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Two simples proofs of Fermat 's last theorem and Beal conjecture

M. Sghiar
9 Allée capitaine Jean Bernard Bossu, 21240, Talant, France
Corresponding Author: M. Sghiar

Abstract : *If after 374 years the famous theorem of Fermat-Wiles was demonstrated in 150 pages by A. Wiles [4], the puspose of this article is to give a simple demonstration and deduce a proof of the Beal conjecture.*
Résumé : *Si après 374 ans le célèbre théorème de Fermat-Wiles a été démontré en 150 pages par A. Wiles [4], le but de cet article est de donner une simple démonstration et d'en déduire une preuve de la conjecture de Beal.*
***Keywords**: Fermat, Fermat-Wiles theorem, Fermat's great theorem, Beal conjecture, Beal.*
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I. Introduction

Set out by Pierre de Fermat [2], it was not until more than three centuries ago that Fermat's great theorem was published, validated and established by the British mathematician Andrew Wiles [4] in 1995.

In mathematics, and more precisely in number theory, the last theorem of Fermat [2], or Fermat's great theorem, or since his Fermat-Wiles theorem demonstration [4], is as follows : There are no non-zero integers a, b, and c such that : $a^n+b^n=c^n$, as soon as n is an integer strictly greater than 2 ".

The Beal conjecture is the following conjecture in number theory : If $a^x+b^y=c^z$ where a, b, c, x, y and z are positive integers with x, y, z > 2, then a, b, and c have a common prime factor. Equivalently, There are no solutions to the above equation in positive integers a, b, c, x, y, z with a, b and c being pairwise coprime and all of x, y, z being greater than 2.

If the famous Fermat-Wiles theorem has been demonstrated in **150 pages** by A. Wiles [4], the purpose of this article is to give a simple proof and deduce a proof of the Beal conjecture.

II. The proof of Fermat 's last theorem

Theorem :

There are no non-zero integers a, b, and c such that: $a^n+b^n=c^n$, with n an integer strictly greater than 2.

Lemma 1 :

If n, a, b and c are a non-zero integers with and $a^n+b^n=c^n$ then:

$$\int_0^b x^{n-1}-\left(\frac{c-a}{b}x+a\right)^{n-1}\frac{c-a}{b}dx=0$$

Proof :

$$a^n+b^n=c^n \Leftrightarrow \int_0^a n x^{n-1}dx+\int_0^b n x^{n-1}dx=\int_0^c n x^{n-1}dx$$

But as :

$$\int_0^c n x^{n-1}dx=\int_0^a n x^{n-1}dx+\int_a^c n x^{n-1}dx$$

So :

$$\int_0^b n x^{n-1}dx=\int_a^c n x^{n-1}dx$$

And as by changing variables we have :

$$\int_a^c n x^{n-1}dx=\int_0^b n\left(\frac{c-a}{b}y+a\right)^{n-1}\frac{c-a}{b}dy$$

Then :

$$\int_0^b x^{n-1} dx = \int_0^b \left(\frac{c-a}{b} y + a\right)^{n-1} \frac{c-a}{b} dy$$

It results:

$$\int_0^b x^{n-1} - \left(\frac{c-a}{b} x + a\right)^{n-1} \frac{c-a}{b} dx = 0$$

Corollary 1 : If N, n, a, b and c are a non-zero integers with and $a^n + b^n = c^n$ then :
$\int_0^{\frac{b}{N}} x^{n-1} - \left(\frac{c-a}{b} x + \frac{a}{N}\right)^{n-1} \frac{c-a}{b} dx = 0$

Proof : It results from the proof of lemma 1 by replacing a, b and c respectively by $\frac{a}{N}$, $\frac{b}{N}$ and $\frac{c}{N}$.

Lemma 2 :
If $a^n + b^n = c^n$, where n, a, b and c are a non-zero integers with $n > 2$ and $a \leq b \leq c$. Then for an integer N big enough we have : $x^{n-1} - \left(\frac{c-a}{b} x + \frac{a}{N}\right)^{n-1} \frac{c-a}{b} \leq 0 \quad \forall \ x \in \left[0, \frac{b}{N}\right]$.

Proof :

Let $f(x, a, b, c, y) = x^{n-1} - \left(\frac{c-a}{b} x + y\right)^{n-1} \frac{c-a}{b}$. with $x, y \in \mathbb{R}^+$.

We have : $\frac{\partial f}{\partial x} = (n-1) x^{n-2} - (n-1) \left(\frac{c-a}{b} x + y\right)^{n-2} \left(\frac{c-a}{b}\right)^2$, $f(0, a, b, c, y) < 0$ and $\frac{\partial f}{\partial x}|_{x=0} < 0$.

So, by continuity, $\exists\, \epsilon > 0$ such that $\forall\ u \in [0, \epsilon]$ we have $\frac{\partial f}{\partial x}|_{x=u} < 0$. So the function f is decreasing in $[0, \epsilon]$ and $\exists\, \epsilon' > 0, \epsilon \geq \epsilon' > 0$ such that we have : $f(x, a, b, c, y) \leq 0 \quad \forall\ x \in [0, \epsilon'], \ \forall\ y \in [0, \epsilon']$.
As $\frac{b}{N} \in [0, \epsilon']$ for an integer N big enough , It follows that $\forall\ x \in \left[0, \frac{b}{N}\right]$ we have :
$f(x, a, b, c, \frac{a}{N}) \leq 0 \quad \forall\ x \in \left[0, \frac{b}{N}\right]$.

Proof of Theorem:
If $a^n + b^n = c^n$, where n, a, b and c are a non-zero integers with $n > 2$ and $a \leq b \leq c$. Then for an integer N big enough, it results from the **lemma 2** that we have :
$f(x, a, b, c, \frac{a}{N}) = x^{n-1} - \left(\frac{c-a}{b} x + \frac{a}{N}\right)^{n-1} \frac{c-a}{b} \leq 0 \quad \forall\ x \in \left[0, \frac{b}{N}\right]$

And by using the **corollary 1**, we have $\int_0^{\frac{b}{N}} x^{n-1} - \left(\frac{c-a}{b} x + \frac{a}{N}\right)^{n-1} \frac{c-a}{b} dx = 0$.

So : $x^{n-1} - \left(\frac{c-a}{b} x + \frac{a}{N}\right)^{n-1} \frac{c-a}{b} = 0 \quad \forall\ x \in \left[0, \frac{b}{N}\right]$

And therefore $\frac{c-a}{b} = 1$ because $f(x, a, b, c, \frac{a}{N})$ is a null polynomial as it have more than n zeros. So $c = a + b$ and $a^n + b^n \neq c^n$ which is absurde .

III. The proof of Beal conjecture

Corollary : **[Beal conjecture]**
If $a^x + b^y = c^z$ where a, b, c, x, y and z are positive integers with x, y, z > 2, then a, b, and c have a common prime factor. Equivalently, there are no solutions to the above equation in positive integers a, b, c, x, y, z with a, b and c being pairwise coprime and all of x, y, z being greater than 2.

Proof :
Let $a^x+b^y=c^z$

If a, b and c are not pairwise coprime, then by posing $a=ka'$, $b=kb'$, and $c=kc'$.

Let $a'=u'^{yz}$, $b'=v'^{xz}$, $c'=w'^{xy}$ and $k=u^{yz}$, $k=v^{xz}$, $k=w^{xy}$

As $a^x+b^y=c^z$, we deduce that $(uu')^{xyz}+(vv')^{xyz}=(ww')^{xyz}$.
So : $k^x u'^{xyz}+k^y v'^{xyz}=k^z w'^{xyz}$
This equation does not look like the one studied in the first theorem. But if a, b and c are pairwise coprime, we have $k=1$ and $u=v=w=1$ and we will have to solve the equation : $u'^{xyz}+v'^{xyz}=w'^{xyz}$
The equation $u'^{xyz}+v'^{xyz}=w'^{xyz}$ have a solution if at least one of the equations : $(u'^{xy})^z+(v'^{xy})^z=(w'^{xy})^z$, $(u'^{xz})^y+(v'^{xz})^y=(w'^{xz})^y$, $(u'^{yz})^x+(v'^{yz})^x=(w'^{yz})^x$, have a solution .
So by the proof given in the proof of the first Theorem we must have : $x\leq 2$ or $y\leq 2$, or $z\leq 2$.
We therefore conclude that if $a^x+b^y=c^z$ where a, b, c, x, y, and z are positive integers with $x,y,z>2$, then a, b, and c have a common prime factor.

IV. Important notes

1- If a, b, and c are not pairwise coprime, someone, by applying the proof given in the corollary like this : $a=u^{yz}$, $b=v^{xz}$, $c=w^{xy}$ will have $u^{xyz}+v^{xyz}=w^{xyz}$, and could say that all the x,y and z are always smaller than 2. What is false: $7^3+7^4=14^3$.
The reason is sipmle: it is the common factor k which could increase the power, for example if $k=c'^r$ in the proof, then $c^z=(kc')^z=c'^{(r+1)z}$. You can take the example : $2^r+2^r=2^{r+1}$ where $k=2^r$.
2- These techniques do not say that the equation $a^n+b^n=c^n$ where $a,b,c\in\,]0,+\infty[$, has no solution since in the proof the equation $X^2+Y^2=Z^2$ can have a sloution. We take $a=X^{\frac{2}{n}}$, $b=Y^{\frac{2}{n}}$ and $C=Z^{\frac{2}{n}}$.
3 – In [3] I proved the **abc conjecture** which implies only that the equation $a^x+b^y=c^z$ has only a finite number of solution with a, b, c, x, y, z a positive integers, a, b and c being pairwise coprime and all of x, y, z being greater than 2.

V. Conclusion

The techniques used in this article have allowed to prove both the Fermat' last theorem and the Beal' conjecture and have shown that the Beal conjecture is only a corollary of the Fermat' last theorem.

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