)^4 = \sum_{y=0}^{n-1}(y-yc)^4 \ldots\ldots\ldots\ldots\ldots (13)$$

$$W_4 = \sum_{x=0}^{n-1}\sum_{y=0}^{n-1}(x-xc)^2\,(y-yc)^2 \ldots\ldots\ldots\ldots\ldots\ldots (14)$$

Where

$$V_1 = \sum_{x=0}^{n-1}\sum_{y=0}^{n-1} S(x,y) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (15)$$

$$V_2 = \sum_{x=0}^{n-1}\sum_{y=0}^{n-1}(x-xc)^2\, S(x,y) \ldots\ldots\ldots\ldots\ldots\ldots\ldots (16)$$

$$V_3 = \sum_{x=0}^{n-1}\sum_{y=0}^{n-1}(y-yc)^2\, S(x,y) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (17)$$

Where *S* represent the sampled image. Where *xc* equal to *yc* that represents a block center of size (*n*×*n*).

4. Create the predicted image sampled$\tilde{S}$as a nonlinear combination polynomial model of coefficients and pixel distance as equation (18), such as [18]:

$$\tilde{S} = a_0 W_1 + a_1(x-xc) + a_2(y-yc) + a_3(x-xc)^2 + a_4(y-yc)^2 + a_5(x-xc).(y-yc) \ldots\ldots\ldots (18)$$

5. Find the residual image Res or prediction error (as difference between original and prediction image), as equation (19).

$$\mathrm{Res}(x,y) = \mathrm{S}(x,y) - \tilde{S}(x,y) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (19)$$

6. Encode the non-linearestimated coefficients and resultant residual image losslessly, using the run length coding (RLC) which is passed through Huffman coding to remove the coding redundancy.

7. Apply linear polynomial coding technique on the original partitioned image$I$ to compute the estimated coefficients (a_0,a_1 and a_2) according to equations (20-22)[19].

$$a_0 = \frac{1}{n \times n}\sum_{x=0}^{n-1}\sum_{y=0}^{n-1} I(x,y) \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots (20)$$

$$a_1 = \frac{\sum_{x=0}^{n-1}\sum_{y=0}^{n-1} I(x,y) \times (y - xc)}{\sum_{x=0}^{n-1}\sum_{y=0}^{n-1} (y - xc)^2} \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots (21)$$

$$a_2 = \frac{\sum_{x=0}^{n-1}\sum_{y=0}^{n-1} I(x,y) \times (x - yc)}{\sum_{x=0}^{n-1}\sum_{y=0}^{n-1} (x - yc)^2} \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots (22)$$

where ***I(x,y)*** is the original image of size *N*x*N* that partitioned into fixed block size *n*×*n.* Where *xc* equal to *yc* that represents a block center of size (*n*×*n)* as equation (7).

8. Using the uniform scalar quantization to quantize the estimated coefficients of the linear base, where each coefficient is quantized using different quantization level, The quantizer/dequantizer as illustrated in equations (23-25).

$$Q_{a_0} = round\left(\frac{a_0}{QL_{a_0}}\right) \rightarrow DQ_{a_0} = Q_{a_0} \times QL_{a_0} \dots\dots\dots\dots\dots\dots\dots. (23)$$

$$Q_{a_1} = round\left(\frac{a_1}{QL_{a_1}}\right) \rightarrow DQ_{a_1} = Q_{a_1} \times QL_{a_1} \dots\dots\dots\dots\dots\dots\dots. (24)$$

$$Q_{a_2} = round\left(\frac{a_2}{QL_{a_2}}\right) \rightarrow DQ_{a_2} = Q_{a_2} \times QL_{a_2} \dots\dots\dots\dots\dots\dots\dots. (25)$$

Where Q_{a_0}, Q_{a_1}, $Q_{a_2}$are represented the quantized value of the coefficients (a_0,a_1 and a_2) respectively, while $QL_{a_0}, QL_{a_1}, QL_{a_2}$ are represented the quantization levels of the parameters. DQ_{a_0}, DQ_{a_1}, DQ_{a_2}represents the dequantization value of the estimated parameters.

9. Create the predicted image value $\tilde{I}$ using the dequantized polynomial coefficients for each encoded block as equation (26).

$$\tilde{I} = DQ_{a_0} + DQ_{a_1}(y - xc) + DQ_{a_2}(x - yc) \dots\dots\dots\dots\dots\dots. (26)$$

10. Find the residual or prediction error as difference between the original image and the predicted one $\tilde{I}$ as equation (27).

$$Residual(x,y) = I(x,y) - \tilde{I}(x,y) \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots (27)$$

11. Apply theuniform scalar quantization to quantize the residual as equation (28) shown quantizer/dequantizer of residual.

$$Q_{Residual} = round\left(\frac{Residual}{QL_{Residual}}\right) \rightarrow DQ_{Residual} = Q_{Residual} \times QL_{Residual} \dots\dots (28)$$

12. Encode the estimated coefficients of linear polynomial model and residual image lossily, also by utilizing both the run length coding (RLC) and Huffman coding.

13. The decoder reconstruct the inverse images, correspond to the identical$\hat{S}$ image and the approximated$\hat{I}$using the nonlinear/linear coefficients and residuals images as equations (29-30).

$$\hat{S}(x,y) = Res(x,y) + \tilde{S}(x,y) \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots.. (29)$$

$$\hat{I}(x,y) = DQ_{Residual}\,(x,y) + \tilde{I}(x,y) \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots.. (30)$$

14. The reconstructed hybrid image of lossy and lossless base as equation (31).

$$\widehat{M}(x,y) = \hat{S}(x,y) + \hat{I}(x,y) \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots. (31)$$

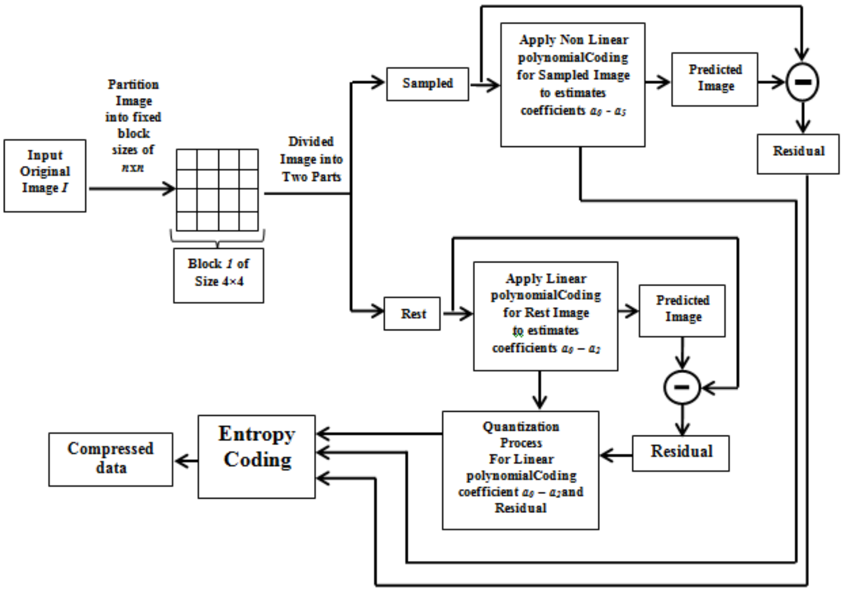

Figure 1- The proposed source coding of compression system.

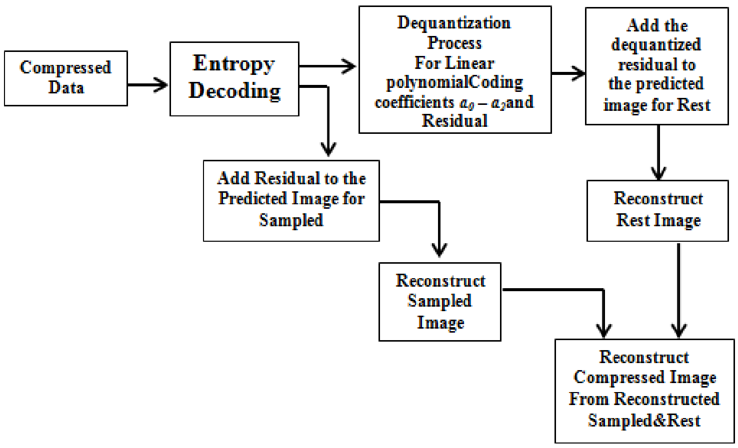

Figure 2- The proposed source decoding of compression system.

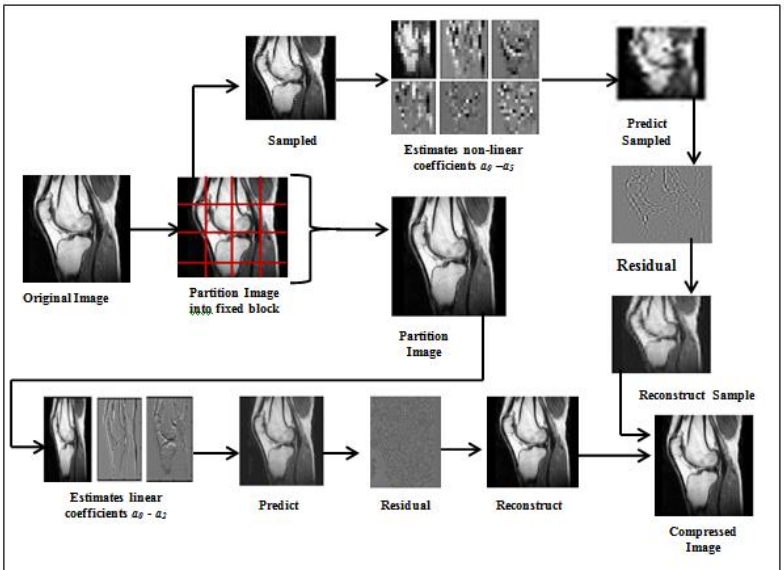

Figure 3- Example of the proposed system structure.

III. Experimental and Results

Three standard medical images are selected for testing the proposed hybrid compression system, the images of 256 gray levels(8 bits/pixel) of size 256×256(see figure 4 for an overview). To evaluate theperformance of the proposed compression system, the compression ratio used (CR)which is the ratio between the original image size and the compressed size (see equation 32), also the peak signal to noise ratio(PSNR) along with normalized root mean square error (NRMSE) adopted, see equations(33-34), where a large PSNR value implicitly means high image quality and close to the original image and vice versa,while NRMSE where the range of the values between 0 and 1, if the value is close to zero refers to high image quality and vice versa [7].

$$Compression\ Ratio = \frac{Size\ of\ Original\ Image}{Size\ of\ Compressed\ Information} \ldots\ldots\ldots..(32)$$

$$PSNR = 10.log_{10}\left[\frac{(255)^2}{\frac{1}{N\times N}\sum_{i=0}^{N-1}\sum_{j=0}^{N-1}\left[\widehat{M}(i,j) - I(i,j)\right]^2}\right] \ldots\ldots\ldots\ldots\ldots.(33)$$

$$NRMSE\left(I,\widehat{M}\right) = \sqrt{\frac{\sum_{x=0}^{N-1}\sum_{y=0}^{N-1}[\widehat{M}(x,y) - I(x,y)]^2}{\sum_{x=0}^{N-1}\sum_{y=0}^{N-1}I(x,y)^2}} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(34)$$

The result of the proposed hybrid compression system indicates that the high image quality is achieved because of utilization of effective non-linear losslesspolynomial coding technique along with the efficient linear lossy polynomial coding technique.

The results showed in table (1) of block sizes 8×8. It is obvious that the blocks size and the quantization step affected the technique performance, where the quantization process utilized for the linear polynomial model only, so the quantization levels of the coefficients and the residual affects the image quality and compression ratio.

Figures 5illustrated the results of the compressed three tested images of block sizes 8×8, and quantization level of three coefficients as, Qa0, Qa1,Qa2={1,2,2} and quantization level ofresidual equal to {30}.

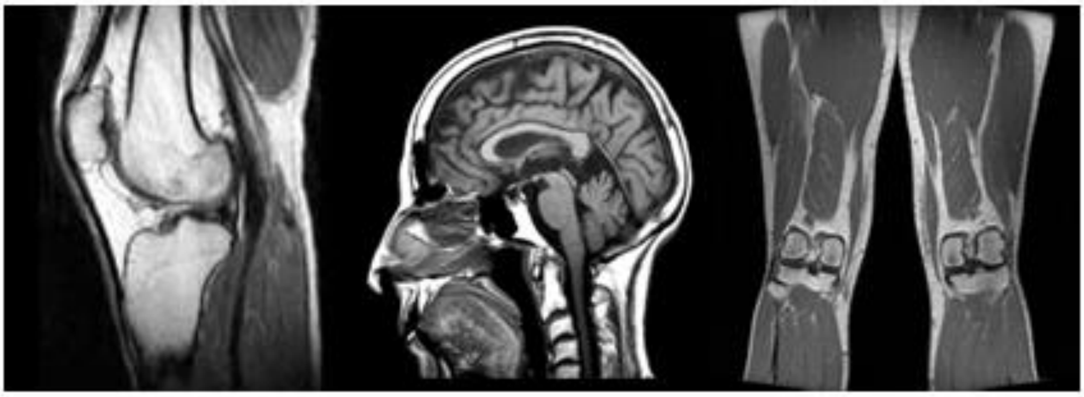

Figure 4- Overview of the tested images.

Table 1- Illustrated the results of three images for block size (8×8) and value of quantization levels of coefficients equals to 1,1,1, and 1,2,2,with different quantization levels of residual image.

Test Images	Quantization Residual	Block Size {8×8} Quantization Coefficients 1,1,1			Block Size {8×8} Quantization Coefficients 1,2,2		
		NRMSE	**PSNR**	**CR**	**NRMSE**	**PSNR**	**CR**
Knee	5	0.0946	27.0276	8.1600	0.0950	26.9927	8.3560
	10	0.0528	32.0880	8.2701	0.0531	32.0379	8.4010
	20	0.0288	37.3597	8.6350	0.0290	37.3014	8.9300
	30	0.0198	40.5962	9.1633	0.0200	40.5307	9.3650
	40	0.0151	42.9651	9.4210	0.0152	42.8877	9.6630
	50	0.0122	44.7930	9.5901	0.0124	44.7101	9.8913
Brain	5	0.2186	21.0490	10.0405	0.2189	21.0374	10.2561
	10	0.1222	26.0966	10.1352	0.1224	26.0822	10.3035
	20	0.0617	32.0426	10.4600	0.0618	32.0247	10.6333
	30	0.0408	35.6381	10.5459	0.0408	35.6235	10.7989
	40	0.0304	38.1716	10.6761	0.0306	38.1395	10.8055
	50	0.0243	40.1437	10.7002	0.0243	40.1277	10.8952
Knees	5	0.1248	27.5953	9.1097	0.1250	27.5805	10.0045
	10	0.0621	33.6622	9.3822	0.0622	33.6355	10.2593
	20	0.0328	39.2129	9.4982	0.0329	39.1631	10.5476
	30	0.0223	42.5402	9.7101	0.0226	42.4497	10.6680
	40	0.0171	44.8702	9.8050	0.0173	44.7642	10.7999
	50	0.0138	46.7091	9.8832	0.0140	46.6091	10.9079

Block size (8×8) **Block size (8×8)** **Block size (8×8)**

Figure 5- Overview the results of the tested three images for block size (8×8), and Coefficients Quantization level, Qa0, Qa1,Qa2={1,2,2} and Quantization Residuallevel = {30}.

IV. Conclusions

This paper attempts to exploit the hybrid image compression system based on lossless non- linear polynomial coding to compress the sampled image and lossy linear polynomial coding to compress the original image. The experimental results clearly showed that when increased the block size the compression ratio increased and also reduced image quality and vice versa. Also shown when increase the quantization levels of residual the image quality increased and CR decreased and vice versa.

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