IOSR Journal of Computer Engineering (IOSR-JCE)
e-ISSN: 2278-0661,p-ISSN: 2278-8727, Volume 19, Issue 1, Ver. III (Jan.-Feb. 2017), PP 53-60
www.iosrjournals.org

Value Based Decision Control: Preferences Portfolio Allocation, Winer and Color Noise Cases

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Abstract: *The paper presents an innovative approach to mathematical modeling of complex systems „human-dynamical process". The approach is based on the theory of measurement and utility theory and permits inclusion of human preferences in the objective function. The objective utility function is constructed by recurrent stochastic procedure which represents machine learning based on the human preferences. The approach is demonstrated by two case studies, portfolio allocation with Wiener process and portfolio allocation in the case of financial process with colored noise. The presented formulations could serve as foundation of development of decision support tools for design of management/control. This value-oriented modeling leads to the development of preferences-based decision support in machine learning environment and control/management value based design.*
Keywords: *Portfolio, Optimal control, Machine Learning, Utility Theory, Human Preferences*

I. Introduction

In the paper is demonstrated an innovative approach and mathematical modeling of complex systems „human-stochastic dynamical financial process" by the use of a preferences based utility objective function. That permit development of value driven control design in complex processes (and management systems) where the human choice is decisive for the final solution. *Value-driven design* is a systems engineering strategy which enables multidisciplinary design optimization. Value-driven design creates an environment that enables optimization by providing designers with an objective function which inputs the important attributes of the system being designed, and outputs a score.

In parallel we compare two optimal stochastic control solutions based on this modeling, portfolio allocation with Wiener process and portfolio allocation in the case of financial process with colored noise. Many systems in the practice are well modeled with dynamic described by differential equation with continuous state vectors. Frequently such systems are in interaction with the environment in the conditions of probability uncertainty. Similarly, the process of optimal portfolio allocation is described by stochastic differential equation model of Black-Scholes [19]. This dynamical model is a standard, basic model in the financial economic. Prevalently such stochastic dynamical systems are discussed in the conditions of Wiener process [9, 19]. But in practice the stochastic processes generally are far away from the cases of Wiener process. From the stochastic control theory point of view this fact could be reflected in two different manners. The first case consists in changing the Black-Scholes model in manner that proceeds to pass from the Ito stochastic differential equation to the Skorokhod stochastic differential equation. This is made by adding an additional term in the stochastic model. Similar transformations of the stochastic models are possible when the real stochastic process is close to the Wiener process. But the expression „close to" is not mathematically precise and is quite vague.

The second case is the case when we approach the discussed problems in the paper with the methods and means of the linear optimal control theory [13]. If there is a colored noise, the control theory proposes a specific mathematical approach which consists on modeling the noise with the help of a linear model with white noise as input. This is the approach chosen in this paper.

We compare two solutions, portfolio allocation with Wiener process and portfolio allocation in the case of financial process with colored noise. The two optimal control processes are graphically shown and compared when using and modeling a real financial process: „*GNP in 1982 Dollars, discount rate on 91-day treasury bills, yield on long term treasury bonds, 1954Q1-1987Q4; source: Business Conditions Digest*". The optimal control law in the case of stochastic financial process with colored noise is described analytically and graphical results are shown.

This investigation is innovative in two directions. The first is preferences based mathematical modeling of the complex system „human-stochastic dynamical financial process" [4, 8]. Such a „human-process" complex systems modeling is a rarity in the contemporary scientific investigations. The second is the investigation and determination of the optimal control law in the case of the Black-Scholes model and stochastic financial process with colored noise.

II. Objective Utility Function And Preferences

People's preferences contain uncertainty due to the cardinal type of the empirical expert information. This uncertainty is of subjective and probability nature. The difficulties that come from the mathematical approach are due to the probability, and subjective uncertainty of the DM expression and the cardinal character of the expressed human preferences. The machine learning evaluation method presented here rests upon the achievements of the theory of measurement (scaling), utility theory and, statistic programming. The so called normative (axiomatic) approach considers the conditions for existence of utility function. The mathematical description follows.

Let X is the set of alternatives and P be a set of probability distributions over X. A utility function $u(.)$ will be any function for which the following is fulfilled: $(p \succ q\ ,\ (p,q)\in P^2\ ) \Leftrightarrow (\int u(.)dp > \int u(.)dq)$.

In keeping with Von Neumann and Morgenstern [6, 16] the interpretation of the above formula is that the integral of the utility function $u(.)$ is a measure for comparison of the probability distributions p and q defined over X. The notation $(\succ)$ expresses the preferences of DM over P including those over X $(X \subseteq P)$. The presumption of existence of a utility function $u(.)$ leads to the "negatively transitive" $(\neg(p \succ t)\ \wedge \neg(t \succ q)) \Rightarrow \neg(p \succ q))$ and "asymmetric" relation $(\succ)$. These properties lead to the existence of: asymmetry $((x \succ y) \Rightarrow (\neg(x \succ y)))$, transitivity $((x \succ y) \wedge (y \succ z\ ) \Rightarrow (x \succ z))$ and transitivity of the "indifference" relation $(\approx)$.The transitivity of the relations $(\succ)$ and $(\approx)$ is violated most often in practice. The violation of the transitivity of the relation $(\succ)$ could be interpreted as a lack of information, or as a DM's subjective mistake. The violation of the transitivity of the relation $(\approx)$ is due to the natural "uncertainty" of the human's preference and the qualitative nature of expressions of the subjective notions and evaluations [1, 3, 7, 17].

There is different utility evaluation methods, all of them based on the "lottery" approach (gambling approach). A "lottery" is called every discrete probability distribution over X. We mark as $<x,y,\alpha>$ the lottery: here α is the probability of the appearance of the alternative x and $(1-\alpha)$ - the probability of the alternative y. The most used evaluation approach is the assessment: $z \approx <x,y,\alpha>$, where $(x,y,z)\in X^3$, $(x \succ z \succ y)$ and $\alpha\in[0,1]$ [5, 8]. Weak points of this approach are the violations of the transitivity of the relations and the so called "certainty effect" and "probability distortion" [3, 8]. Additionally, it is very difficult to determine the alternatives x (the best) and y (the worst) on condition that $(x \succ z \succ y)$, where z is the analyzed alternative. The measurement scale of the utility function $u(.)$ originates from the previous mathematical formulation of the relations $(\succ)$ and $(\approx)$. It is accepted that $(X \subseteq P)$ and that the set of the probability distributions P is a convex set $((q,p)\in P^2 \Rightarrow (\alpha q+(1-\alpha)p)\in P$, for $\forall\alpha\in[0,1])$. The utility function u(.) over X is determined with the accuracy of an affine transformation (i.e. interval scale) [6, 16].

We assume that the outcome set X is a two-attribute product set $V\times W$, with generic element $x=(v,w)$. The sets V and W are attribute sets where W designates the second attribute, the quantity of money in BGN's and V designates the first attribute - the amount $w\pi_t$, $(w\pi_t\in V,\ w\in W,\ \pi_t\in[0,1]\ )$ invested in a risky process. The aggregation of the two attributes in a multiattribute utility function needs investigation of the *Utility independence* in between the risky investment v $(v=w\pi_t)$ and the quantity of money (w) [8]. A implication of preference independence is that changing v does not affect rank-ordering in „lotteries" over W. Assume that the image of the function $F(w) \rightarrow U(v, w\ )$ is an interval for all v, where $U(v,w)$ is two attribute utility function. Then w is utility independent if and only if:

$U(v, w\ ) = f(v)p(w) + g(v)$ for some functions f, p, g and f is positive.

Let W be relevant over a range w^o to w^* and V over a range v^o to v^* and assume that $U(v, w^*)>U(v, w^o)$ for all v and $U(v^*,w)>U(v^o, w)$ for all w. We may rewrite this independency condition as:

$U(v, w\ ) = U(v, w^o)+ [U(v, w^*) -U(v, w^o)]\ U(v^o, w\ )$.

We know that the measuring scale is the interval scale. That fact permits more detailed functional description of the utility function:

$U(v, w\ ) = (a_1+b_1g(v)) + [(a_2+b_2f(v)) - (a_3+b_3g(v))]((a_4+b_4p(w)))$.

The functions $g(v)$, $f(v)$ and $p(w)$ are normed between 0 and 1. The notations a_i and b_i are coefficients, where $i=1 \div 4$.

The single-attribute utility functions $U(v, w^*)$, $U(v, w^o)$ and $U(v^o, w)$ are evaluated by a stochastic recurrent algorithm and the objective utility function is shown in figure 1 [14, 15]. We underline that is supposed preferences associated with the quantity of BGN's (set W) are utility independent from the level of risky investment π (set V). Starting from the properties of the preference relation $(\succ)$ and indifference relation $(\approx)$ and from the weak points of the "lottery approach" we propose the following stochastic approximation procedure for evaluation of the utility function. It is assumed that $(X\subseteq P)$, $((q,p)\in P2 \Rightarrow (\alpha q+(1-\alpha)p)\in P$, for $\forall\alpha \in[0,1])$ and that the utility function u(.) exists. It is defined two sets:

$Au^*=\{(\alpha,x,y,z)/(\alpha u^*(x)+(1-\alpha)u^*(y))>u^*(z)\}$,

$Bu^*=\{(\alpha,x,y,z)/(\alpha u^*(x)+(1-\alpha)u^*(y))>u^*(z)\}$.

The approximation of the utility function is constructed by pattern recognition of the set Au*[14]. The assessment process is a machine-learning approach based on the DM's preferences. The machine learning is a probabilistic pattern recognition procedure because ($Au^* \cap Bu^* \neq \varnothing$) and the utility evaluation is a stochastic approximation with noise (uncertainty) elimination.

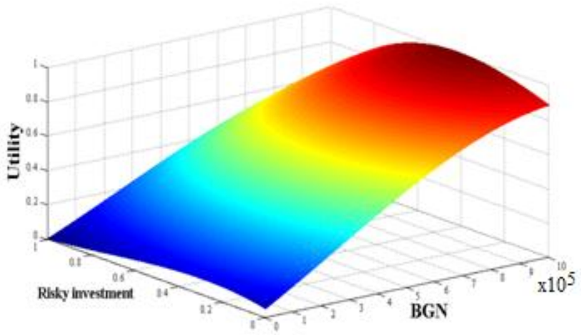

Fig.1 Utility:1≈1000000BGN's

The following presents the evaluation: DM compares the "lottery" <x,y,α> with the simple alternative z, z∈X ("better-$\succ$, f(x,y,z,α)=1", "worse-$\prec$, f(x,y,z,α)=(-1)" or "can't answer or equivalent- ~ , f(x,y,z,α)=0". The discrete function f(.) denotes the qualitative DM's answer expressed as preference. This determine a learning point ((x,y,z,α), f(x,y,z,α)). The following recurrent stochastic algorithm constructs the utility polynomial approximation based on the learning points:

$$u(x) = \sum_i c_i \Phi_i(x),$$

$$c_i^{n+1} = c_i^n + \gamma_n \left[f(t^{n+1}) - \overline{(c^n, \Psi(t^{n+1}))} \right] \Psi_i(t^{n+1}),$$

$$\sum_n \gamma_n = +\infty,\ \sum_n \gamma_n^2 < +\infty, \forall n, \gamma_n > 0.$$

The notations are (based on Au*):

t=(x,y,z,α), $\psi_i(t)=\psi_i(x,y,z,\alpha) = \alpha\Phi_i(x)+(1-\alpha)\Phi_i(y)-\Phi_i(z)$, where ($\Phi_i(x)$) is a family of polynomials.

The line above the scalar product $\bar{v} = \overline{(c^n, \Psi(t))}$ means: ($\bar{v}=1$), if (v>1), ($\bar{v}=-1$) if (v< -1), and $\bar{v}=v$ if (-1<v<1). The coefficients c_i^n take part in the polynomial representation:

$$g^n(x) = \sum_{i=1}^{n} c_i^n \Phi_i(x),\ (c^n, \Psi(t)) = \alpha g^n(x) + (1-\alpha) g^n(y) - g^n(z),$$

$$(c^n, \Psi(t)) = G^n(x, y, z, \alpha),\ \textit{where}\ (c^n, \Psi(t))\ \textit{is a scalar multiplication}.$$

The mathematical procedure describes the following assessment process. The learning points are set with a pseudo random sequence. The expert relates intuitively the "lottery" (x,y,z,α)) to the set Au*with probability $D_1(x,y,z,\alpha)$ or to the set Bu* with probability $D_2(x,y,z,\alpha)$. The probabilities $D_1(x,y,z,\alpha)$ and $D_2(x,y,z,\alpha)$ are mathematical expectations of f(.) over Au* and Bu* respectively:

($D_1(x,y,z,\alpha)=M(f/x,y,z,\alpha)$) if ($M(f/x,y,z,\alpha)>0$), ($D_2(x,y,z,\alpha)=(-)M(f/x,y,z,\alpha)$) if ($M(f/x,y,z,\alpha)<0$).

Let $D'(x,y,z,\alpha)$ is the random value: $D'(x,y,z,\alpha)=D_1(x,y,z,\alpha)$ if ($M(f/x,y,z,\alpha)>0$); $D'(x,y,z,\alpha)=(-D_2(x,y,z,\alpha))$ if ($M(f/x,y,z,\alpha)<0$); $D'(x,y,z,\alpha)=0$ if ($M(f/x,y,z,\alpha)=0$). We approximate $D'(x,y,z,\alpha)$ by $G(x,y,z,\alpha)=(\alpha g(x)+(1-\alpha)g(y)-g(z))$. The coefficients c_i^n take part in the approximation of the function $G(x,y,z,\alpha)$ $G^n(x,y,z,\alpha)=(c^n,\Psi(t))$.

The function $G^n(x,y,z,\alpha)$ is positive over Au* and negative over Bu* depending on the degree of approximation of $D'(x,y,z,\alpha)$. *The function* $g_n(x)$ *is the approximation of the utility function* u(.). The proof of the convergence is based on the extremal approach of the potential function method [14].

The utility U(v,w) and the single attribute utility functions *g*(v), *f*(v) and *p*(w)) are normed between 0 and 1 and are measured in the interval scale [6, 14, 16]. These facts permit determination of coefficients a_i and b_i by a very easy procedure. The coefficients in the utility formula are determined by comparisons of lotteries of the following type [14]:

$$\left\{ \begin{matrix} U(x_1)\times\alpha \\ + \\ U(x_2)\times(1-\alpha) \end{matrix} \right\} \begin{matrix} \succ \\ \approx \\ \prec \end{matrix} \quad U(x_3).$$

But now x_1, x_2 and x_3 are fixed and is fulfilled ($x_1 \succ x_3 \succ x_2$) [8, 14]. The questions to the decision maker are like lotteries in which we vary only the values by $\alpha \in [0,1]$. Two of them x_1 and x_2 are with fixed values (utilities) and the aim is to determine the third value $U(x_3)$. The stochastic procedure is of the type of Robins-Monro but the convergence is much quicker. The procedure is the following [14]. Lets α is a uniformly distributed random value in [0, 1]. We define the following random vector $\chi = (\eta_1, \eta_2, \eta_3)$, where:

a) If $(\prec) \Rightarrow \chi=(\eta_1=1, \eta_2=0, \eta_3=0)$;
b) If $(\succ) \Rightarrow \chi=(\eta_1=0, \eta_2=0, \eta_3=1)$;
c) If indiscernibility $(\approx) \Rightarrow \chi=(\eta_1=0, \eta_2=1, \eta_3=0)$.

Let χ^n is a learning sequence of independent random values with equal to χ distribution. The stochastic recurrent procedure is the following:

$$\left(\lambda_1^{n+1}, \lambda_2^{n+1}, \lambda_3^{n+1}\right) = \mathrm{Pr}_P\left[(\lambda_1^n, \lambda_2^n, \lambda_3^n) - \gamma_n((\eta_1^n, \eta_2^n, \eta_3^n) - (\lambda_1^n, \lambda_2^n, \lambda_3^n))\right],$$

$$\sum_1^\infty \gamma_n = \infty, \sum_1^\infty \gamma_n^2 < \infty, \gamma_n \geq 0, \forall n \in N.$$

The notation Pr_P has the meaning of projection over the set:

$P = \{(\lambda_1, \lambda_2, \lambda_3) / \lambda_1 \geq 0, \lambda_2 \geq 0, \lambda_3 \geq 0, \lambda_1 + \lambda_2 + \lambda_3 = 1\}$.

The searched value is determined in the end as the following:

$U(x_3) = U(x_2) + (\lambda_1 + \lambda_2/2)(U(x_1) - U(x_2))$.

The proposed procedure and its modifications are machine learning. The DM is comparatively fast in learning to operate with the procedure: session with 128 questions (learning points) takes approximately 30 minutes and requires only qualitative answers “yes”, “no” or “equivalent ”.The learning points $((x,y,z,\alpha), f(x,y,z,\alpha))$ are set with a Sobol’s pseudo random sequence [14].

III. Value Driven Modelling Based On Measurement Of Human Preferences

The Computational Economics area in *Operations Research* recognizes that current developments in computational methodology are of essential importance to new research into the application of economic principles to the solution of practical policy problems. The author presents two value driven policy problem solutions that are based on measurement of human preferences [4, 8, 14]. The first of them concerns the construction of optimal portfolio utility allocation in the case of Wiener process. The second application presents the construction of optimal portfolio utility allocation in the case of colored noise. These problems are discussed repeatedly in the scientific literature [2, 12, 13, 19].

Generally the construction of the objective utility function is out of discussion. In this investigation is considered construction of a polynomial objective function, which is based on evaluations of decision maker’s preferences as is shown in figure 1. This model includes the human decisive opinion as a utility function. The utility function together with the Black-Scholes differential model comes to an integral mathematical description of the complex problem „human- portfolio policy determination”. The portfolio optimal control is in agreement with the DM’s preferences.

IV. Optimal Portfolio Allocation, Wiener Process

Consider a non-risky asset S_0 and a risky one S. The Black-Scholes dynamic model represents a stochastic differential equation determined by:

$$dS_t^0 = S_t^0 r dt \quad \text{and} \quad dS_t = S_t \mu dt + \sigma dW.$$

In the formula r, μ and σ are constants ($r=0.03$, $\mu=0.05$ and $\sigma=0.3$) and W is one dimensional Brownian motion. By X_t we denote the wealth, the state space vector of the controlled process. The investment policy is defined by a progressively adapted process $\pi=\{\pi_t, t\in[0,T]\}$ where π_t represents the risky amount ($X_t\pi_t$, $\pi_t \in [0,1]$) invested at any moment t. The invested amount is ($w\pi_t$, $\pi_t \in [0,1]$, $w \in W$) in agreement with the notations of the previous paragraph. The remaining wealth ($X_t - \pi_t X_t$) is non-risky investment. The time period T is 30 weeks. The self-financing satisfies the following stochastic differential equation [15, 19]:

$$dX_t^\pi = \pi X_t^\pi \frac{dS_t}{S_t} dt + (X_t^\pi - \pi X_t^\pi) \frac{dS_t^0}{S_t^0} = (rX_t^\pi + (\mu - r)\pi X_t^\pi)dt + \sigma \pi X_t^\pi dW_t$$

The amount $X^{t,x,\pi}$ is the solution of the stochastic differential equation with initial wealth (x) at time t. It is obvious that $\mathrm{E}\int_0^T (\pi X_t^\pi)^2 dt < \infty$, where E denote mathematical expectation. The main objective of the investor (DM) is maximization of the expected DM’s utility $U(.)$ at the final moment T:

$$V(t,x) := \sup_{\pi\in[0,1]} \mathrm{E}[U(X_T^{t,x,\pi})].$$

The DM's objective utility function is shown on the figure 1. The control input is π_t and determines the partition amount ($X_t\pi_t$) invested in the risky process at any moment t. The optimal control could be determined step by step from the Hamilton-Jacobi-Bellman partial differential equation in agreement with the dynamical programming principle [18, 19]:

$$\frac{\partial B}{\partial t}(t,X)+\sup_{\pi\in[0,1]}[(rX+(\mu-r)\pi X)\frac{\partial B}{\partial X}(t,X)+\frac{1}{2}\sigma^2\pi^2x^2\frac{\partial^2 B}{\partial^2 X}(t,X)]=0.$$

We underline that the coefficients in the stochastic differential equation are continuous, the objective function U(.) is continuous and that the optimal control is continuous by parts. This determines that there is a smooth by parts solution of the HJB partial differential equation [9, 19]. Following the presentations in [9] and passing through generalized solution of the Black-Scholes stochastic differential equation we found polynomial approximations of the Hamilton-Jacobi-Bellman (HJB) function *B*(t, X) and of the control manifold π(t,X) [10, 18]. We show them in figures 2 and 3.

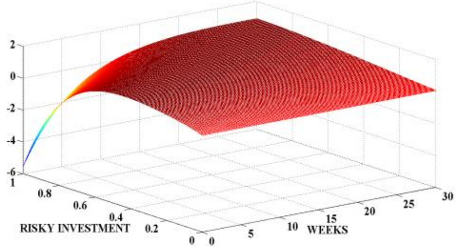

Fig.2. HJB function

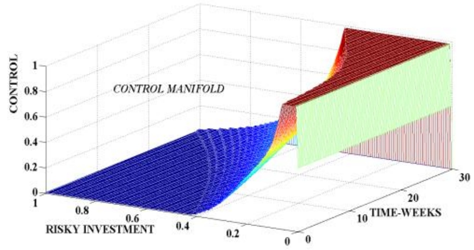

Fig.3. Optimal control manifold

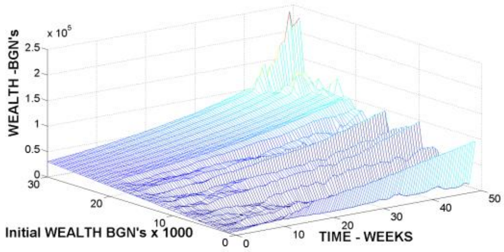

Fig.4. Scale in BGN's

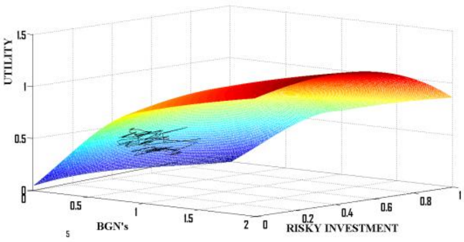

Fig.5. Utility: 1≈1000000 BGN's

The stochastic process is started in 30 different initial points; from 1000 BGN's to 30000 BGN's and the optimal control solutions could be seen in figure 4 [15]. We underline that in figure 4 is shown the optimal solution whit final wealth X_T evaluated in BGN's. The black seesaw line under the objective utility function in figure 5 is a sample of stochastic optimal control process flow.

V. Colored Noise And Optimal Portfolio Utility Allocation

The Wiener process is an abstraction, some time far away from the reality. The white noise assumption is too strong. In this paragraph the optimal portfolio control allocation in the case of a finacial process with non-white colored noise will be investigated. Data from a real process are used, available in internet: „*GNP in 1982 Dollars, discount rate on 91-day treasury bills, yield on long term treasury bonds, 1954Q1-1987Q4; source: Business Conditions Digest*". The noise of the real financial process (figure 6) is far away from white noise as could be seen by the correlation function in figure 7. The approximation of the real correlation function is shown in also in figure 7.

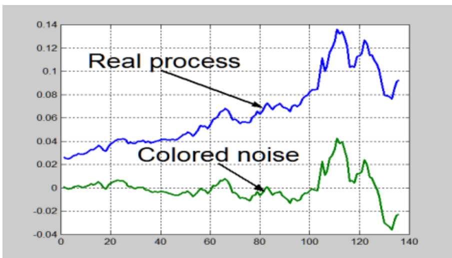

Fig.6. Experimental data

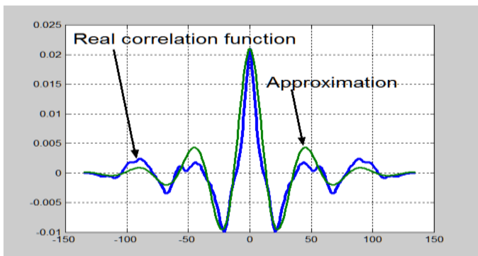

Fig.7. Correlation functions

The problem of colored noise modeling and optimal filtering in linear control theory is repeatedly discussed in the scientific forums and has practical significance [11]. The noise in the real financial process could be approximated by colored noise as is shown in figure 7. This admission leads to modifications in the Black-Scholes model. The stochastic differential equation is appended to a three dimensional differential equation and becomes:

$$dX_t^{\pi} = (rX_t^{\pi} + (\mu - r)\pi X_t^{\pi})dt + \pi X_t^{\pi} N_1 \quad .$$

$$dN_1 = N_2 dt + 0.0028 dW_t$$

$$dN_2 = -(0.135 N_1 + 0.07 N_2)dt + 0.0047 dW_t$$

It is obvious that the noise *N* appears autonomously in the second and the third row and that this is a description of a linear system. The wealth X_t appears only with its first derivative in the Hamilton-Jacobi-Bellman (HJB) partial differential equation [18, 19].

$$\frac{\partial B}{\partial t}(t, X, N_1, N_2) + \sup_{\pi\in[0,1]}[(rX + (\mu - r)\pi X) + \pi X N_1]\frac{\partial B}{\partial X}(t, X, N_1, N_2)$$

$$+ N_2 \frac{\partial B}{\partial N_1}(t, X, N_1, N_2) - (0.135\frac{\partial B}{\partial N_1}(t, X, N_1, N_2) + 0.07\frac{\partial B}{\partial N_2}(t, X, N_1, N_2)) +$$

$$+\frac{1}{2}(0.0028)^2 \frac{\partial^2 B}{\partial^2 N_1}(t, X, N_1, N_2) + \frac{1}{2}(0.0047)^2 \frac{\partial^2 B}{\partial^2 N_2}(t, X, N_1, N_2) = 0.$$

The main objective of the DM is maximization of the expected DM's utility *U*(X) at the final moment T:

$$V(t, x) := \sup_{\pi\in[0,1]} \mathrm{E}[U(X_T^{t,x,\pi})].$$

The formula above has the meaning of mathematical expectation of *U*(*X*) at the final moment where *U*(.) is the objective utility function. The optimal control law could be determined from the following mathematical expression:

$$\sup_{\pi\in[0,1]}[(rX_t + (\mu - r)\pi_t X_t) + \pi X_t N_{1_t}]\frac{\partial B}{\partial X}(t, X, N_1, N_2) \quad .$$

This formula show that the determination of the optimal control law needs the determination of the partial derivative on *X* of the Belman's function B(t, X, N_1, N_2). These observations permit a decomposition of the HJB partial differential equation to a partial differential equation of the first degree with variables X_t and t and to an autonomous HJB partial differential equation with variables N_1 and N_2. We will look for a solution of the HJB partial differential equation of the form B_1(t, X)B_2(t, N_1, N_2), where B_2(t, N_1,N_2,) is a positive smooth function, solution of the following partial differential equation:

$$\frac{\partial B_2}{\partial t}(t, N_1, N_2) + N_2\frac{\partial B_2}{\partial N_1}(t, N_1, N_2) - [0.135\frac{\partial B_2}{\partial N_1}(t, X, N_1, N_2) + 0.07\frac{\partial B_2}{\partial N_2}(t, N_1, N_2)] +$$

$$+\frac{1}{2}(0.0028)^2\frac{\partial^2 B_2}{\partial^2 N_1}(t, N_1, N_2) + \frac{1}{2}(0.0047)^2\frac{\partial^2 B_2}{\partial^2 N_2}(t, N_1, N_2) = 0.$$

The function B_1(T, X) is chosen to be equal to the DM's utility function *U*(*X*) in the final moment T. This function is solution of the partial differential equation:

$$\frac{\partial B_1}{\partial t}(t, X, N_1, N_2) + \sup_{\pi\in[0,1]}[(rX + (\mu - r)\pi X) + \pi X N_1]\frac{\partial B_1}{\partial X}(t, X, N_1, N_2) = 0$$

This decomposition permits determination of the partial derivative on X of the Belman's function B(t, X, N_1, N_2) as follows:

$$\frac{\partial B}{\partial X}(t, X, N_1, N_2) = \frac{\partial U}{\partial X}(t, X)e^{\int_t^T (r + (\mu - r)\pi_s + \pi_s E(N_1(s)))ds} B_2(t, N_1, N_2), \quad .$$

In the formula π_t, t∈[0,T] is the optimal control policy and $E(N_1(t))$ is the mathematical expectation of the colored noise at moment t. We remind that the color noise is generated by the linear system with Gauss white noise as input. Now is clear that the two partial derivatives have the same sign and the formula of the optimal control law becomes:

$$\sup_{\pi\in[0,1]}[(rX_t + (\mu - r)\pi_t X_t) + \pi_t X_t N_{1_t}]sign(\frac{\partial U}{\partial X}(t, X)) \quad .$$

The stochastic process is started in 30 different initial points; from 1000 BGN's to 30000 BGN's. The solutions could be seen in figure 8. In the next figure 9 are shown the same solutions but with classical control law in the case of Wiener process. We underline that in figure 8 and figure 9 are shown the optimal solutions, the wealth evaluated as utilities U(X). We remember that the solutions shown in figure 4 are in the scale of BGN's. It is seen that the solution that renders an account of the colored noise gives much better results.

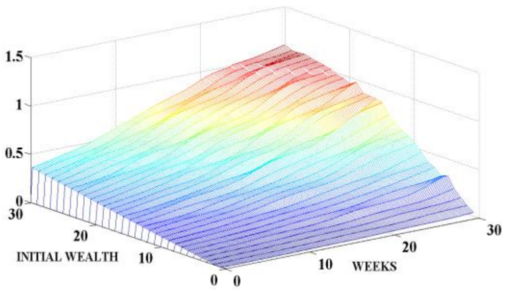

Fig.8. Utility: 1≈1000000 BGN's

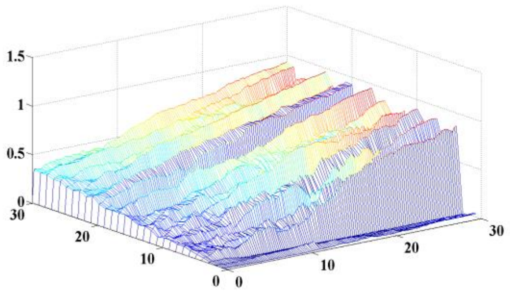

Fig.9. Utility: 1≈1000000 BGN's

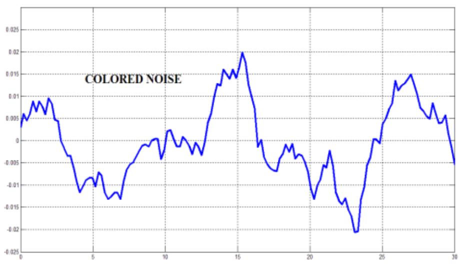

Fig.10. Colored noise sample

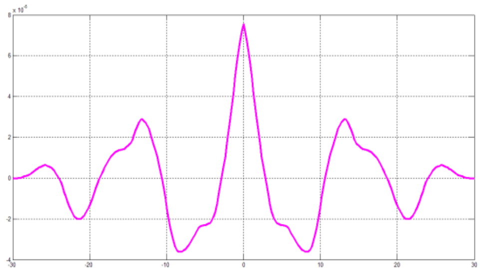

Fig.11. Correlation function

The noise in the numerical modeling is approximated by colored noise as output of a linear system with Gauss white noise input [11]. A sample of the colored noise and its correlation function are shown in figure 10 and figure 11. The linear system is constructed based on the approximation of the correlation function shown in figure 7.

VI. Conclusions

The Computational Economics recognizes that current developments in computational methodology are of essential importance. Advances in numerical methods now permit a genuine analysis of important and difficult economic problems without compromises. In the paper a system engineering value driven approach within the problem of determination of the optimal portfolio allocation modeled with Black-Scholes dynamic is demonstrated. It provides two optimal control solutions; the first solution is in the condition of a Wiener process and the second solution discusses the approach with inclusion of a colored noise in the stochastic financial process. It is important to emphasize that the optimal portfolio controls are specified based on the individual consumers' preferences.

In the paper is discussed an approach that allows practitioners to take advantage of individual application of the achievements of decision making theory in various fields of human activities. The analytical presentations of the expert's preferences as value or utility function allow the inclusion of the decision maker mathematically in the model "Human-process". The suggested approach can be regarded as a realization of the *prescriptive decision making*. The utility function is an abstraction presented in the limits of the normative approach, the axiomatic systems of Von Neumann. The mathematical expectation measured in the interval scale on the base of the DM's preferences over lotteries is an approximation of the Von Neumann's utility function.

References

[1]. Allais, M.: Le comportement de l'homme rationnel devant le risque: Critique des postulats et axiomes de l'école américaine, *Econometrica 21*, 503-546 (1953).

[2]. Charness, G. and U. Gneezy: Portfolio choice and risk attitudes: An experiment, *Economic Inquiry*, 48, 133–146, (2010).

[3]. Cohen, M., Jaffray, J.Y.: Certainty Effects versus Probability Distortion: An Experimental Analysis of Decision Making Under Risk. *Journal of Experimental Psychology: Human Perceptions and Performance* 14(4), 554-560 (1988).

[4]. Collopy, P., Hollingsworth, P.: Value-driven design. *AIAA Paper* 2009-7099, American Institute of Aeronautics and Astronautics, Reston, VA (2009).

[5]. Farquhar, P.: Utility Assessment Methods. *Management Science* 30, 1283-1300 (1984).

[6]. Fishburn, P.: *Utility theory for decision-making*. Wiley, New York, (1970).

[7]. Kahneman, D,Tversky, A.: Prospect theory: An analysis of decision under risk. *Econometrica* 47, 263-291 (1979).

[8]. Keeney, R., Raiffa, H.: *Decision with multiple objectives: Preferences and value trade-offs*, Cambridge University Press, Cambridge & New York (1999).

[9]. Krylov N. V.: *Controlled Diffusion Processes*, Springer-Verlag, New York (1980).

[10]. Kushner H., Dupuis P. G.: *Numerical Methods* for Stochastic Control Problems in Continuous Time (Applications of Mathematics: Stochastic Modelling and and Applied Probability), Springer-Verlag, New York, *(*2001).

[11]. Kwakernaak H., Sivan R.: *Linear optimal control systems*, Wiley, (1972).

[12]. Mania M., Tevzadze R.: Backward Stochastic Partial Differential Equations Related to Utility Maximization and Hedging, *Journal of mathematical science*s,Vol.153, N3, 2008.

[13]. Pang T.: Sotochastic Portfolio Optimization with Log Utility, *International Journal of Theoretical* and *Applied Finance*, Vol. 09, No. 06, 2006.

[14]. Pavlov, Y., Andreev, R.: *Decision Control, Management, and Support in Adaptive and Complex Systems: Quantitative Models*, IGI Global, Hershey, PA (2013)

[15]. Pavlov, Y.: Rational Portfolio investment based on Consumer's Preferences: Black-Scholes Model and Stochastic Control, *Journal of Communication and Computer*, 12, (2015)

[16]. Pfanzagl, J.: *Theory of Measurement*, Physical-Verlag, Wurzburg-Wien (1971)

[17]. Shmeidler,D.: Subjective probability and expected utility without additivity, *Econometrica* 57(3), 571-587 (1989)

[18]. Smears, I.: Hamilton-Jacobi-Bellman Equations Analysis and Numerical Analysis, *PhD Theses*, Durham University, UK (2010), http://maths.dur.ac.uk/Ug/projects/highlights/PR4/ Smears_HJB_report.pdf **.**

[19]. Touzi, N., Tourin, A.: *Optimal Stochastic Control, Stochastic Target Problems and Backward SDEs*, Fields Institute Monographs 29, Springer- Business & Economics (2012)