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Study of Average Transaction Confirmation time in a Blockchain using Queueing Model

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***Abstract*:**
Blockchain technology was designed to support cryptocurrency system. It is peer-to-peer(P2P) distributed ledger technology for creating trust and agreement. It has received considerable attention in the current world such as banking, business, industry, transportation, healthcare, and so on. Detailed performance assessments are required to study the key performance indicators and some limitations like average transaction confirmation time of this technology. They provide further research for what this technology can be used for. However, the existing set of literature focuses on blockchain development and execution, with some work done on mathematical models, performance analysis, and optimization of blockchain systems. In this research, a queueing model that allows for a detailed examination of blockchain system properties is analyzed.

***Key Word*:** *Blockchain, Distributed ledger, Transaction Confirmation time, Queueing model,Performance analysis.*

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I. Introduction

Blockchain has gained a considerable interest in the recent few years from the industry, government, and academia. It is the core technology of the world's most famous cryptocurrency, Bitcoin, as suggested by Satoshi Nakamoto in his white paper [15]. A 'Blockchain' is defined as 'chain of blocks'. As compared to the traditional payment system such as credit cards and debit cards, one differentiating aspect of the crypto system is its decentralised nature. Cryptocurrency does not have any central authority to manage transactions. All the transactions are recorded in a ledger known as the blockchain. Because each block that contains a whole record list is linked to the next, a chain is formed. Which is maintained by a volunteer – based peer to peer (P2P) network. Volunteer nodes enters the P2P network, who have the same replica of the blockchain. They are allowed to check transaction consistency.

A block is made up of many transactions. By solving a puzzle which is crypto-math puzzle, a newly formed block is confirmed. This confirmation procedure is known as mining, and it involves several nodes called miners compete to solve this crypto-math puzzle. Only confirmed transactions, which are transactions that are contained in blocks on the blockchain, are accepted as valid.

Different blockchain deployment approaches can be followed depending on the application areas. Based on this there are three types of Blockchain environment. Public Blockchain, Private Blockchain, Hybrid or Consortium Blockchain.

- Public Blockchain: In this type of environment there are no access restrictions. Anyonecan use, access, and participate in this. It is also known as permissionless blockchain.The Bitcoin blockchain and the Ethereum blockchain are examples of the most known largest public blockchain.
- Private Blockchain: This is also known as a permissioned blockchain. Access is restricted to only participants and validators. Hyperledger and R3 Corda are examples of the private blockchain.
- Hybrid Blockchain or Consortium Blockchain: It has a combination of centralized and decentralized features. In this type of environment where instead of a single organization, multiple organizations govern the platform. It is not a public platform rather a permission platform, such as Dragon chain.

Blockchain has many advantages. Immutability, better security, fault tolerance, and transparency are just a few of the benefits of blockchain. But due to its nature of decentralisation, the performance (e.g., throughput and latency) of blockchain is restricted. For example, throughput of Bitcoin is 6 to 7 transactions per second and transaction confirmation time is around 10 minutes. Therefore, to find solution of this performance issue is aim of the researchers in study of the Blockchain. Very few works have been done on basic theory of Blockchain system. To develop the mathematical model of Blockchain performance evaluation in terms of Markov process, Markov decision processes, Queueing networks, Petri networks, Game models and so on, are interesting topics in contemporary research.

II. Literature Review

As of now, blockchain hasbeing used in numerous genuine applications. So far hardly any work has been done on execution demonstrating of blockchain frameworks. Many people work in Queueing model of Blockchain system to increase its performance.

Kasahara and Kawahara[9] (May 2016) studied transaction-confirmation time by taking priority queueing analysis. It is observed that the demand of transactions with low fees affects the transaction-confirmation time. For this they exhibited the transaction – confirmation process of Bitcoin as a queueing system in which priority mechanism and batch service are considered.

Kawase and Kasahara[10] (2017) showed the mining process with a M/G^B/1 queue with batch service and analysed the transaction-confirmation time. They derived that the block-size affects the transaction-confirmation time.

Lin et. al. [12] (Sep. 2018) developed queueing theory of blockchain systems and provide system performance evaluation. To accomplish this, they designed a Markovian batch-service queueing method with two distinct service stages that are appropriate to express the mining process in the miners pool as well as the progress of a new blockchain. They obtained a system stable state and express three main performance measurements using the matrix-geometric solution:

a. The average number of transactions in the queue
b. The average number of transactions in a block
c. The average transaction-confirmation time.

To ensure that the theoretical results are computable, they use numerical examples. Even though their queueing model is only simple under exponential or Poisson assumptions, their analytic approach will open the door to a series of potentially fruitful research in blockchain queueing theory.

Bowden et. al. [2] (Jan 2018) examined time-inhomogeneous behaviour of the block arrivals in the bitcoin blockchain. He observed that the block generation process is influenced by multiple key factors like solving difficulty level of crypto-math puzzle, transaction fee, mining rewards, mining pools and so on. The proof-of-work technique is of particular interest in this paper, since it accepts those who want to contribute to do so. He supposed the block arrival as Poisson Process.

Papadis et. al. [16] (April 2018) applied the time inhomogeneous block arrivals to set up some Markov processes. They also developed a stochastic model for the progress and dynamics of Blockchain network. They examined Blockchain characteristic such as the number of miners, their hashing power, block dissemination delays among distributed nodes and block confirmation rules.

Memon et. al. [14] (Feb 2019) proposed a model to imitate a blockchain using queuing theory. The suggested model is created by using M/M/1 queue as a memory pool, and an M/M/c queue as a mining pool. This model is a lucid but efficient way to reveal many important indices such as (a) The Number of Transactions per block (b) The Mining Time of Each Block (c) System Throughput/Transactions per second (d) Memory-pool count (e) Waiting Time in Memory-pool (f) Number of Unconfirmed Transactions in Whole System (g) Total Number of Transactions and (h) Number generated of Blocks.

Li et. al. [13] (April 2019) established a more general context of block-structured Markov processes in the queueing study of blockchain systems. It provides analysis both for the stationary performance measures and for the sojourn times of any transaction or block.

Reddy and Sharma[17] (Oct. 2019) developed the delay diameter (D) and double spending attack in Erdos-Renyi random network topology as constraints.

Caxiang et. al. [4] (July 2020) surveyed existing blockchain performance evaluation, sorted empirical and analytical evaluation methods. In empirical analysis, they reviewed current blockchain evaluation methodologies along with benchmark evaluation monitoring, experimental analysis, and simulations. In analytical modelling, they compared the methods: Markov chains, Queuing models and Stochastic Petri notes. They also surveyed the performance evaluation studies and detected the bottlenecks of major blockchain platforms. They ended with the identification of open issues and ascertainment of future research directions.

The aim of this paper is to analyse the transaction confirmation time using queueing theory. In this paper, Blockchain Queueing theory is being theoretically analysed for the block-generation and blockchain-building processes. In this, the sum of the block-generation and blockchain-building times can be considered as the transaction-confirmation time of a block.

III. Model Description

In Blockchain environment transactions, blocks and nodes are basic elements. It is essential to develop amathematical model that properly captures the stochastic characteristics of transactions. It is important to notethateach block-size are different yet limited.

Arrival process: Transactions arrived in the system according to Poisson process with arrival rateis λ. Whentransactions take place, ithas to wait in queue as memory pool. Transactions are stored in a block from memorypool. The transactions are stored into a block based on the First Come First Service (FCFS) with respect to the transaction arrivals.

When modelling the blockchain with a queueing system, it is critical to build up the service process through study of the mining management, which is related to the consensus mechanism.

Service process: When transactions are entered in the system, it has to wait in queue. Transactions are stored in a block and generate a block. Miners confirms the block based on consensus. Consensus algorithm establishes the rules and requires all nodes to obey them in order to reach a consensus (e.g., transaction confirmation) on blockchain.Thus, Consensus affect transaction confirmation time.

A freshly created block is confirmed by solving a computationally hard cryptographic Hash problem, which is referred to as mining, The winner will be awarded reward and can append the block in the Blockchain.In Bitcoin proof-based consensus proof-of-work (PoW) is used. PoW consensus algorithm is very time consuming and need high energy consumption. There for the classic PoW protocol has a poor efficiency on processing transactions.

Once the block is confirmed, is add to the blockchain. The process of block-generation and block-building is regardedas service process. The block-generation along with blockchain-building times can be considered as service time.Service time follows general distribution.**Table 1** describes different notations used in this paper.

Table 1 : Terminology

λ	Arrival rate of a transaction
μ	Service rate of transaction
X_n	The number of transactions in the system immediately after nth transactionconfirmed
$N(t)$	Total Number of transactions in the system at time t
$N_q(t)$	Number of transactions in queue at time t
$N_s(t)$	Number of transactions in system at time t
$E[N]$	Mean number of transactions
$E[N_q]$	Mean number of transactions in queue
T_q	Time a transaction spends to wait in a queue
T	Total time a transaction spends in the system
S	Random Service time with general distribution
$E[T_q]$	Mean waiting time in queue
$E[T]$	Mean waiting time in the system
$E[S]$	Mean Service time

In this paper, queueing theory is modelledfor Blockchain system. M/G/1 queue with discrete time Markov chain (DTMC)is analysed. This isapplied to our blockchain system.This system is considered in terms of arrival and departure. This random transaction is tagged on its arrival to the queue. And the system is observed at its arrival. The transactions in the queue at departure are those transactions that arrived in the queue during sojourn time.The inter-arrival times are exponential, but the service times are general distribution.

IV. Analysis

Here we analyze the blockchain system at a point of a time just after a transaction complete its service(transaction – confirmation time (Departure times)). Considering the queue at departure points give rise to a discrete time Markov chain. Let X_n be the number of transactions in the system immediately after the nth transaction confirmed.

Assume that $\{Y(t), t \geq 0\}$ will be semi – Markov process having embedded discrete time Markov chain (DTMC)$\{X_n, n = 0, 1, \ldots\}$. The one-step transition probabilities of the embedded DTMC $\{X_n, n = 0, 1, \ldots\}$ to be computed toobtain steady-state probabilities of $Y(t)$.

Let $N(t)$ be the number of transactions in the system at time t. The cumulative density function (CDF) of servicetime is $F(t)$ with mean $\frac{1}{\mu}$.

Let A_n be a random variable denoting the number of transactions that arrive during the service time of the n^{th} transaction. `

We have

$X_{n+1} = \begin{cases} A_{n+1}X_n = 0 \\ X_n - 1 + A_{n+1}X_n \geq 1 \end{cases}$ (1)

The service times of all the transactions are denoted by $B(.)$. Which has same distribution of A_n for all n.
Denoting, for all n,

$$a_r = \Pr(A_{n+1})$$

$= \int_0^\infty \frac{e^{-\lambda t}(\lambda t)^r}{r!} dF(t), \qquad r = 0, 1, \ldots$ (2)

Therefore,

$$P_{ij} = \Pr(X_{n+1} = j | X_n = i)$$

$= \begin{cases} a_j, & j \geq 0, i = 0 \\ a_{j-i+1}, & i \geq 1, j \geq i-1 \\ 0, & i \geq 1, j \geq i-1 \end{cases}$ (3)

Denoting $P = [P_{ij}] = \begin{bmatrix} a_0 & a_1 & a_2 & \cdot & \cdot & \cdot \\ a_0 & a_1 & a_2 & \cdot & \cdot & \cdot \\ 0 & a_0 & a_1 & \cdot & \cdot & \cdot \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \end{bmatrix}$

Here X_n is irreducible Markov chain. When $\rho = \frac{\lambda}{\mu} < 1$, the chain is positive recurrent. Hence the Markov chainisergodic.

The limiting probabilities

$v_j = \lim_{n\to\infty} P_{ij}^{(n)}$, $j = 0, 1, 2, \ldots$ (4)

exist and are independent of the initial state i.

The probability vector $v = [v_0, v_1, \ldots]$ is given as the unique solution of

$v = vP$ and $\sum_j v_j = 1$. (5)

The solution can be obtained by using n generating fractions of v_j's and a_j's.

Let,

$A(z) = \sum_{j=0}^{\infty} a_j z^j$ and $V(z) = \sum_{j=0}^{\infty} v_j z^j$ (6)

$$A(z) = \sum_{j=0}^{\infty} a_j z^j$$

$$= \sum_{j=0}^{\infty} z^j \left(\int_0^\infty \frac{e^{-\lambda t}(\lambda t)^r}{r!} dF(t)\right)$$

$$= \int_0^\infty e^{-\lambda t} e^{\lambda z t} f(t)dt$$

$$= \int_0^\infty e^{-t(\lambda - \lambda z)} f(t)dt$$

$= f^*(\lambda - \lambda z)$ (7)

$f^*(s)$ is the Laplace transform of $f(t)$.

Using equation (7), expected arrival is given by

$$E(Arrival) = A'(1) = [\frac{d}{dz}A(z)]_{z=1}$$

$= [\frac{d}{dz} f^*(\lambda - \lambda z)]_{z=1}$

$$= [\frac{df^*}{ds}\frac{ds}{dz}]_{z=1}$$

$= [\frac{df^*}{ds}]_{s=0} [\frac{d(\lambda - \lambda z)}{dz}]_{z=1}$

$$= [-\lambda \frac{d}{ds} \left(\int_0^\infty e^{-st} f(t)dt\right)]_{s=0}$$

$$= -\lambda \left(\int_0^\infty t\, f(t)dt\right)$$

$= \frac{\lambda}{\mu} = \rho$ (8)

Now, from equation (3) & (5)

$v_0 = v_0 a_j + \sum_{i=0}^{j+1} v_j\, a_{j-i+1}, j = 0,1,2, \ldots$ (9)

Multiplying by z^j on both sides and taking the sum, we get

$$\sum_{j=0}^{\infty} V_j z^j = \sum_{j=0}^{\infty} v_j a_j z^j + \sum_{j=0}^{\infty} (\sum_{i=0}^{j+1} v_j \, a_{j-i+1}) z^j$$

From equation (6)

$$V(z) = v_0 \sum_{j=0}^{\infty} a_j z^j + \sum_{i=1}^{\infty} (\sum_{j=i-1}^{\infty} v_j \, a_{j-i+1} z^j)$$

$$= v_0 A(z) + \frac{1}{z}[V(z) - v_0] A(z)$$

$$V(z) - \frac{1}{z} A(z) V(z) = v_0 A(z) - \frac{1}{z} v_0 A(z)$$

$V(z) = \frac{v_0 A(z)(z-1)}{z-A(z)}$(10)

We know that $V(1) = A(1) = 1$.

$V(1) = \lim_{z \to 1} v_0 \; (\frac{A'(z)(z-1)+A(z)}{1-A'(z)})$

$$= \frac{v_0 A(1)}{1 - A'(1)}$$

$$= \frac{v_0}{1 - A'(1)} \quad (11)$$

Provided$A'(1)$ we get, $v_0 = 1 - \rho$.

Hence, $V(z) = \frac{(1-\rho)(z-1) f^*(\lambda(1-z))}{z - f^*(\lambda(1-z)}$(12)

Now, the average number of transactions in the system in steady – state is given by,

$$E[N] = L_s = [\frac{dV(z)}{dz}]_{z=1}$$

More generally,

$$E[N] = L_s$$

$= \rho + \frac{\lambda^2 E[S^2]}{2(1-\rho)}$(13)

$E[S^2]$ is the second order moment about the origin for the service time. This result holds true for all scheduling disciplines in which the server is busy if the queue is non-empty.

Average time a transaction spent wait in a queue:

When transaction is arrived in the system it has to wait. The Delay of transaction is determined by the transactions which are already in the system. Each transaction which are in the queue ahead of it contributes, on average,$E[S]$ to delay. The average number of transactions in queue is $E[N_q]$ when it enters in the system. Hence, average delay due to these transactions is $E[N_q] * E[S]$.

Now, the transactions who are in service also contributes a different amount of delay. This transaction has completed some of the service already, so contribution on delay is remaining service time not the total service time.

$E[T_q] = E[N_q] * E[S] + P\{Serverbusy\} * E[residualservicetime|serverbusy]$(14)

$P\{Serverbusy\}$ is the probability that arriving transaction finds the server busy, which is given by ρ.

Using the formula $E[N_q] = \lambda E[T_q]$ and by eliminating $E[N_q]$in (14) we get,

$$E[T_q] = \frac{\rho E[residualservicetime|serverbusy]}{1 - \rho} \quad (15)$$

Since the arrival time is equivalent to a randomly selected time, the remaining service time can be viewed as that obtained for a renewal sequence consisting of generic random variables S. So, to find the expected residual service time, conditional on the arrival finding the server busy can be return as

$$E[residualservicetime|serverbusy] = \frac{E[S^2]}{2E[S]}$$

$=\frac{1+C_B^2}{2} E[S]$(16)

From the above equations

$$E[T_q] = \frac{\rho}{1-\rho} \frac{1 + C_B^2}{2} E[S]$$

The above equation shows that the expected waiting time increases with the variability of the service process.Which can be also written as,

$$E[T_q] = \frac{\lambda E[S^2]}{2(1-\rho)} \quad (17)$$

And from the Little's formula the other performance measures like the average number of transactions in queue, theaverage number of transactions in system and expected time in the system are

$$E[N_q] = \frac{\lambda^2 E[S^2]}{2(1-\rho)} \quad (18)$$

$$E[N] = \frac{\lambda^2 E[S^2]}{2(1-\rho)} + \rho \quad (19)$$

$$E[T] = \frac{\lambda E[S^2]}{2(1-\rho)} + \rho \quad (20)$$

It is important to observe the common term, $\frac{1}{(1-\rho)}$, found in all the above equations. This term expresses the rate of growth of the queue as a function of its utilization and is a salient feature of Blockchain system.
For ρ> 1, i.e.$\lambda > \mu$the queue grows without bound. When transaction arrival is more and comparatively service is slower in Blockchain system, server is always busy.
When $\lambda = \mu$, the value of ρ became 1. There for the performance measures for this case becomes infinite.
Some general observations about M/G/1 queue with this Blockchain system provides average transaction confirmation time, average time of transaction in queue, average number of transactions in the system as well as in queue, utilization factor.

V. Numerical Examples

Toanalyzeaverage transaction confirmation time and other performance measure of Blockchain system, numerical experiments of theoretical results are provided.These numerical experiments are done in python.
When arrival rate $\lambda = 0.5$ all the parameters will be infinite. So random λ are taken for numerical analysis.
For arrival rate $\lambda = 0.48$ and $= 0.7$,service rateμ ∈ [0.5,1.5] the parameters like$E[S]$,$E[S^2]$, $\rho$and other performance measures are obtained.

Table 2 shows the values of performance measures of blockchain system for transaction arrival rate $\lambda = 0.48$.

Table 2: Values for $\boldsymbol{\lambda = 0.48}$

μ	ρ	$E[S^2]$	$E[N_q]$	$E[N]$	$E[T_q]$	$E[T]$
0.5	0.96	8.0	23.040	24.000	48.000	48.960
0.61	0.787	5.375	2.905	3.692	6.053	6.840
0.72	0.667	3.858	1.333	2.0	2.778	3.444
0.83	0.578	2.903	0.793	1.371	1.652	2.231
0.94	0.511	2.263	0.533	1.043	1.110	1.621
1.06	0.453	1.780	0.375	0.828	0.781	1.234
1.17	0.410	1.461	0.285	0.696	0.595	1.005
1.28	0.375	1.221	0.225	0.6	0.46875	0.845
1.39	0.345	1.035	0.182	0.527	0.379	0.723
1.5	0.32	0.889	0.151	0.471	0.314	0.634

In figure 1(a).shows the transaction confirmation time. As service rate increases average confirmation time of transaction decreases. And in figure 1(b). average number of transactions in system decreases as service rate increases. And from graph 1(c) and 1(d), average transaction time in queue and average number of transactions in queue is analyzed. This number of transactions in queue are stochastically equal to the number of transactions in the system, at a random point.

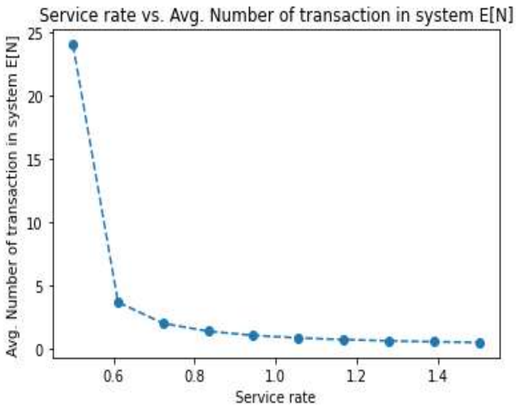
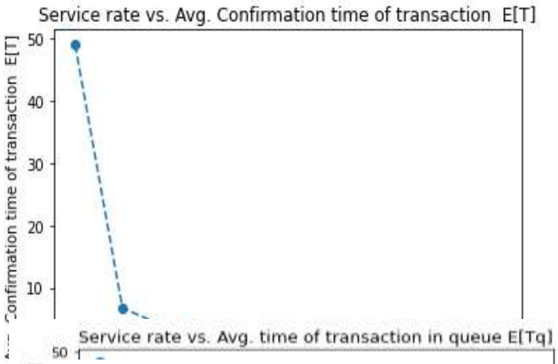


Figure 1(b). Graph for Avg. number of transactions in system



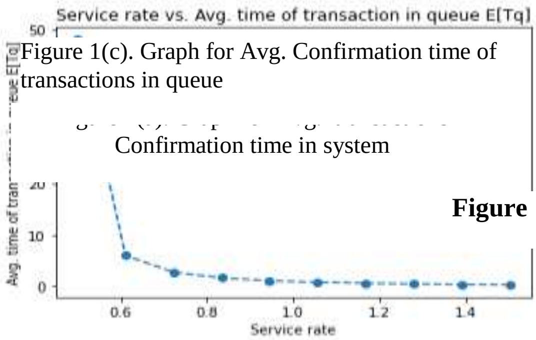


Figure 1(c). Graph for Avg. Confirmation time of transactions in queue

Confirmation time in system

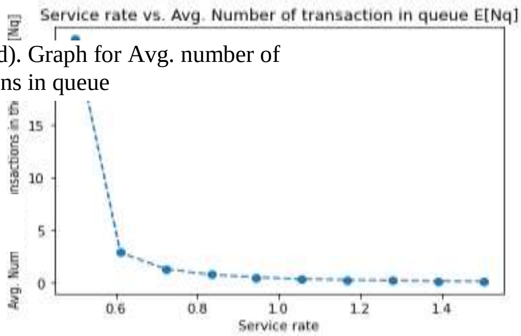


Figure 1(d). Graph for Avg. number of transactions in queue

Figure 1: Graph for λ = 0.48

Table 3

shows the values of performance measures of blockchain system for transaction arrival rate $\lambda = 0.7$.

Table 3: Values for $\lambda = 0.7$

μ	ρ	$E[S^2]$	$E[N_q]$	$E[N]$	$E[T_q]$	$E[T]$
0.5	1.4	8.0	-4.9	-3.500	-7.000	-5.600
0.61	1.148	5.375	-8.925	-7.778	-12.750	-11.603
0.72	0.972	3.858	34.028	35.000	48.611	49.583
0.83	0.843	2.903	4.541	5.385	6.487	7.331
0.94	0.745	2.263	2.172	2.917	3.103	3.848
1.06	0.660	1.780	1.284	1.944	1.834	2.495
1.17	0.598	1.461	0.891	1.489	1.273	1.871
1.28	0.547	1.221	0.660	1.207	0.943	1.490
1.39	0.504	1.035	0.511	1.014	0.730	1.233
1.5	0.467	0.889	0.408	0.875	0.583	1.050

In figure 2(a) it is observed that transaction confirmation time is negative for the values of $\mu < \lambda$,which shows that queue is imbalanced. When $\mu = \lambda$, transaction confirmation time is at the peak. When $\mu > \lambda$i.e.,average confirmation time of transaction decreasesincreasing service rate.

Similarly, the same thing happens when other three performancesi.e.,average transaction confirmation time in queue, average number of transactions in system and average number of transactions in the system are being measured.

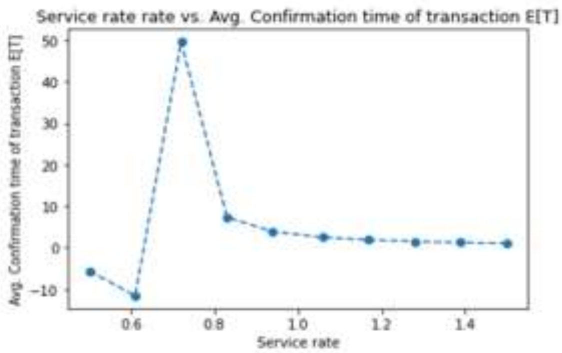


Figure 2(a). Graph for Avg. transactions Confirmation time in system

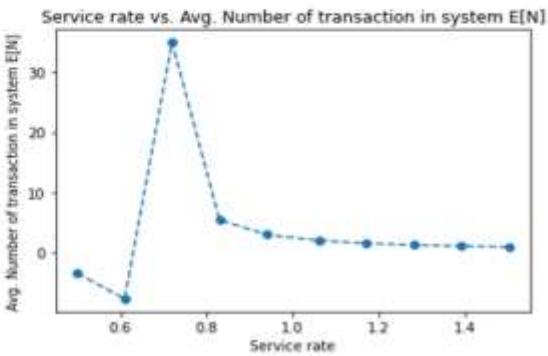


Figure 2(b). Graph for Avg. number of transactions in system

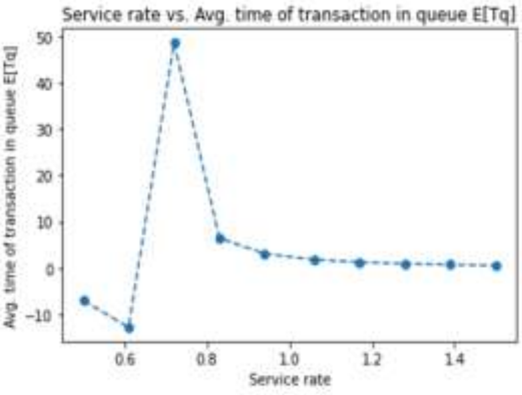


Figure 2(c). Graph for Avg. Confirmation time of transactions in queue

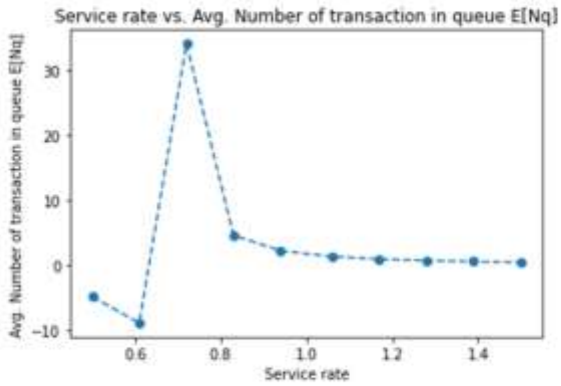


Figure 2(d). Graph for Avg. number of transactions in queue

Figure 2: Graph for λ = 0.7

It is important to note that the work done for the system is moregeneral\for M/G/1 queue. A system in termsofitsinput and output propertiesi.e.,transaction arrived at system and depart after taking the service are observed. We donot need to specify how the transactions are managed within the system.

VI. Conclusion

In this paper, Blockchain Queueing theory is analysed theoretically for the transaction-confirmation time of a block. The Blockchain system is examined at arrival as well as departure point. This paper is reflected on M/G/1 queue and to examine the Blockchain system using queueing theory. Equation (13) is the expected steady-state size at system departure points. By using equation (17) and (20) one can measure the performance of Blockchain.

Numerical analysis of this work is performed in python. This result can be applied on Practical Blockchain. It is observed that the number of transactions in queue at arrival point and the number of transactions at departure points have minor difference.Any random transaction arrival rate and its service rate can be analysed by the results described in this paper.

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