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Mathematical Model Of The Dynamics Of Blood Pressure In The Small Circle Of Blood Circulation And The Study Of The Nature Of Its Interaction With The Electrical Dynamics Of The Heart In A Case Of Model Pneumonia

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Abstract: *The mathematical model of the blood pressure dynamics in the pulmonary circulation has been developedand investigated the nature of its interaction with electrical dynamics of the heart in norm and in case of pneumonia. The model dynamics of blood pressure in the right and left ventricles of the heart were described in the framework of phase-shifted solutions of the coupled the Van der Pol equations, modeling the ECG pattern. In its turn, the dynamics of blood pressure in the pulmonary veins coming from the right ventricle of the heart and the pulmonary veins coming to the left ventricle were described in the framework of nonlinear differential equations of the second order. Taking into account the shift of initial phases in the dynamics of blood pressure in the right and left ventricles, the multipliers for conservative terms of differential equations describing the dynamics of blood pressure in the pulmonary veins and arteries were written as terms of the expanded into odd and even series sine and cosine functions correspondingly. A computer study of the model of the blood dynamics in a small circle of blood circulation resulted in different solutions, depending on the value of the coefficients of expansion into even and odd series with conservative terms of the equations describing the dynamics of blood pressure in the pulmonary veins and arteries. At the same time, the mixing of members of even and odd series and the value of their coefficients in the equations were identified with a model violation of the gas exchange process in the lungs associated with the development of model pneumonia. In addition, during the study of the model, it was found that the value of the difference between the squares of the sine and cosine components of the Fourier harmonics amplitudes of ECG representations could illustrate the degree of development of model pneumonia. It should be emphasized that as a resultof the study of the proposed model, it was also shown that the course of model pneumonia is characterized by a quasi-periodic re-pumping of the stochastic energy components of the Fourier representation of the ECG into the energy of the Fourier harmonics amplitudes ECG spectrum, leading to their increase. These properties allow us to attribute this process to the complex Fermi-Paste-Ulam (FPU) recurrence [1], as well as to the intermittent dynamics of the Ginzburg-Landau chain [2].*

Keywords*: Dynamics of blood pressure in the pulmonary veins and ventricles of the heart, solutions of the coupled Van der Pol equations, model pneumonia, stochastic component of the harmonics amplitudes of the Fourier series of ECG representations, complex FPU recurrence, intermittent dynamics of the Ginzburg-Landau chain.*

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I. Introduction

Pneumonia - inflammation of the lung tissue, usually of infectious origin with a predominant lesion of the alveoli (the development of inflammatory exudation in them) and interstitial lung tissue. The term "pneumonia" combines a large group of diseases. Pneumonia is the first cause of death from infections. The number of its victims is estimated in the millions every year, and this was the case before COVID -19. Pneumonia can be asymptomatic. In this regard, computer modeling of the development of this disease can find new approaches for both its diagnosis and treatment.

Consider the small circulatory system, which includes the lungs and heart. The small circle of blood circulation (Fig.1) begins in the right ventricle, which releases venous blood saturated with carbon dioxide into the pulmonary trunk. The pulmonary trunk is divided into the right and left pulmonary arteries. The pulmonary arteries branch into lobular, segmental, and sub-segmental arteries. Sub-segmental arteries are divided into arterioles that break up into capillaries. The outflow of blood, enriched with oxygen, goes through the veins, which are collected in reverse order and in the amount of four pieces flow into the left atrium, where the small circle of blood circulation ends. Blood circulation in the small circle of blood circulation occurs in 4-5 seconds.

HYPOTHESIS

Accounting the fact that the cycle of the small circle of blood circulation in a normal state is 4 -5 seconds (0.2-0.25 Hz), the period of heart contraction is about 1 second (1 Hz), while the initial times of ventricular contractions are shifted in phase relative to each other by about 0.1-0.2 seconds, pneumonia, causing inflammation of the alveoli, will lead to an increased respiratory rate and a decrease the time period between the initial phases of ventricular contraction. In this case, there is a narrowing of the width of the contraction impulse, which in the simplest case is similar to the pattern of an electrocardiogram (ECG) without stochastic components. In this regard, the specified dynamics of narrowing the width of the ventricular contraction pattern in pneumonia can be described in the framework of the difference between the squaresof sine and cosine forms of the Fourier representation of the ECG. At the same time, the value of this difference in the low frequency range of 0 – 6 Hz will be less, the greater the development of pneumonia due to the re-pumping of the energy of stochastic components of the sinusoidal and cosine Fourier representations of ECG into the energy of the Fourier harmonics, increasing their amplitude. However, it should be emphasized that this process is quasi-periodic in nature, and therefore it could be attributed to the fundamental processes – the complex Fermi-Paste-Ulam recurrence[1] and the intermittent dynamics of the Ginzburg-Landau chain [2].

MATHEMATICAL MODEL

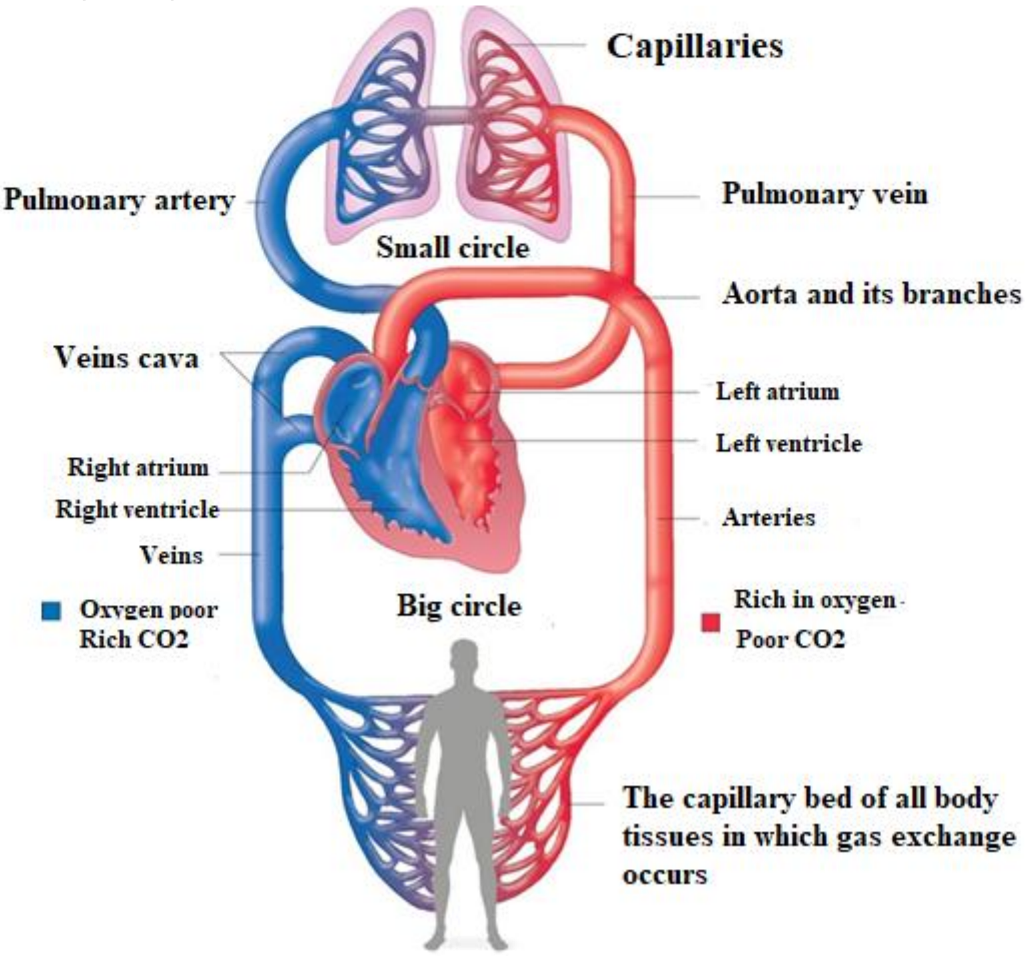

Fig. 1. Dynamics of blood in the small circulatory circle

When modeling the dynamics of venous and arterial blood flows in the small circulatory circle, it is necessary to simulate not the electrical activity of the heart, but the nature of the dynamics of blood pressure in the right and left ventricles, which is shifted in the initial phase of the contraction time relative to each other. In this regard, it was necessary to modify the model of electrical activity of the heart in the form of the FPU auto recurrence [1] to describe the dynamics of pressure in both ventricles. For this purpose, the coefficients in the coupled Van der Pol equations were changed and as a result, a model of the dynamics of blood pressure in the right and left ventricles was obtained. It turned out that the pressure patterns are identical to ECG patterns, but their initial phases are shifted by 0.1-0.2 seconds relative to each other. The dynamics of blood pressure in the pulmonary veins and arteries was described in the framework of nonlinear differential equations of the second order. Taking into account the shift of the initial phases in the dynamics of blood pressure in the right and left ventricles, the multipliers for the conservative terms of differential equations describing the dynamics of blood pressure in the pulmonary veins and arteries were described as terms of expansion into odd and even series of sine and cosine functions. In the computer study of the model of blood dynamics in the small circle of blood circulation, various solutions were obtained, depending on the value of the coefficients of expansion into even and odd series. At the same time, the mixing of the terms of even and odd series and the value of their coefficients in the equations was identified with the development of model pneumonia.

The mathematical model looks like this:

$$\frac{d^2P_1}{dt^2} - a_1(1-Y_1)\frac{dP_1}{dt} + \omega_1{}^2(1+\alpha_1 P_2)P_1 = c_1\frac{d^2P_2}{dt^2} + d_1F_1 + d_2F_2$$

$$b_1Y_1 + T_1\frac{dY_1}{dt} = P_1{}^2$$

$$\frac{d^2m_1}{dt^2} + \frac{dm_1}{dt} + \omega^2(1 + e_1m_1{}^2 + e_2m_1{}^4 + e_3m_1{}^6)m_1 = k_1P_2 + k_2Y_2 \qquad (1)$$

$$\frac{d^2P_2}{dt^2} - a_2(1-Y_2)\frac{dP_2}{dt} + \omega_2{}^2(1+\alpha_2 P_1)P_2 = c_2\frac{d^2P_1}{dt^2} + d_1F_1 + d_2F_2$$

$$b_2Y_2 + T_2\frac{dY_2}{dt} = P_2{}^2$$

$$\frac{d^2m_2}{dt^2} + \frac{dm_2}{dt} + \omega^2(j_1m_2 + j_2m_2{}^3 + j_3m_2{}^5 + j_4m_2{}^7)m_2 = k_4m_1$$

Where is P_1 - a value proportional to the pressure of the blood in the left ventricle, P_2 - is a value proportional to the pressure of the blood in the right ventricle, m_1 - is a value proportional to the pressure of the blood in the artery emanating from the left atrium into the lungs, m_2 - is a value proportional to the pressure of the blood in the artery coming into the right atrium from the lungs , Y_1 -is a value proportional to the time delay in the propagation of blood flow between the left atrium and the left ventricle., Y_2 - is a value proportional to the time delay in the propagation of blood flow between the right atrium and the right ventricle, b_1 - is a value proportional to the area of lateral surface of the artery emanating from the left atrium into the lungs, b_2 - is a value proportional to the area of lateral surface of the artery coming from the lungs to the right atrium , T_1 - is a value proportional to the time of contraction of the left ventricle, T_2 - is a value proportional to the time of contraction of the right ventricle, ω -breathing frequency, ω_1, ω_2 heart contraction frequency, F_1 - function of the impact of random variations of the heart rate dynamics, F_2 - function of the impact of thermal fluctuations on the dynamics of the blood, $a_1, a_2, e_1, e_2, e_3, \alpha_1, \alpha_2\, c_1, c_2, d_1, d_2, j_1, j_2, j_3, j_4, j_5$ - constants.

This mathematical model (1) was numerically studied in order to model the occurrence of abnormal conditions in the lungs, in particular, pneumonia. In the course of the study it was found that the value of the difference between the squares of the sine and cosine components of the Fourier harmonics amplitudes of the ECG representations, which have a pattern similar to the solutions of the coupled Van der Pol equations describing the dynamics of blood pressure in the left and right ventricles of the heart, may reflect the degree of development of the model pneumonia. In particular, during the study of the proposed model it was shown, that the current model of pneumonia is characterized by a quasi-periodic re-pumping of the energy of the stochastic components of the Fourier harmonics amplitudes of the Fourier representation of the ECG into the energy of its Fourier harmonics amplitudes without stochastic components. And that led to increasing of the harmonics amplitudes.

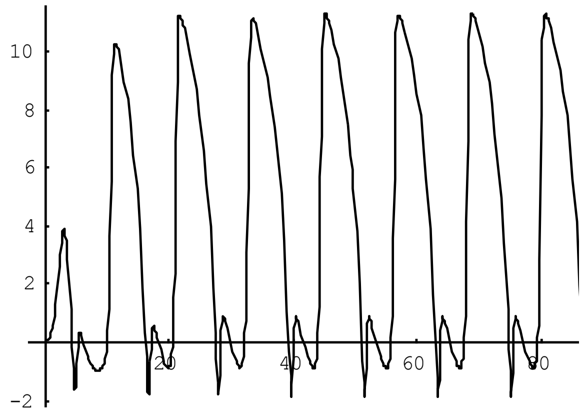

Fig. 2. Form of dynamics of blood pressure in the left ventricle at the model norm. Vert. axis is the amplitude. Horiz. axis-is the frequency. Conditional units.

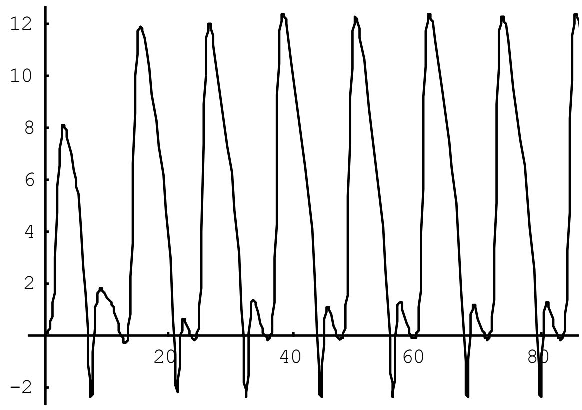

Fig. 3. Form of blood pressure dynamics in the right ventricle at the model norm. Vert. axis is the amplitude. Horiz. axis- is the frequency. Conditional units.

At the first stage of the numerical study of the model, thevariations in the phase relations between solutions of coupled Van der Pol equations describing the form of blood pressure dynamics in the left and right ventricles were studied. It was found that in a normally functioning heart and lungs, the initial phase ratios of the initial times of ventricular contractions in the model vary from a quarter to one-eighth of the period (2Pi) (Fig. 2, Fig. 3).Together with this, it was found that the value of the difference between the squares of the sine and cosine components of the Fourier amplitudes of ECG representations, which have a pattern close to the solutions of the coupled Van der Pol equations describing the dynamics of blood pressure in the left and right ventricles of the heart, changes quasi-periodically, reducing and increasing the share of energy of the stochastic component of the Fourier representation of the ECG. This dynamic is typical for the complex FPU recurrence, as well as for the intermittent dynamics of the Ginzburg-Landau chain. In this connection, we can roughly distinguish three types of differences in the squares of the sine and cosine components of the Fourier amplitudes of ECG representations, reflecting the complex return of the FPU. Figs. 4,5,6 show different energy shares of the stochastic component at the model norm.

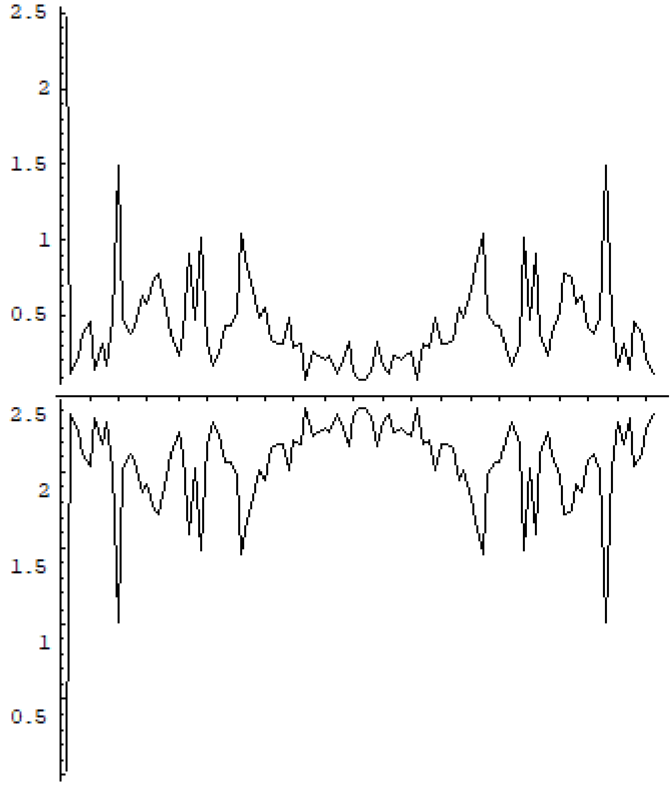

Fig. 4. Differences in the squares of the sine and cosine components of the Fourier harmonicsamplitudes of ECG representations that have the maximum value of stochastic componentenergy at the model norm. Vert. axis is the amplitude. Horiz. axis-is the frequency. Conditional units.

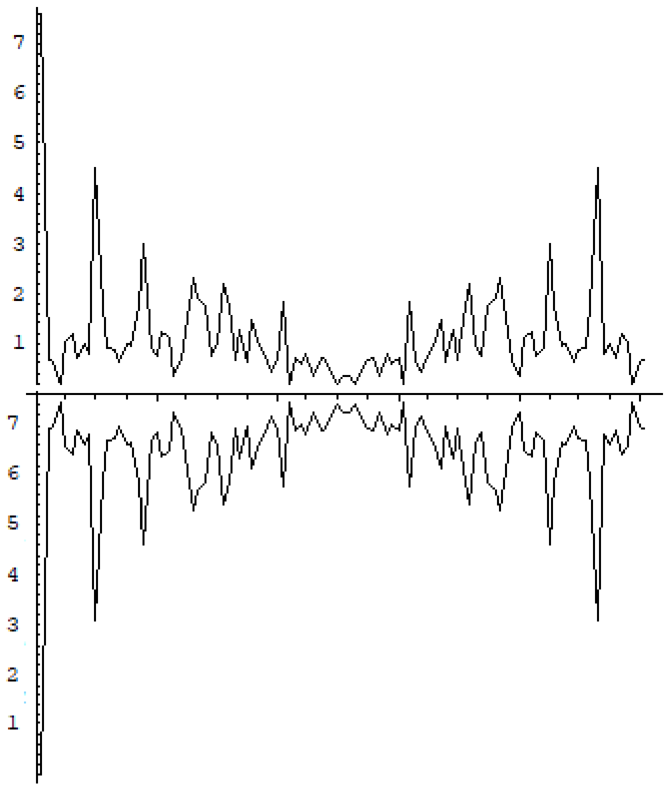

Fig. 5. Differences in the squares of the sine and cosine components of the Fourier harmonicsamplitudes of ECG representations with the average stochastic componentenergy at the model norm. Weert. The axis is the amplitude. Horiz. axis-is the frequency. Conditional units.

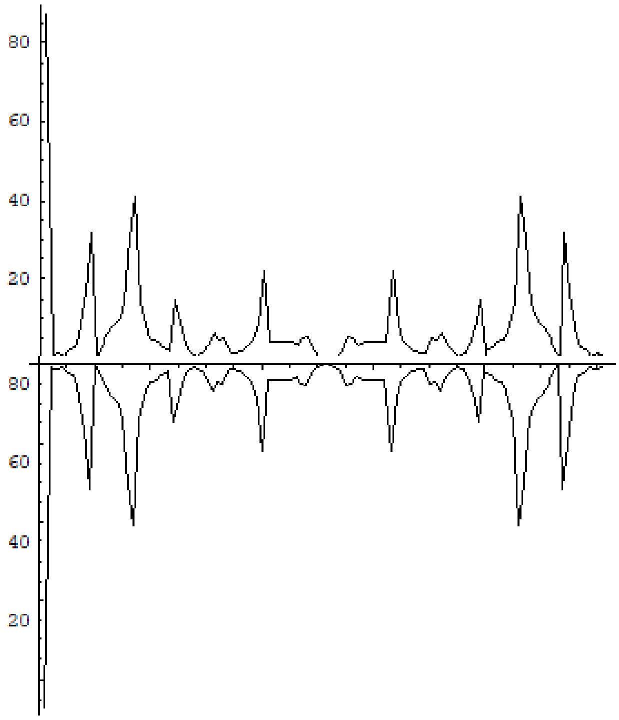

Fig. 6. Differences in the squares of the sine and cosine components of the Fourier harmonics amplitudes of ECG representations that have a minimal stochastic component energy at the model norm. Vert. axis is the amplitude. Horiz. axis-is the frequency. Conditional units.

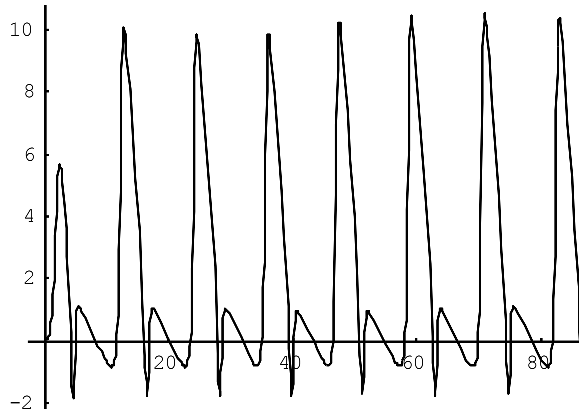

Fig. 7. Form of dynamics of blood pressure in the left ventricle in model pneumonia. Vert. axis is the amplitude. Horiz. axis-is the frequency. Conditional units.

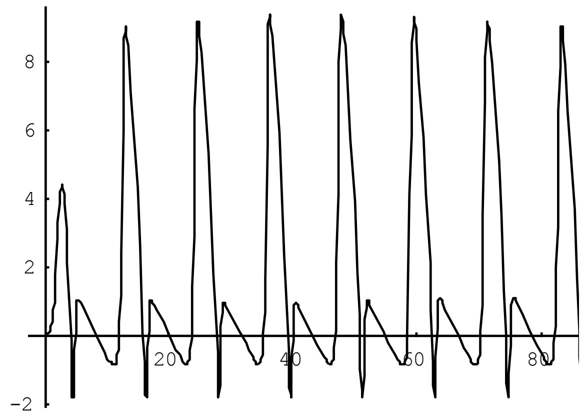

Fig. 8. Form of blood pressure dynamics in the right ventricle in model pneumonia. Vert. The axis is the amplitude. Horiz. axis-is the frequency. Conditional units.

At the second stage of the numerical study of the model, the variations in the phase relations between solutions of the coupled Van der Pol equations describing the form of blood pressure dynamics in the left and right ventricles in the case of model pneumonia were studied. Model pneumonia was created in the model by adding high-degree even variables to equations with odd polynomials and vice versa. It was found that in model pneumonia, the phase ratios within the model vary from a quarter to zero (Fig. 7, Fig. 8). Furthermore, it was found that the value of the difference between the squares of the sine and cosine components of the Fourier amplitudes of ECG representations, which have a pattern close to the solutions of the coupled Van der Pol equations describing the dynamics of blood pressure in the left and right ventricles of the heart, changes quasi-periodically, reducing and increasing the energy of the Fourier harmonics amplitudes of the ECG representation. However, in contrast to the model of a healthy heart and lungs, the share of the stochastic component energy in the differences was much smaller. This dynamic is typical for the complex FPU recurrence, as well as for the intermittent dynamics of the Ginzburg-Landau chain. In this connection, we can roughly distinguish three types of differences in the squares of the sine and cosine components of the Fourier amplitudes of ECG representations, reflecting the complex FPU recurrence. In Fig. 9,10,11. various energies of amplitudes of differences in model pneumonia are shown.

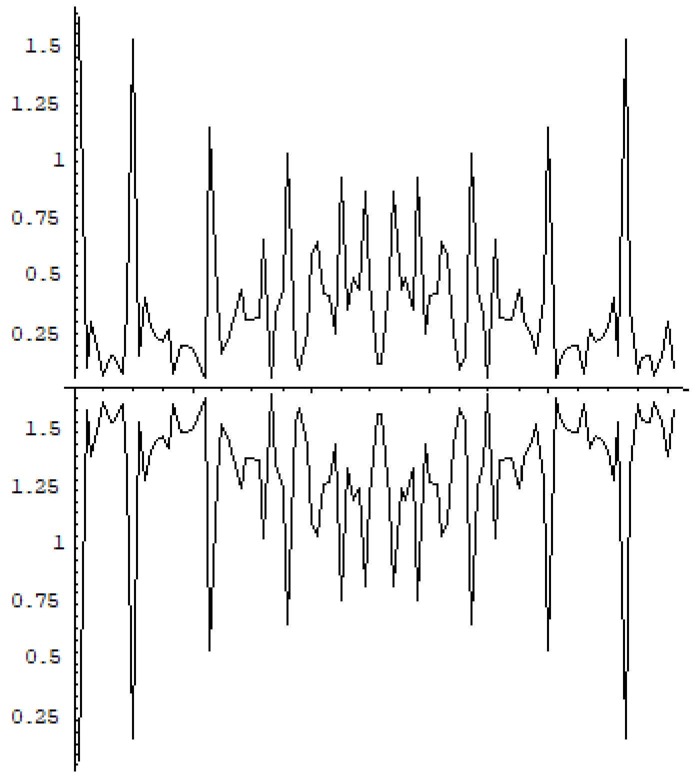

Fig. 9. Differences in the squares of the sine and cosine components of the Fourier harmonics amplitudes of ECG representations, which have different energies of the difference amplitudes and a small proportion of stochastic components energy in model pneumonia. Vert. axis is the amplitude. Horiz. axis-is the frequency. Conditional units.

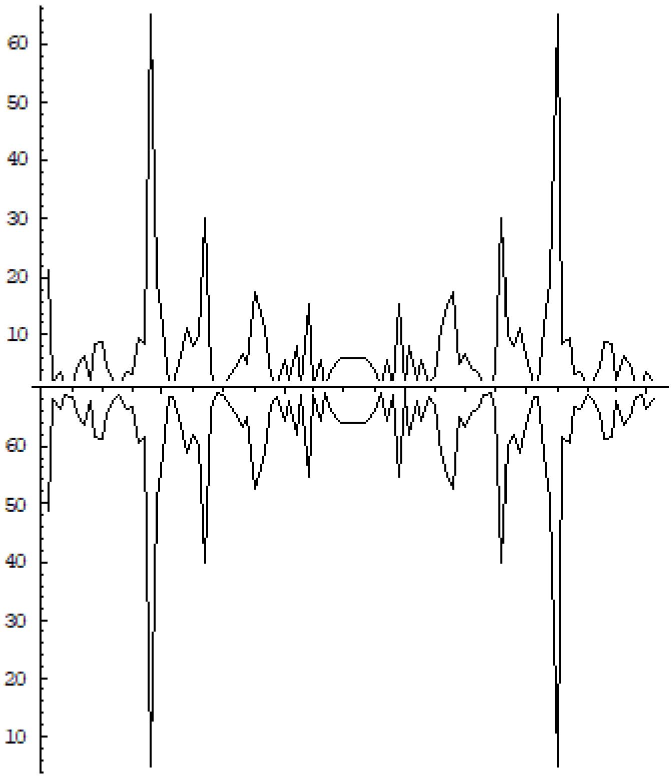

Fig. 10. Differences in the squares of the sine and cosine components of the Fourier harmonics amplitudes of ECG representations that have significant energies of the difference amplitudes and a minimal share of stochastic componentsenergy in model pneumonia. Vert. axis is the amplitude. Horiz. axis-is the frequency. Conditional units.

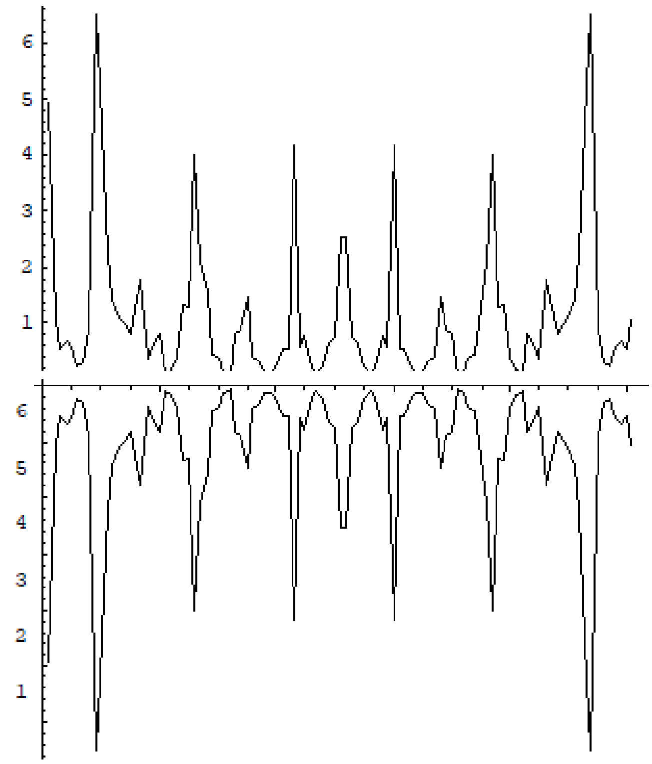

Fig. 11. Differences in the squares of the sine and cosine components of the Fourier harmonics amplitudes of ECG representations that have different maximum energies of the difference amplitudes and the practical absence of stochastic components energy in model pneumonia. Vert. axis is the amplitude. Horiz. axis-is the frequency. Conditional units.

II. Discussion

The main result of the study of the mathematical models of the pulmonary circulation should be considered as a confirmation of the hypothesis that the dynamics of blood in the pulmonary circulation as in the model norm, and the dynamics of the blood in the pneumonia model represent fundamental processes – the Fermi-Pasta-Ulam recurrence [1] and the intermittent dynamics of Ginzburg-Landau chain [2]. The difference in the parameters of these processes allows to use the solutions of the mathematical model for diagnosing pneumonia and monitoring the process of its treatment. However, this requires a ,RFsignificant clinical research [3].

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