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(1, 2) Burst-Correcting Optimal Codes Over GF (3)

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Abstract :.*In this paper we obtain a lower bound on the number of parity-check digits in an (n, k) linear codes over GF(3) which are optimal in a specific sense i.e. the codes are capable to correcting single errors in the first sub-block of length n_1 and bursts of length 2 or less in the second sub-block of length n_2; $n = n_1 + n_2$*
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I. Introduction

Burst is the most common error in the history of coding theory and the literature is full with different type of burst error correcting codes. In many communication channels, occurrence of burst error is more frequent than random errors. So, from applications point of view, burst error correcting codes are more e useful as well as economical in digital communications. Dass and Tyagi [7] studied such codes in two sub-blocks of length n_1 and n_2, $n_1+n_2=n$ by using the definition of burst due to Chien and Tang [2] . Such codes were termed as (1, 2) binary optimal codes. Later Buccimazza, Dass, Iembo and Jain studied these codes over GF(3), GF(5) and GF(7) [1], [3] and [4].

Our objective in this paper is to explore the possibility of the existence of linear codes of length n which are sub divided into two sub-blocks of length n_1 and n_2, $n_1 + n_2 = n$. These codes are capable of correcting bursts of length 1 in the first sub-block of length n_1 and bursts of length 2 or less in the second sub-block of length n_2 over GF(3).The distance between vectors as well as the weight of the vector shall be considered in the Hamming sense. Here, we consider the definition of burst given by Fire [8] according to which **'**a burst of length b or less has been considered as an n-tuple whose only non-zero components are confined to some b consecutive positions, the first and the last of which is non-zero'.

The paper is organized into five sections. In Section 2, we state necessary condition for the existence of such (1,2)-optimal codes whereas Section 3 presents possibilities of occurrence of these codes. In Section 4 we discuss these codes with the help of example. Finally we give conclusion of the paper and open problem in Section 5.

II. NECESSARY CONDITION

As mentioned earlier, in this section we obtain necessary bound on the number of parity check digits required for the existence of (1, 2) burst-correcting optimal linear codes over GF(q) by using well known Fire's bound [8].

Theorem: The number of parity check digits in an ($n=n_1+n_2$, k) linear code over GF(q) correcting all burst errors of length b_1 or less in the first block of length n_1 and all burst errors of length b_2 or less in the second block of length n_2 is at least

$$\log_q\left\{1 + \left[n_1(q-1) + \sum_{i=1}^{b_1}(n_1 - i + 1)(q-1)^2 q^{i-2}\right] + \left[n_2(q-1) + \sum_{j=1}^{b_2}(n_2 - j + 1)(q-1)^2 q^{j-2}\right]\right\}.$$

In other words,

$$q^{n-k} \geq 1 + \left[n_1(q-1) + \sum_{i=1}^{b_1}(n_1 - i + 1)(q-1)^2 q^{i-2}\right] + \left[n_2(q-1) + \sum_{j=1}^{b_2}(n_2 - j + 1)(q-1)^2 q^{j-2}\right]. \qquad (1)$$

Proof: The result for ($n=n_1+n_2$, k) linear code over GF(q) will be proved by enumerating all possible bursts of length b_1 or less in the first block of length n_1 and all possible bursts of length b_2 or less in the second block of length n_2. Then in view of the fact that all these correctable error vectors should belong to different cosets shall be compared with q^{n-k} which is nothing but the total number of available cosets.

All possible bursts of length b_1 or less in the first block of length n_1

Number of bursts of length 1 in the first bock of length $n_1 = n_1(q-1)$.

Number of bursts of length 2 in the first bock of length $n_1 = (n_1-1)(q-1)^2$.

Number of bursts of length 3 in the first bock of length $n_1 = (n_1-2)(q-1)^2q$.

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Number of bursts of length i in the first bock of length n_1 =$(n_1-i+1)(q-1)^2q^{i-2}$.
Therefore, total number of bursts of length b_1 or less in the first block of length n_1, is

$$n_1(q-1)+\sum_{i=1}^{b_1}(n_1-i+1)(q-1)^2q^{i-2}.$$

Also the code is capable of correcting all burst errors of length b_2 or less in the second block of length n_2, the number of all such burst patterns in the different cosets is

$$n_2(q-1)+\sum_{j=1}^{b_2}(n_2-j+1)(q-1)^2q^{j-2}.$$

Thus, the total number of error patterns to be corrected, including the vector of all zero, is

$$1+\left[n_1(q-1)+\sum_{i=1}^{b_1}(n_1-i+1)(q-1)^2q^{i-2}\right]+\left[n_2(q-1)+\sum_{j=1}^{b_2}(n_2-j+1)(q-1)^2q^{j-2}\right].$$

As we know that the total number of cosets is q^{n-k}. So, we must have

$$\mathbf{q^{n-k}} \;\square\; 1+\left[n_1(q-1)+\textstyle\sum_{i=1}^{b_1}(n_1-i+1)(q-1)^2q^{i-2}\right]+\left[n_2(q-1)+\textstyle\sum_{j=1}^{b_2}(n_2-j+1)(q-1)^2q^{j-2}\right].$$

which implies that

n-k

$$\square\; \mathbf{\log_q}\{1+\left[n_1(q-1)+\textstyle\sum_{i=1}^{b_1}(n_1-i+1)(q-1)^2q^{i-2}\right]+\left[n_2(q-1)+\textstyle\sum_{j=1}^{b_2}(n_2-j+1)(q-1)^2q^{j-2}\right]\}.$$

Hence the theorem.

III. Optimal codes

For optimality of the linear codes, the inequality (1) should be considered as equality. This gives us

$$\mathbf{q^{n-k}}=\mathbf{1+\left[n_1(q-1)+\textstyle\sum_{i=1}^{b_1}(n_1-i+1)(q-1)^2q^{i-2}\right]+\left[n_2(q-1)+\textstyle\sum_{j=1}^{b_2}(n_2-j+1)(q-1)^2q^{j-2}\right]} \quad \textbf{(2)}$$

The values of the parameters that satisfy (2) results into codes that are optimal in the sense that the number of burst errors to be corrected length 1 in the first block of length n_1 and all burst errors of length 2 or less in the second block of length n_2 in such codes equals the total number of cosets viz. 3^{n-k}. Such codes are termed as ternary (1,2) burst-correcting optimal linear codes .

For b_1 = 1 and b_2 =2, equality (2) becomes,

$$\mathbf{q^{n-k}=1+n_1(q-1)+n_2(q-1)+(n_2-1)(q-1)^2.} \quad \textbf{(3)}$$

For q = 3, the the equality in (3) reduces to

$$\mathbf{3^{n-k}=2n_1+6n_2-3.} \quad \textbf{(4)}$$

Now we examine the possibilities of the existence of codes for different values of the parameters n_1, n_2 and k satisfying (4) in such a way that $n_1+n_2\leq 119$ and $r = n-k \leq 5$.We also note that the values of n_1 satisfying (4) should always be the multiple of 3 in order to obtain integer solution. It can be verified that for n_1 = 1 , 2 , 4,5,7,8,… etc. x ≠3n , ∀ n ϵ N. This shows that the above equation does not have any integer solution for n_2.
Therefore (1,2)-burst error correcting optimal linear code for n_1 = 1,2,4,5,7,8,….,x ≠3n , ∀ n ϵ N, cannot exist.
Let n_1 =3.The equation (4) reduces to

$$\mathbf{3^{n_2-k}=\frac{1}{9}(1+2n_2).} \quad \textbf{(5)}$$

Then the values of parameters n_2 and k for $r\leq 5$ satisfying (5) are (4, 4), (13, 12) and (40,38).
This gives us to the possibilities of the existence of (3+4, 4), (3+13, 12) and (3+40, 38) ternary codes.
Let n_1 =6.
The equation (4) reduces to

$$\mathbf{3^{n_2-k}=\frac{1}{243}(3+2n_2).} \quad \textbf{(6)}$$

Then the various values of parameters n_2 and k for $r\leq 5$ satisfying the above equation are
(n_2, k) = {(3,6), (12,14) and (39,40)}.
This shows the possibilities of the existence of (3+3,6), (3+12,14) and (3+39,40) ,(1,2)-burst –error correcting codes over GF(3).

TABLE 1

n_1	n_2	k
3	4	4
	13	12
	40	38
6	3	6
	12	14
	39	40

9	**2 11 38**	**8 16 42**
12	**10 37**	**18 44**
15	**9 36**	**20 46**
18	**8 35**	**22 48**
21	**7 34**	**24 50**
24	**6 33**	**26 52**
27	**5 32**	**28 54**
30	**4 31**	**30 56**
33	**3 30**	**32 58**
36	**2 29**	**34 60**
39	**28**	**62**
42	**27**	**64**
45	**26**	**66**
48	**25**	**68**
51	**24**	**70**
54	**23**	**72**
57	**22**	**74**
60	**21**	**76**
63	**20**	**78**
66	**19**	**80**
69	**18**	**82**
72	**17**	**84**
75	**16**	**86**
78	**15**	**88**
81	**14**	**90**
84	**13**	**92**
87	**12**	**94**
90	**11**	**96**
93	**10**	**98**
96	**9**	**100**
99	**8**	**102**
102	**7**	**104**
105	**6**	**106**
108	**5**	**108**
111	**4**	**110**
114	**3**	**112**
117	**2**	**114**

IV. Discussion

Example 1: For various values of the parameters n_1 =6, n_2 =3 and k = 6, the matrix (7) may be considered as the parity check matrix for an (6+3, 3) code for q =3 where first sub-block n_1 of length 6 corrects all bursts of length 1 and the second sub-block n_2 of length 3 corrects all bursts of length 2 or less.

$$H = \begin{bmatrix} 1 & 1 & 2 & 1 & 0 & 0 & 1 & 1 & 0 \\ 2 & 0 & 0 & 1 & 1 & 0 & 1 & 0 & 1 \\ 1 & 2 & 0 & 1 & 0 & 1 & 0 & 1 & 1 \end{bmatrix}. \qquad (7)$$

It can be verified from the following error pattern-syandrom table 2 that the (9,6) code corrects all burst of length 1 in the first block of length n_1 and all burst of length2 or less in the second block of length n_2 over GF(3).

TABLE 2

Error-Pattern	Syndrome	Error-Pattern	Syndrome
100000 000	121	000000 120	012
010000 000	102	000000 011	112
001000 000	200	000000 012	120
000100 000	111	000000 220	122
000010 000	010	000000 210	021
000001 000	001	000000 022	221
200000 000	212	000000 021	210

020000 000	**201**	**000000 100**	**110**
002000 000	**100**	**000000 010**	**101**
000200 000	**222**	**000000 001**	**011**
000020 000	**020**	**000000 200**	**220**
000002 000	**002**	**000000 020**	**202**
000000 110	**211**	**000000 002**	**022**

Example 2: For values of the parameters n_1 =12, n_2 =10 and k = 18, the following matrix (8) may be considered as parity – check matrix for (12+10, 18) for ternary (1,2) – burst correcting optimal code .

$$H = \begin{bmatrix} 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 2 & 1 & 1 & 1 & 0 & 0 & 0 & 1 & 1 & 1 & 0 \\ 1 & 2 & 2 & 2 & 1 & 2 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 2 & 1 & 0 & 0 & 1 & 0 & 1 & 0 & 1 \\ 2 & 2 & 1 & 2 & 0 & 0 & 2 & 2 & 0 & 0 & 1 & 0 & 2 & 0 & 2 & 0 & 1 & 0 & 0 & 1 & 1 & 0 \\ 2 & 0 & 1 & 2 & 2 & 1 & 1 & 2 & 1 & 2 & 2 & 2 & 0 & 2 & 1 & 1 & 0 & 0 & 0 & 2 & 1 & 2 \end{bmatrix}. \tag{8}$$

The existence of (22,18) code can be verified from the following error pattern-syndrome table 3.

Table 3

Error-Pattern	Syndrome	Error-Pattern	Syndrome
100000000000 0000000000	0122	000000000000 0000000120	0101
010000000000 0000000000	1220	000000000000 0000000012	1212
001000000000 0000000000	1211	000000000000 2200000000	1111
000100000000 0000000000	1222	000000000000 0220000000	1010
000010000000 0000000000	1102	000000000000 0022000000	2211
000001000000 0000000000	1201	000000000000 0002200000	0022
000000100000 0000000000	1021	000000000000 0000220000	0220
000000010000 0000000000	1022	000000000000 0000022000	2200
000000001000 0000000000	1001	000000000000 0000002200	1221
000000000100 0000000000	1002	000000000000 0000000220	1210
000000000010 0000000000	1012	000000000000 0000000022	2220
000000000001 0000000000	2202	000000000000 2100000000	0212
200000000000 0000000000	0211	000000000000 0210000000	0222
020000000000 0000000000	2110	000000000000 0021000000	2210
002000000000 0000000000	2122	000000000000 0002100000	0012
000200000000 0000000000	2111	000000000000 0000210000	0120
000020000000 0000000000	2201	000000000000 0000021000	1200
000002000000 0000000000	2102	000000000000 0000002100	0112
000000200000 0000000000	2012	000000000000 0000000210	0202
000000020000 0000000000	2011	000000000000 0000000021	2121
000000002000 0000000000	2002	000000000000 1000000000	1020
000000000200 0000000000	2001	000000000000 0100000000	1202
000000000020 0000000000	2021	000000000000 0010000000	1121
000000000002 0000000000	1101	000000000000 0001000000	0001
000000000000 1100000000	2222	000000000000 0000100000	0010
000000000000 0110000000	2020	000000000000 0000010000	0100
000000000000 0011000000	1122	000000000000 0000001000	1000
000000000000 0001100000	0011	000000000000 0000000100	1112
000000000000 0000110000	0110	000000000000 0000000010	1011
000000000000 0000011000	1100	000000000000 0000000001	0102
000000000000 0000001100	2112	000000000000 2000000000	2010
000000000000 0000000110	2120	000000000000 0200000000	2101
000000000000 0000000011	1110	000000000000 0020000000	2212
000000000000 1200000000	0121	000000000000 0002000000	0002
000000000000 0120000000	0111	000000000000 0000200000	0020
000000000000 0012000000	1120	000000000000 0000020000	0200
000000000000 0001200000	0021	000000000000 0000002000	2000
000000000000 0000120000	0210	000000000000 0000000200	2221
000000000000 0000012000	2100	000000000000 0000000020	2022
000000000000 0000001200	0221	000000000000 0000000002	0201

V. Conclusion And Open Problem

As we know that optimal codes improve the efficiency of the communication channels as well as the rate of transmission. So, these codes are very useful from application point of view. In this paper, we have investigated the solutions of the equation (4) for $r \leq 5$ and for $n_1 \leq 117$. We noticed that equation (4) has solutions only for $n_1 = 3x$, $1 \leq x \leq 29$ and no integer solutions for n_1 = 1,2,4,5,7,…………….., 116. We have been able to obtain two codes (6+3,3) and (12+10,18) corresponding to the solutions. This justifies existence of such (1, 2) burst-correcting optimal linear codes over GF(3).

However, in view of the existence of other solutions of the equation (4), the existence of corresponding codes is an open problem. Also, it would be interesting to find such codes for $b_1 \geq 1$ and $b_2 \geq 2$.

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References

[1] Buccimazza, B., Dass, B.K. and Jain, S., Ternary (1,2)- optimal linear codes, Journal of Interdisciplinary Mathematics, 7(1), 2004, 71-77
[2] Chein, R.T. and Tang, D.T., On definition of a Burst, IBM Journal Research Development, 9 (4), 1965, 292-293.
[3] Dass, B.K., Iembo, R. and Jain, S., (1,2)- optimal linear codes over GF(5), Journal of Interdisciplinary Mathematics, 9(2), 2006, 319-326.
[4] Dass, B.K., Iembo, R. and Jain, S., (1,2)- optimal linear codes over GF(7), Reliability and Information Technology: Trends and Future Directions, Narosa Pub. House, Delhi, India, 2005, 371-376.
[5] Dass, B.K. and Das, P.K., On perfect like binary and non binary perfect codes- A brief survey, Bull. Malays. Maths. Sci. Soc.(2), 32(2), 2009, 187-210.
[6] Das, P.K., (1,3) Optimal Linear Codes on Solid Bursts, International Journal of Information Security Science, 1(4), 2012, 106-111.
[7] Dass, B.K. and Tyagi, V., A New type of (1,2)-optimal codes over GF(2), Indian Journal of Pure and Applied Mathematics, 13(7),1982, 750-756
[8] Fire, P., A class of multiple error correcting binary codes for non independent errors, Sylvania Reports R SL-E-2, 1959.
[9] Hamming, R.W., Error detecting and error correcting codes, Bell Syst. Tech. J., 29, 1950, 147-160.
[10] Peterson, W.W. and Weldon, E.J. (Jr.),(Error correcting codes, John Wiley, M.I.T. Press, 1972).
[11] Reiger, S.H., Codes for the correction of 'clustered' errors, IRE Trans. Information Theory, 6, 1960, 16-21.

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