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Critical Path Method in the Network Analysis with Parametric Fuzzy Activity

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***Abstract**: In this paper, a method is presented to find fuzzy critical path using parametric interval valued function. Here network model has been developed in fuzzy environment. All activities are considered as a Trapezoidal fuzzy number. This fuzzy number has been transform as an interval number using Nearest Interval Approximation method. In this work, we develop a parametric interval-valued functional form of an interval number and then solve the parametric network problem. To illustrate the technique, an airport's cargo ground operation system is considered here.*
***Keywords**: Network Problem, Critical Path Method, Fuzzy Number, Trapezoidal Fuzzy Number.*

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I. Introduction:

In today's highly competitive business environment, one of the most challenging jobs that any manager can take on is the management of a large-scale project that requires coordinating numerous activities throughout the organization to maintain competitive priorities such as on time delivery and customization. A myriad of details must be considered in planning how to coordinate all these activities, in developing a realistic schedule, and then in monitoring the progress of the project. Fortunately, two closely related operations research techniques, PERT (program evaluation review technique) and CPM (critical path method), are available to assist the project manager in carrying out these responsibilities. These techniques make heavy use of networks to help plan and display the coordination of all the activities. They also normally use a software package to deal with all the data needed to develop schedule information and then to monitor the progress of the project. In order to solve these types of problems several methods exists in the literature. For instance, Chen[1] proposed an approach to critical path analysis for a project network with activity times being fuzzy numbers, in that membership function of fuzzy total duration time is constructed which is based on the extension principle and linear programming. Chen and Hsueh[2] presented a simple approach to solve the critical path method problem with fuzzy activity times on the basis of the linear programming formulation and the fuzzy number ranking method that are more realistic than crisp ones. They also defined that the most critical path and the relative path degree of criticality which are easy to use in practice. Sireesha and Shankar [3] presented a new method based on fuzzy theory for solving fuzzy project scheduling in fuzzy environment. Shankar et al. [4] presented an analytical method for measuring the criticality in a fuzzy project network, where the duration time of each activity is represented by a trapezoidal fuzzy number. Yakhchali et al.[5] assumed time lags which are common practice in the different projects, are imprecise and they discussed the problems of possibly critical path in the networks with interval valued activity and time lag duration. Yao and Lin [6] proposed a method for ranking fuzzy numbers without the need for any assumptions and used both positive and negative values to define ordering which then applied to critical path method. Liang and Han [7] presented an algorithm to perform fuzzy critical path analysis for project network problem. They showed that the ambiguities involved in the assessment activity times in a project network can be effectively improved and thus a more convincing and effective project management decision-making can be obtained. Zielinski [8] extended some results for interval numbers to the fuzzy case for determining the possibility distribution describing latest starting times for activity. He also proposed the time algorithm for computing the intervals of the possible values of the latest starting times of an activity in general networks with interval durations and extended the results to the networks with fuzzy duration. In this paper, we are finding an alternative way to deal with the situation, where duration can be large and in order to find a specific and optimized range, we made a concept of fuzziness in which each and every instances of duration is made a set with the given intervals by putting the function. The optimized duration is further taken by the decision makers.

II. Prerequisite Mathematics

Fuzzy sets were first introduced by Zadeh [9]in 1965 as a mathematical way of representing impreciseness or vagueness in everyday life.

2.1. Fuzzy Set

A fuzzy set $\tilde{A}$ in a universe of discourse X is defined as the following set of pairs $\tilde{A}=\left\{\left(x,\mu_{\tilde{A}}(x)\right):x\in X\right\}$.Here $\mu_{\tilde{A}}:X\to[0,1]$ is a mapping called the membership function of the fuzzyset $\tilde{A}$ and $\mu_{\tilde{A}}(x)$ is called the membership value or degree of membership of $x\in X$ in the fuzzy set $\tilde{A}$.The larger $\mu_{\tilde{A}}(x)$ is the stronger the grade of membership form in $\tilde{A}$.

2.2.Convex Fuzzy Set

A fuzzy set $\tilde{A}$ of the universe of discourse X is convex if and only if for all x_1,x_2 in X,

$\mu_{\tilde{A}}\left(\lambda x_1+(1-\lambda x_2)\geq \min\left(\mu_{\tilde{A}}(x_1),\mu_{\tilde{A}}(x_2)\right)\right)$ when $0\leq\lambda\leq 1$.

2.3.Interval Number

Interval analysis is a new and growing branch of applied Mathematics. It provides necessary calculus called interval arithmetic for interval numbers. An interval number can be thought as an extension of the concept of a real number and also as a subset of real numbers.

An interval number A is defined by an ordered pair of real numbers as follows:

$A=[a_L,a_R]=\{x:a_L\leq x\leq a_R, x\in\Re\}$

where a_L and a_R are the left and right bounds of interval A respectively.

2.4.Fuzzy Number

A fuzzy number is a special case of a fuzzy set. Different definitions and properties of fuzzy numbers are encountered in the literature but they all agree on that fuzzy number represents the conception of a set of 'real numbers close to a' where 'a' is the number being fuzzy field. A fuzzy number is a fuzzy set in the universe of discourse X that is both convex and normal. A fuzzy number $\tilde{A}$ is a fuzzy set of the real line $\Re$ whose membership function $\mu_{\tilde{A}}(x)$ has the following characteristic with $-\infty<a_1<a_2<a_3<a_4<\infty$

$$\mu_{\tilde{A}}(x)=\begin{cases}\mu_L(x) & for\ \ a_1\leq x<a_2\\ 1 & for\ \ a_2\leq x\leq a_3\\ \mu_R(x) & for\ \ a_3<x\leq a_4\\ 0 & for\ \ otherwise\end{cases}$$

where $\mu_L(x):[a_1,a_2]\to[0,1]$ is continuous and strictly increasing ; $\mu_R(x):[a_3,a_4]\to[0,1]$ is continuous and strictly decreasing.

The general shape of a fuzzy number following the above definition is shown below

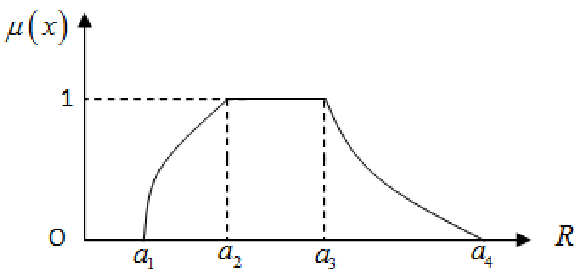

Fig-1 Fuzzy Number

2.4.1.Triangular Fuzzy Number (TFN)

Let $F(\Re)$ be a set of all triangular fuzzy numbers in real line $\Re$. A triangular fuzzy number $\tilde{A} \in F(\Re)$ is a fuzzy number with membership function $\mu_{\tilde{A}} : \Re \to [0,1]$ (Fig-2) parameterized by a triplet $(a_1, a_2, a_3)_{TFN}$. Where a_1 and a_3 denote the lower and upper limits of support of a fuzzy number $\tilde{A}$:

$$\mu_{\tilde{A}}(x) = \begin{cases} \left(\dfrac{x-a_1}{a_2-a_1}\right) & for \;\; a_1 \le x \le a_2 \\ 1 & for \;\; x = a_2 \\ \left(\dfrac{a_3-x}{a_3-a_2}\right) & for \;\; a_2 \le x \le a_3 \\ 0 & otherwise \end{cases}$$

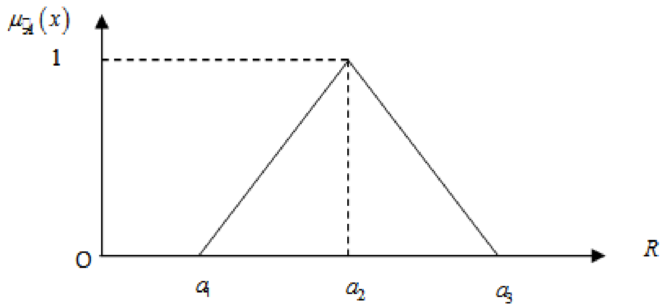

Fig-2 Triangular Fuzzy Number

2.4.2.Trapezoidal Fuzzy Number (TrFN)

Let $F(\Re)$ be a set of all triangular fuzzy numbers in real line $\Re$. A trapezoidal fuzzy number $\tilde{A} \in F(\Re)$ is a fuzzy number with membership function $\mu_{\tilde{A}} : \Re \to [0,1]$ (Fig-3) parameterized by a quadruple $(a_1, a_2, a_3, a_4)_{TrFN}$.

$$\mu_{\tilde{A}}(x) = \begin{cases} \left(\dfrac{x-a_1}{a_2-a_1}\right) & for \;\; a_1 \le x \le a_2 \\ 1 & for \;\; a_2 \le x \le a_3 \\ \left(\dfrac{a_4-x}{a_4-a_3}\right) & for \;\; a_3 \le x \le a_4 \\ 0 & otherwise \end{cases}$$

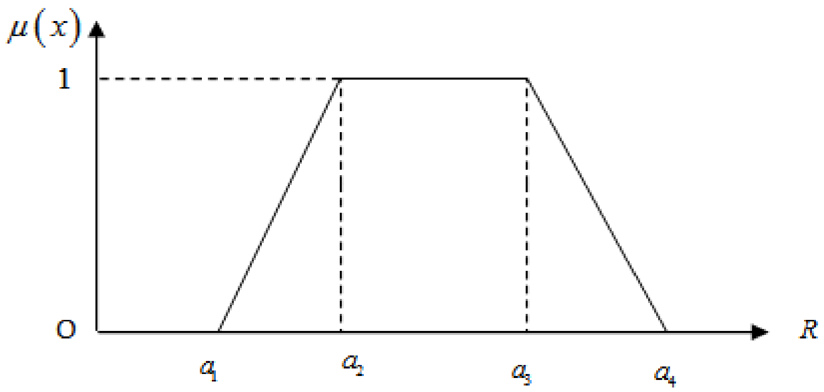

Fig-3 Trapezoidal Fuzzy Number

2.4.3. α -cut of a Fuzzy Number

The α -level of a fuzzy number $\tilde{A}$ is defined as a crisp set $A_\alpha = \{x : \mu_{\tilde{A}}(x) \geq \alpha, x \in X, \alpha \in [0,1]\}$. A_α is non-empty bounded closed interval contained in X and it can be denoted by $A_\alpha = [A_L(\alpha), A_R(\alpha)]$, $A_L(\alpha)$ and $A_R(\alpha)$ are the lower and upper bounds of the closed interval , respectively. Fig-2.5 shows a fuzzy number $\tilde{A}$ with $\alpha-$cuts $A_{\alpha_1} = [A_L(\alpha_1), A_R(\alpha_1)]$, $A_{\alpha_2} = [A_L(\alpha_2), A_R(\alpha_2)]$. It is seen that if $\alpha_2 \geq \alpha_1$ then $A_L(\alpha_2) \geq A_L(\alpha_1)$ and $A_R(\alpha_2) \geq A_R(\alpha_1)$.

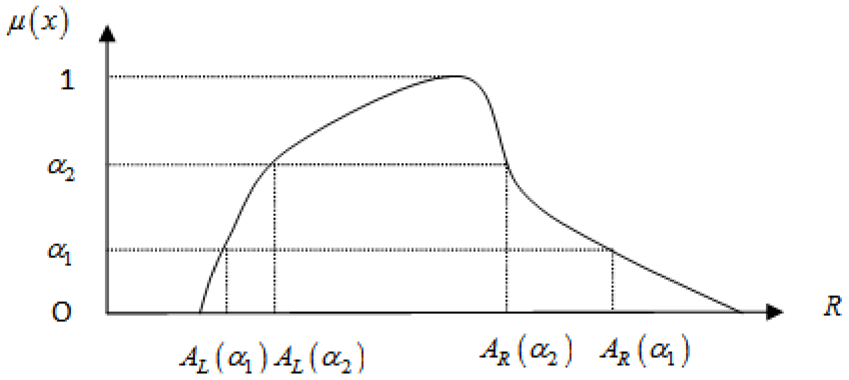

Fig-4 Fuzzy number $\tilde{A}$ with α -cut

III. Nearest Interval Approximation

Here we want to approximate a fuzzy number by a crisp model. Suppose $\tilde{A}$ and $\tilde{B}$ are two fuzzy numbers with $\alpha-$cuts are $[A_L(\alpha), A_R(\alpha)]$ and $[B_L(\alpha), B_R(\alpha)]$ respectively.

Then the distance between $\tilde{A}$ and $\tilde{B}$ is

$$d(\tilde{A},\tilde{B}) = \sqrt{\int_0^1 (A_L(\alpha) - B_L(\alpha))^2 d\alpha + \int_0^1 (A_R(\alpha) - B_R(\alpha))^2 d\alpha}.$$

Given $\tilde{A}$ is a fuzzy number. We have to find a closed interval $C_d(\tilde{A})$, which is the nearest to $\tilde{A}$ with respect to metric d. We can do it since each interval is also a fuzzy number with constant $\alpha-$cut for all $\alpha \in [0,1]$. Hence $(C_d(\tilde{A}))\alpha = [C_L, C_R]$. Now we have to minimize

$$d(\tilde{A}, C_d(\tilde{A})) = \sqrt{\int_0^1 (A_L(\alpha) - C_L)^2 d\alpha + \int_0^1 (A_R(\alpha) - C_R)^2 d\alpha}.$$

with respect to C_L and C_R.

In order to minimize $d(\tilde{A}, C_d(\tilde{A}))$, it is sufficient to minimize the function $D(C_L, C_R)\left(= d^2(\tilde{A}, C_d(\tilde{A}))\right)$. The first partial derivatives are

$$\frac{\partial}{\partial C_L}(D(C_L, C_R)) = -2\int_0^1 A_L(\alpha) d\alpha + 2C_L.$$

and $\frac{\partial}{\partial C_R}(D(C_L, C_R)) = -2\int_0^1 A_R(\alpha) d\alpha + 2C_R$.

Solving $\frac{\partial}{\partial C_L}(D(C_L, C_R)) = 0$ and $\frac{\partial}{\partial C_R}(D(C_L, C_R)) = 0$, we get

$C_L = \int_0^1 A_L(\alpha) d\alpha$ and $C_R = \int_0^1 A_R(\alpha) d\alpha$.

Again since $\frac{\partial^2}{\partial C_L^2}(D(C_L^*, C_R^*)) = 2 > 0$, $\frac{\partial^2}{\partial C_R^2}(D(C_L^*, C_R^*)) = 2 > 0$ and

$$H\left(C_L^*,C_R^*\right)=\frac{\partial^2}{\partial C_L^2}\left(D\left(C_L^*,C_R^*\right)\right).\frac{\partial^2}{\partial C_R^2}\left(D\left(C_L^*,C_R^*\right)\right)-\left(\frac{\partial^2}{\partial C_L \partial C_R}\left(D\left(C_L^*,C_R^*\right)\right)\right)^2=4>0.$$

So $D(C_L,C_R)$ i.e. $d\left(\tilde{A},C_d\left(\tilde{A}\right)\right)$ is global minimum. Therefore, the interval $C_d\left(\tilde{A}\right)=\left[\int_0^1 A_L(\alpha)\,d\alpha,\int_0^1 A_R(\alpha)\,d\alpha\right]$ is the nearest interval approximation of fuzzy number $\tilde{A}$ with respect to the metric d.

Let $\tilde{A}=(a_1,a_2,a_3,a_4)$ be a trapezoidal fuzzy number. The $\alpha-$cut interval of $\tilde{A}$ is defined as $A_\alpha=\left[A_L(\alpha),A_R(\alpha)\right]$ where $A_L(\alpha)=a_1+\alpha(a_2-a_1)$ and $A_R(\alpha)=a_4-\alpha(a_4-a_3)$. By nearest interval approximation method the lower limit of the interval is $C_L=\int_0^1 A_L(\alpha)\,d\alpha=\int_0^1\left[a_1+\alpha(a_2-a_1)\right]d\alpha=\frac{a_1+a_2}{2}$ and the upper limit of the interval is $C_R=\int_0^1 A_R(\alpha)\,d\alpha=\int_0^1\left[a_4-\alpha(a_4-a_3)\right]d\alpha=\frac{a_3+a_4}{2}$.

IV. Parametric Interval-valued Function

Let $[a,b]$ be an interval, where $a>0,b>0$. From analytical geometry point of view, any real number can be represented on a line .Similarly; we can express an interval by a function. The parametric interval-valued function for the interval $[a,b]$ can be taken as $g(s)=a^{1-s}b^s$ *for* $s\in[0,1]$ which is strictly monotone, continuous function and its inverse exits. Let ψ be the inverse of $g(s)$, then $s=\frac{\log\psi-\log a}{\log b-\log a}$.

V. Method for Finding the Fuzzy Critical Path

The FCP of a project network can be obtained by using the following steps:
Step-1.Identify activities in a project
Step-2.Establish precedence relationships of all activity
Step-3. Estimate the fuzzy activity time with respect to each activity.
Step-4.Using nearest interval approximation method, convert each and every fuzzy activity into interval valued function.
Step-5. Construct the project network.
Step-6. Let $\text{invf ES}_1=0$ and calculate $\text{invf ES}_j=\text{Maximum}\left\{\text{invf ES}_i+\text{invf ET}_{ij}\,\middle|\, i\in \text{NP}(j), j\neq 1, j\in \text{N}\right\}$, $j=1,2,...,n$.
Step-7.Let $\text{invf LF}_n=\text{invf ES}_n$ and calculate $\text{invf LF}_j=\text{Minimum}\left\{\text{invf LF}_k-\text{invf ET}_{jk}\,\middle|\, k\in \text{NS}(j), j\neq n, j\in \text{N}\right\}$.
Step-8. Calculate $\text{invf TS}_{ij}=\text{invf LF}_j-\left(\text{invf ES}_i+\text{invf ET}_{ij}\right), 1\le i<j\le n, j\in \text{N}$ with respect to each activity in a project network by using.
Step-9. Calculate total float for each activity. The activities having zero total float value are critical activities. Critical path is obtained by sequence of critical activities.

VI. Application of method to airport's cargo ground operation system

As the volume of cargo traffic has grown and the demand for cargo transport continues to rise, surface congestion has become an increasing problem, within an airport's cargo terminal. If an airport terminal's internal operations and service systems are inefficient, there will be a delay in ground operations. Therefore, cargo operations time needs to be shortened and passengers' luggage must be processed before cargo goods in order to maintain customer satisfaction. Fig..shows an international airport cargo terminal's ground operation procedures network. With the set of node $\text{N}=\{1,2,3,4,5\}$, the fuzzy activity time for each activity is shown in table 1.

Table 1: Fuzzy activity time for each activity

Activity A_{ij}	Fuzzy activity time (minutes)	Nearest interval approximation	Interval valued function
A_{12}	(10,15,15,20)	[12.5,17.5]	$12.5^{1-t}17.5^{t}$
A_{13}	(30,40,40,50)	[35,45]	$35^{1-t}45^{t}$
A_{23}	(30,40,40,50)	[35,45]	$35^{1-t}45^{t}$
A_{14}	(15,20,25,30)	[17.5,27.5]	$17.5^{1-t}27.5^{t}$
A_{25}	(60,100,150,180)	[80,165]	$80^{1-t}165^{t}$
A_{35}	(60,100,150,180)	[80,165]	$80^{1-t}165^{t}$
A_{45}	(60,100,150,180)	[80,165]	$80^{1-t}165^{t}$

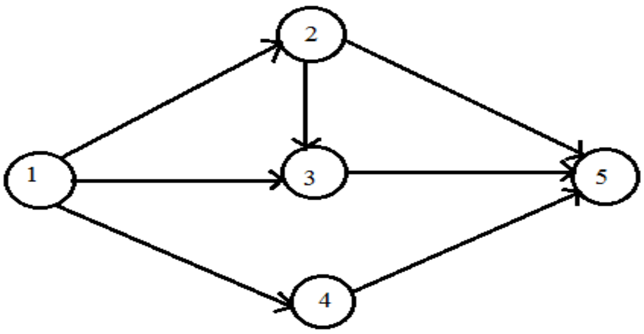

Fig-5. Airport cargo terminal's ground operation network

Solution: The optimal solution of the fuzzy project network model by interval-valued parametric function is presented in table 2.

Table 2 optimal solution of the fuzzy Airport cargo terminal's ground operation network

t	A_{12}	A_{13}	A_{23}	A_{14}	A_{25}	A_{35}	A_{45}	Critical Path	Duration
0	12.5	35	35	17.5	80	80	80	1-2-3-5	115
0.1	12.92	35.89	35.89	18.80	86	86	86	1-2-3-5	134.81
0.2	13.37	36.80	36.80	19.15	92.46	92.46	92.46	1-2-3-5	142.63
0.3	13.82	37.74	37.74	20.04	99.40	99.40	99.40	1-2-3-5	150.96
0.4	14.30	38.70	38.70	20.96	106.86	106.86	106.86	1-2-3-5	159.86
0.5	14.79	39.68	39.68	21.93	114.89	114.89	114.89	1-2-3-5	169.36
0.6	15.29	40.69	40.69	22.95	123.51	123.51	123.51	1-2-3-5	179.49
0.7	15.81	41.73	41.73	24.01	132.79	132.79	132.79	1-2-3-5	190.33
0.8	16.36	42.79	42.79	25.12	142.75	142.75	142.75	1-2-3-5	201.90
0.9	16.92	43.88	43.88	26.28	153.47	153.47	153.47	1-2-3-5	214.27
1	17.5	45	45	27.5	165	165	165	1-2-3-5	227.40

The critical path is 1-2-3-5. For t = 0, the lower bound of the interval value of the parameter is used to find the optimal solution. For t = 1, the upper bound of the interval value of the parameter is used to for the optimal solution. The main purpose of the proposed technique is that decision maker can get the intermediate optimal result using proper value of t

VII. Conclusion

This paper presents a simple approach to solve the CPM problem with fuzzy number (trapezoidal fuzzy number) activity times that are more realistic than crisp ones. On the basis of nearest interval approximation, the fuzzy CPM problem is transformed into an interval number. We than transform these interval number into a parametric interval-valued functional form. Without formation as a pair of two-level critical path problem, we can find the total duration time for the different value of the parameter t.

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