IOSR Journal of Computer Engineering (IOSR-JCE)
e-ISSN: 2278-0661,p-ISSN: 2278-8727, Volume 25, Issue 1, Ser. II (Jan. – Feb. 2023), PP 29-42
www.iosrjournals.org

Design of Vertical Curve with Circular and Transition Curve

Brindabon Samanta

Abstract: *Vertical curve is designed in India based on the Indian Roads Congress (IRC): SP-23 -1993, AASHTO, UK and other Guide lines. Generally parabolic curve is used for the determination of elevation of summit curve and valley curve. When speed of vehicle is more than 200 even 400 kmph for innovative transport system like Hyperloop one, vertical curve could be designed with transition curve on either side of conventional vertical curve to avoid jerk due to such high speed with sudden changes in grade.This paper presents a guideline for design of vertical curve using transition similar to horizontal alignment and discus design methodology, benefit of user/ passenger while travelling on road.*

Date of Submission: 11-02-2023 Date of Acceptance: 23-02-2023

I. Introduction

Vertical curves to effect gradual changes between tangent grades may be any one of the crestor sag types. Vertical curves should be simple in application and should result in a design that is safe and comfortable in operation, pleasing in appearance, and adequate for drainage. The major control for safe operation on crest vertical curves is the provision of ample sight distances for the design speed; while research (Fambro et. al. 1997) has shown that vertical curves with limited sight distance do not necessarily experience safety problems, it is recommended that all vertical curves should be designed to provide at least the stopping sight distances.Wherever practical, more liberal stopping sight distances should be used. Furthermore, additional sight distance should be provided at decision points. For driver comfort, the rate of change of grade should be kept within tolerable limits. This consideration is most important in sag vertical curves where gravitational and vertical centripetal forces act in opposite directions. Appearance also should be considered in designing vertical curves. A long curve has a more pleasing appearance than a short one; short vertical curves may give the appearance of a sudden break in the profile due to the effect of foreshortening (AASHTO 2004).

Drainage of curbed roadways on sag vertical curves needs careful profile design to retain a grade of not less than 0.5 percent or, in some cases, 0.30 percent for the outer edges of the roadway. Although not desirable, flatter grades may be appropriate in some situations(AASHTO 2004).

Even larger vertical curve is avoided where horizontally reverse curve is in place. At this location to achieve the superelevation at the middle part of reverse curve becomes horizontal or nearly horizontal for a certain length i.e., no slope across the road alignment and due to larger length of vertical curve this part becomes flat vertically i.e., no longitudinal slope, hence becomes critical location to drain out the surface water neither across the width of carriageway nor along the alignment results in accumulation on the road surface.

Standard Design of Vertical Curves as per IRC: SP:23-1993

To attain the primary objectives of safety and comfort in travelling over different grades the provision of vertical curves is introduced and accordingly due attention has been given on the design of vertical curves.

Due to speed when a change takes place in the direction of motion of a body a considerable force acts on the body. When a motor vehicle changes its direction in the vertical plane a force is involved and if this change is not affected gradually the vehicle will be subjected to jerk and occupants of vehicle will feel jerk as well, hence will experience discomfort.Hence require appropriate remedial measure of introducing vertical curve where direction changes occur.

Vertical curves are classified into two types viz.

1. Summit curve to ease of intersection convex upwards
2. Valley curve to ease of intersection concave upwards

When a vehicle approaches to a summit curve the view of the road is cut off beyond the summit. Therefore, to secure the required sight distance the intersection of the two grades should be eased of by interposing a properly designed vertical curve.

For valley curve visibility is not a problem during day time. However, for night travel the design must ensure that the roadway ahead is illuminated by vehicle headlight to a sufficient length enabling the vehicle to break to a stop if necessary.

Formatted: Font: (Default) Times New Roman, 11 pt, Not Bold, Italic

Formatted: Font: (Default) Times New Roman, 10 pt

Sight Distance:
Three types of sight distance are relevant for the design of summit curve.

1. Stopping sight distance
2. Intermediate sight distance which is basically double of sight distance for more safety in design
3. Overtaking sight distance.

For valley curve the design is governed by night visibility which is reckoned in terms of headlight sight distance. This distance should at least equal the safe stopping sight distance.

Formatted: Font: (Default) Times New Roman, 10 pt

Summit/crest curve:
The dynamics of movement over a summit curve is of little consequence. There are two considerations.

1. The centrifugal force generated by the movement of the vehicle along the curve acts particularly in opposition to the force of gravity and is therefore beneficial in so far as it relieves the pressure on the tyres and springs of the vehicle.
2. Vertical deviation angle on road is so small because the summit curve prescribed by the sight distance is so long and easy that shock is automatically rendered imperceptible to the travellers.

It therefore follows that on summit curve transition is not essential and simple circular arc is good enough. Since a circular arc has a constant radius of curvature throughout its length, it gives a constant sight distance all along. From this view point the alternative of a curve fully transitional and symmetrical about the intersection is unsuitable, as the radius of the curve decreases towards its apex and the visibility on a vertical transition curve varies from point to point and is smallest across its apex. At a given intersection of gradient, a transition curvewill have to be much longer than a circular arc for equal visibility across the apex. Because of this disadvantage a transition curve is not recommended.

Formatted: Font: (Default) Times New Roman, 10 pt

Valley/sag curve:
Dynamics of movement on valley curve is somewhat difference.
On valley curve the gravitational and centrifugal forces act combinedly resulting in extra pressure on the tyres and spring on the vehicle. The effect of this on travel comfort depends on several factors such as the vehicle body suspension, tyre flexibility and weight carried etc.
For establishing the length of valley curve most commonly used criteria are:

1. Head light sight distance
2. Rider comfort

For valley curve the designed is mainly governed by night visibility which is reckoned in terms of head light sight distance. This distance should at least equal the safe stopping sight distance.

Formatted: Font: (Default) Times New Roman, 10 pt

Requirement of Vertical curve with transition for Hyperloop one:
When speed is more than 200 even 400 kmph for innovative transport system like **Hyperloop one**, vertical curve could be designed with transition curve on either side of conventional vertical curve to avoid jerk due to such high speed with sudden changes in grade.
Circular curve with transition at either end instead of fully transitional as mentioned in IRC: SP23 -1993 will avoid problems in smallest visibility near apex since circular curve close to apex will have constant radius of curvature hence visibility won't vary and constant visibility will remain around apex. However, from a constant gradient travel to a circular curve will cause passenger discomfort but providing a transition in between will reduce the impact.
Hyperloop is a high-speed linear infrastructure system that will transport passengers and goods in pods through a low-pressure tube. The technology is envisioned to improve on traditional transportation modes-sea, rail, road and air-by offering energy efficient, on demand, and cost-effective service at aircraft speeds. The Hyperloop service will offer customers convenience with travel speeds comparable to aircraft and low wait times in stations, it is also envisioned to transform urban planning and development, improve productivity, and revolutionize global supply chains.

Formatted: Font: (Default) Times New Roman, 10 pt

Virgin Hyperloop One
Hyperloop is designed by Virgin Hyperloop One (VHO). VHO is building a point to point, on demand and autonomous transport system. It is designed to be fast, reliable, and cost effective, with lower environmental impact than existing modes of travel.
VHO, in coordination with Pune Metropolitan Region Development Authority (PMRDA) and the Government of Maharashtra, identified the opportunity to connect and integrate the economies of Mumbai and Pune through the VHO system therefore substantially reducing travel time and improving mobility with high speed connectivity.

Formatted: Font: (Default) Times New Roman, 10 pt

Designer's Requirement:

Circular curve shall be used in the vertical profile design. Parabolic curve is not permissible. Vertical circular curve shall be sized based on the limitations of vertical acceleration in the passenger reference frame. In certain cases, when large grade angles are used, the vertical curve radius may be reduced. This is due to a portion of the vertical acceleration transforming longitudinally by the grade angle. However, the reduction in the radius of vertical curve is negligible, and thus, gradeeffects shall not be considered when determining the minimum vertical curve radius.

Parabolic Curve as per IRC:
In actual practice a simple parabolic curve is used instead of the circular arc. The reasons are:
1. A simple parabola is nearly congruent with a circular arc between the same tangent points, because on road work the vertical deviation angle is very small and length of curve is very great.
2. A parabola is very easy of arithmetical manipulation for computing ordinates.

Deficiency in Parabolic curve:
1. In parabolic curve, there is no such deficiency since deviation angle is small and length is large. Only deficiency is sudden movement from grade to a curve which results in passenger discomfort due to sudden jerk.
2. There is no constant sight distance as obtained in case of circular curve.
3. No transition curve in between grade and parabola. Hence change in level from point to point along the curve is very fast/steeper causing passenger's discomfort.

Circular curve with clothoid transition in Hyperloop project:
Vertical clothoid transition curve shall be used and shall accompany all vertical circular curve. The length of vertical transition curve is dictated by the vertical jerk in the passenger reference frame (PRF: is the alignment reference frame rotated about the "x" axis by the total Bank Angle).

Combined horizontal and vertical curve as per IRC:
Vertical curvature superimposed upon horizontal curvature gives a pleasing effect. As such the vertical and horizontal curve should coincide as far as possible and their length should be more or less equal. If this is difficult for any reason, the horizontal curve should be somewhat longer than the vertical curve.

Combined horizontal and vertical curve in the design of Hyperloop:
If horizontal and vertical curve are combined special consideration shall be taken that the lateral acceleration in the passenger reference frame (The transformed curvilinear coordinate system in which the passenger sits. Transformer is angular from the global reference frame, due to the total bank angle.) remains fully compensated, the vertical acceleration in the passenger reference frame does not exceed the limit, and the roll rate does not exceed the limit.

Design of Parabolic curve as per IRC:
Design of vertical curves following IRC Guidelines which depicts about parabolic curve. Even in other countries the vertical curve is designed considering parabolic shape with Equation of $Y = x^2/a = u^2/a$.

Design of Sag curve as per IRC:
On sag curve the gravitational and centrifugal forces act combinedly resulting in extra pressure on the tyres and spring on the vehicle. The effect of this on travel comfort depends on several factors such as the vehicle body suspension, tyre flexibility and weight carried etc. the broad calculation from limited observations show that for riding comfort on valley curve, the radial acceleration should not exceed $0.3\ m/s^2$.

Design of Circular curve with transition in design of Hyperloop:
In on-/off -ramp areas, the design speed through a transition-in, circular curve, transition out will change with distance from the start of the alignment. This is applicable for both horizontal and vertical alignment design. Special consideration is applied here since the minimum circular curve radius will be designed for the speed leaving the circular curve. Similarly, the transition curve will need to be lengthened to accommodate the increasing speed along each transition.

Minimum length of vertical curvein Design of hyperloop:
The minimum length of vertical alignment shall be dictated by the maximum value obtained from the following constraints:
1. Attenuation time and
2. Vertical jerk

For vertical grades and circular curve, the minimum length is dictated by attenuation time. For vertical clothoid transition the minimum length is dictated by vertical jerk.

Speed:
Speed is basically calculated from the basic equation as mentioned below:

$$v^2 - u^2 = 2 f s \quad (1)$$

Where,
u is the initial speed;
v is the final speed;

s is the distance travel; and
f is the horizontal acceleration.
Here one criterion may be mentioned that acceleration is varying with the distance. If distance increases acceleration decreases. Acceleration varies from 2.45 $m/s^2$to 0.70 m/s^2. Here maximum acceleration/maximum speed is determined from the system including pod's size and shape, maximum banking angle, vacuum pressure etc. Depending upon the varying acceleration various speed according to distance is calculated which is shown below (VHO 2018).
Distance versus speed is presented in **Fig.1**

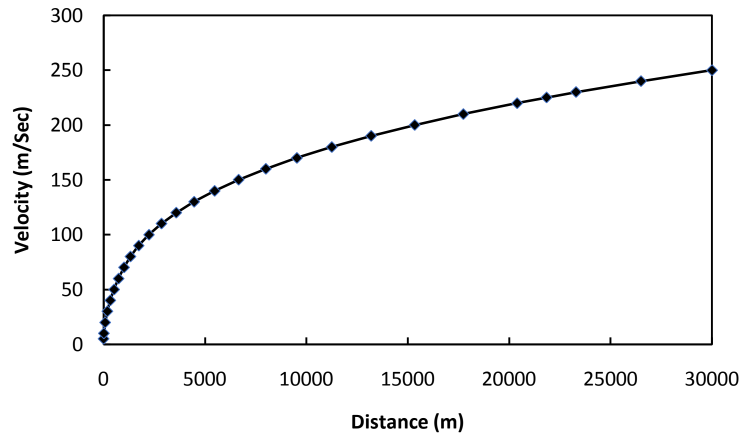

Fig.1: Distance VS Velocity

Distance versus Acceleration is presented in **Fig.2**.

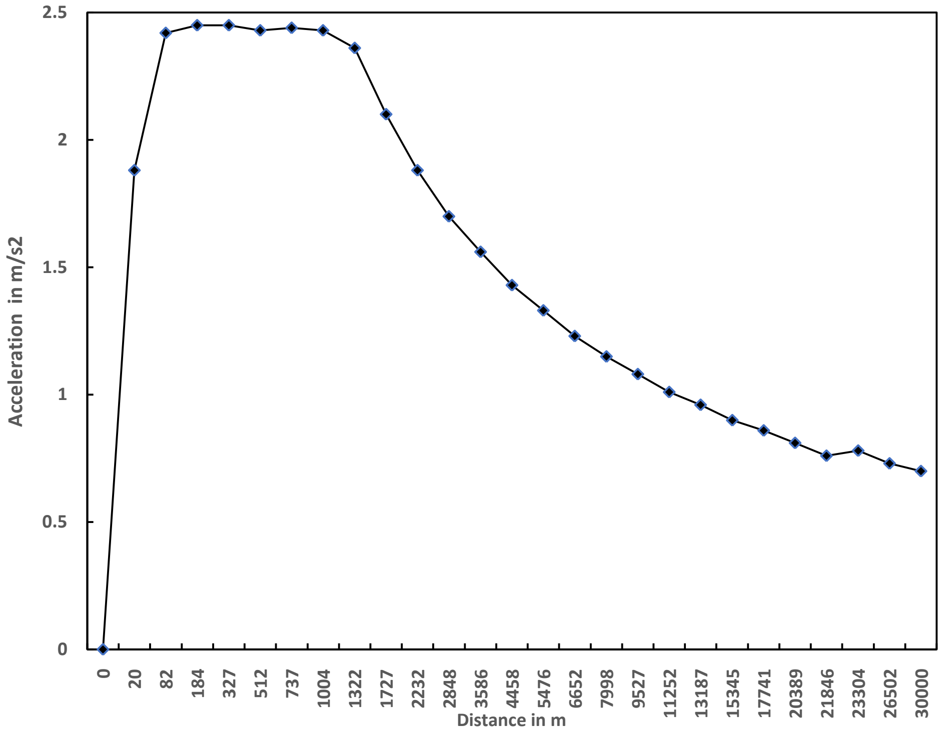

Fig.2: Distance VS Acceleration

Calculation of radius of vertical curve:
Radius of vertical curve is calculated from the basic equation as presented below:

$R_v = v^2/f_n$ (2)

Where,

R_v is radius of vertical curve in m;
V is speed of pod in m/sec;and
f_n is vertical acceleration in m /sec^2.

Actual Vertical acceleration is calculated from maximum permissible of vertical acceleration in terms of “g” i.e., acceleration due to gravity.

Design of Sag curve in Hyperloop:
On sag curve the gravitational and centrifugal forces act combinedly resulting in extra pressure on the tyres and spring on the vehicle. The effect of this on travel comfort depends on several factors such as the vehicle body suspension, tyre flexibility and weight carried etc. Hence vertical acceleration for sag curve in such case should not exceed 0.30xg (where g is gravitational acceleration = 9.81m/s^2 i.e.,2.943m/s^2) (VHO 2018).However, for safer design maximum vertical acceleration is provided in this design 0.2g i.e. 1.962 m/s^2.

Design of Crest curve in Hyperloop:
On crest curve the gravitational and centrifugal forces act in opposite direction. Hence vertical acceleration for crest curve in such case should notbe exceededby 0.20xg (where g is gravitational acceleration = 9.81m/s^2) i.e.,1.962m/s^2. However, for safer design maximum vertical acceleration is provided in this design 0.05g i.e.,0.4905m/s^2. Depending upon passenger comfort vertical acceleration can vary from 0.4905 to 1.962 m/s^2. Depending upon speed and acceleration various transition curves proposed and shown in **Fig.3**.

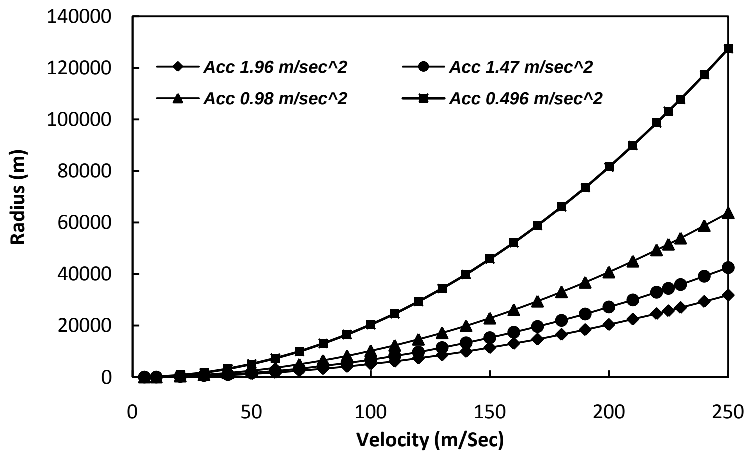

Fig.3: Proposed Radius for Different Speed

Values of proposed radius of curvature are rounded to the nearest 5 metres of the calculated curve.

Transition curve as per IRC:

IRC does not recommend for transition curve for vertical curve, however only reference has been made and suggested about fully transitional curve on either side of intersection points and referred about disadvantages of providing fully transitional curve due to smallest sight distance close to apex.

Transition curve in Design of Hyperloop:

Here in geometric design it is clearly specified that Vertical clothoid transition curves shall be used and shall accompany all vertical circular curve. The length of the vertical transition is dictated by the vertical jerk in the passenger reference frame(VHO 2017).Here in hyperloop design one important criteria was to keep all the element of alignment like straight distance, transition length of horizontal and vertical curve, length of circular horizontal and vertical curve should be minimum of design speed in numeric figure.

As example if for particular section speed is 60m/s then for that section of horizontal curvature, vertical curvature and straight between two horizontal and vertical curves exist then transition length of horizontal and vertical curvature, straight distance between two horizontal and vertical curvature, and straight distance between one horizontal and one vertical curvature, and minimumlength of circular horizontal and vertical curve should be 60 m.

Length of transition curve is proposed based on designed speed and experience as presented in **Fig.4**.

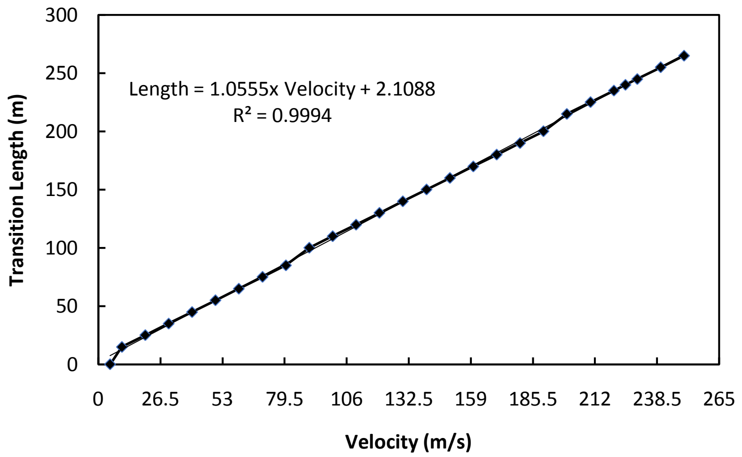

Fig.4: Length of Transition Curve against velocity

Generally, transition length is provided considering the speed only irrespective of radius of curvature.

II. Discussion

Velocity Vs Distance graph has been plotted and presented in **Fig.1**. From **Fig.1**, it is noticed that vehicle travel distance increases non-linearly with positive slope.
Acceleration Vs Distance graph has been plotted and presented in **Fig.2**. From **Fig.2**, it is noticed that acceleration increases with travel distance increases non-linearly with positive slope up to 245m travel distance. Beyond 245 m, acceleration decreases with increasing travel distance.
For a constant acceleration, radius Vs Velocity graph has been prepared and presented in **Fig.3**.From **Fig.3**, it is noticed that radius increases with increasing velocity nonlinearly with positive slope. It depends on acceleration values as shown in the same curve.
Length of Transition Curve Vs velocity has been prepared and presented in **Fig.4**. From **Fig.4**, it is observed that length of transition curve increases with increasing speed with positive slope-upward direction. Equation of curve is shown in Fig.4 with R^2 value of 0.999. This indicates that curve is statistically significance and can be used for other speed for determination transition length of the vertical curve.
An example is presented in **Annexure 1**.The**TableA1**of sample calculation presented in **Annexure 1** it is noticed that the levels computed from parabolic curve and circular curve are nearly equal when only circular curve is designed in place of parabolic curve since grade difference is very small and length of curve is great. Therefore, it justifies the statement mentioned in IRCSP:23-1993.
However, from **TableA 1** and **Fig. A 3** it is seen that when entire Parabolic curve is replaced by Transition and circular curve it is found that change of level for combined transition and circular curve is smoother than that for only parabolic curve. For Parabolic curve it is seen that there is sudden change in level from tangent gradient to curve whereas for transition with circular curve change is gradual and smooth. Hence for high speed transportation system where users' safety and comfort are utmost concerned of a transport system transition curve with circular vertical curve should be designed instead of design with parabolic curve**.**

III. Conclusion

There is no guide line in India for hyper loop of speed train 200 kmph or more speed. For highway vehicle maximum speed is in the range of 120 kmph.Vertical curve design is not major issue for this case. IRC: SP-23 recommended simple parabolic curve is sufficient for vertical curve. When speed exceeds 200 kmph, this equation produces substandard vertical profile and passenger and driver feel discomfort after entering vertical curve. To avoid this discomfort or reduce discomfort, vertical curve with transition may be an alternative / better solution. The proposed methodology mentioned in this paper may be useful and gives a guideline for design of vertical curve and can be plotted for execution during construction of high-speed rail corridor.Following conclusions may also be drawn from this present study.

- Velocity increases with increasing distance as shown in **Fig.1**. This will be useful for the designer during taking decision for a particular distance for vertical curve design.Acceleration increases with increasing distance up to 245 m and beyond this distance acceleration decreases with increasing distance.
- For a given acceleration, radius of curve increases with increasing velocity.
- Length of transition curve increases with increasing speed of vehicle linearly with positive upward slope.
- All the curves will be useful for design of vertical curve for high speed vehicle vertical alignment.
- When entire Parabolic curve is replaced by Transition and circular curve it is found that change of level for combined transition and circular curve is smoother than that for only parabolic curve. For Parabolic curve it is seen that there is sudden change in level from tangent gradient to curve whereas for transition with circular curve change is gradual and smooth. Hence for high speed transportation system where users' safety and comfort are utmost concerned of a transport system transition curve with circular vertical curve should be designed instead of design with parabolic curve.

Sample Calculation for level of Vertical Curve
A sample calculation of vertical curve for all combination has been worked out based ontransition and Parabola and presented in **Annexure 1** attached at the end of this paper.
Conclusion

References:

[1]. Fambro, D. B., K. Fitzpatrick, and R. J. Koppa. Determination ofStopping Sight Distances,
[2]. NCHRP Report 400, Washington, D.C.: Transportation Research Board, 1997.
[3]. IRC: SP-23-1993-Vertical Curve for Highways.
[4]. IRC:38-1988-Design of Horizontal Curves for Highways.

[5]. Virgin Hyperloop One (VHO 2018) -System Overview and Infrastructure Design Guidelines: Mumbai-NMIA-Pune Detailed Project Report.
[6]. AASHTO (2004) - Geometric Design

Annexure 1

Calculation Based on IRC design:
Development of Basic equation for Parabolic Curve
Calculating the ordinate of summit Curve:
For calculating the level along the curve, length is divided into a number of equal chords and ordinated to the curve calculated at the ends of these chords.
Ordinates y_1, y_2, y_3………$y_r$at station 1, 2, 3…..r (**Fig A 1**) are calculated as under:

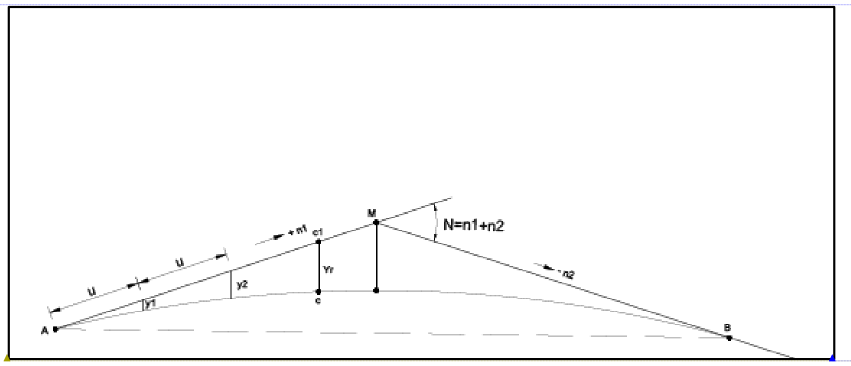

Fig.A 1 Typical Parabolic Curve

Since $y = x_2/a$ ($a = 2L/N$)
L is the length of the curve and N is the grade difference
$y_1 = u^2/a$, (where u is the chosen length of the chord)
$y_2 = (2u)^2/a$ or $y_1 x 2^2$
$y_3 = y_1 x 3^2$
$y_r = y_1 x r^2$
Let C be the point on the road surface curve at the end of the r^{th} sub chord. Let C1 be the point on the grade line vertically above C. let the Reduced level of the tangent point A be 100.00.
Then R L of C1 = $100 + r\times(u\times n_1)$
R L of C = R L of $C_1 - y_r = 100 + rx(u\times n_1) - y_1 x r^2$
n_1 = in-grade
Calculating the ordinate of sag curve:
For calculating the level along the curve, length is divided into a number of equal chords and ordinated to the curve calculated at the ends of these chords.
Let C be the point on the road surface curve at the end of the r^{th} sub chord. Let C1 be the point on the grade line vertically above C. let the Reduced level of the tangent point A be 100.00.
Then R L of $C_1 = 100 - r\times(u\times n_1)$
R L of C = R L of C_1 + yr
Where u is the chosen length of the chord
$Y_r = y_1 \times r_2$
n_1 = in-grade
Calculation based on Hyperloop Design:
Basic Equation of Transition and circular curve are developed which are explained below and sample calculation with values are also shown for reference.

Development of Basic Equation of circular vertical crest curve with transition (clothoid) at either end:
1^{st} step for transition curve:
Say a line of up-grade n_1 meets at a line of downgrade n_2 at point M (**Fig. A 2**).

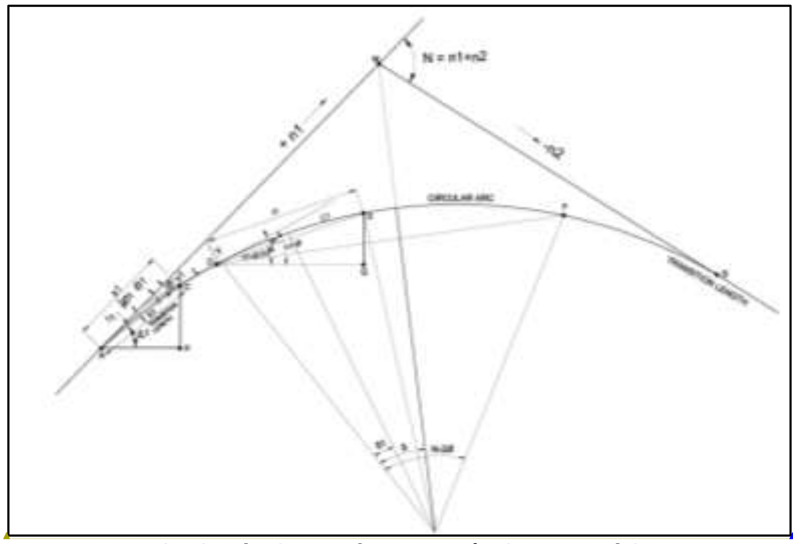

Fig. A 2 Circular Curve with Transition for Crest Vertical Curve

Grade difference is N = n1+n2
Transition curve starts from point A and ends at point E
Circular curve starts at point E and ends at point F
As sample calculation for transition curve level at one point within transition curve and at the end of transition curve are shown.
Similarly, for circular curve at one point within circular curve and at the end of circular curve are shown.
Say point on transition curve is C and length of transition is l_1.
From basic equation of transition $\varphi = x^2/2LR$ angle with ingrade at distance of l_1 is $\varphi_1 = l_1^2/2LR$.
At C radius of curvature $R_1 = LR/l_1$ (for clothoid $A^2 = RL$, where A is constant, hence $LR = l_1R_1$)
Angle of l_1 with horizontal is assumed as $\Delta_1 = n_1 - 1/3\varphi_1$
Abscissa along the ingrade for l_1 is assumed as $x_1$and ordinate is assumed as y_1
From clothoid transition $x_1 = l_1 - l_1^3/40R_1^2$
And $y_1 = l_1^2/6R_1 - l_1^4/336R_1^3$
Chord length for transition length l1 is $s_1 = \sqrt{(x_1^2+y_1^2)}$
Level at C = Level at A + $s_1\sin\Delta_1$
Similarly, level at end of transition (length of transition is L) and start of circular curve (radius of circular curve is R) i.e. level at D is
Level at D = level at A + $s\sin\Delta$
Where $\Delta = n1 - 1/3\ \varphi$
$\varphi = L/2R$
$s = \sqrt{(x^2+y^2)}$
$x = L - L^3/40R^2$
And $y = L^2/6R - L^4/336R^3$
2nd step for circular curve
Circular curve starts from point D as mentioned above. Say level at point E on circular curve is calculated following ways:
Say length of circular arc at point E is C_1
Say C_1 makes angle at centre of circular curve is $2\delta_1$
Then $\delta_1 = C_1/2R$
Say t_1 is chord length for circular arc C_1
Then $t_1/2 = R\sin\delta_1$
Therefore $t_1 = 2R\sin\delta_1$
Level at E = level at D + $t_1\sin(n1 - \varphi - \delta_1)$
Similarly, level at end point of circular curve (length of circular curve $C = R(N - 2\varphi)$) i.e. level at point F is
Level at point F = level at point D + $t\sin(n1 - \varphi - \delta)$
Where $\delta = C/2R$

Formatted: Font: (Default) Times New Roman, 11 pt, Not Bold, Italic

$t = 2R\sin\delta$

Calculating the ordinate of transition curve in a Crest Curve in design of hyperloop:

For calculating the level along the curve, length is divided into a number of equal chords and ordinated to the curve calculated at the ends of these chords. Ordinate is calculated in following ways:

For clothoid transition the equation if $A^2=R\times L$

Where R is Radius of curvature at L length from start of transition

Radius of curvature R = 3265m proposed for vertical acceleration of 0.491m/s^2 and speed 40m/s

Length of transition L = 45m

A is a constant

$A^2 = 146925$

Say length is divided in equal chords of 5 m length

At the end of 5 m chord $R = A^2/5 = 29385$m

$\varphi_1 = L/2R = 5/(2\times29385)$

Angle with tangent of this 5 m chord $1/3\varphi_1 = 1/3\times(5/(2\times29385)$ in radian $= 2.83591\times10^{-5c}$

$\Delta_1 = n_1 - 1/3\varphi_1$

X ordinate of 5 m chord along tangent $= 5 - 5^3/(40\times29385^2) = 5.0$m

Y ordinate of 5 m chord $= 5^2/(6\times29385) - 5^4/(336\times29385^3) = 0.000142$ m

Distance between start point and end of chord is $s_1 = \text{Sqrt}(5.0^2+0.000142^2) = 5.0$m.

Say level at start of transition 100.000m

Ingraden$_1 = 0.041095129^c$

Level at the end of 5 m chord would be

Level at C = Level at A + $s_1\sin\Delta_1$ = $100+5.0\times\sin(0.041095129-2.83591\times10^{-5}) = 100.2053$m.

Similarly, levels along transition curve at certain interval will be calculated measuring the length from start point of transition curve and at the end,level of end of transition curve will be obtained.

Calculating the ordinate of transition curve in a Sag Curve in design of hyperloop:

For calculating the level along the curve, length is divided into a number of equal chords and ordinated to the curve calculated at the ends of these chords. Ordinate is calculated in following ways:

For clothoid transition the equation if $A^2=R\times L$

Where R is Radius of curvature at L length from start of transition

Radius of curvature R = 7900m proposed for vertical acceleration of 0.491m/s^2 and speed 60m/s, however here larger radius is provided

Length of transition L = 65m

A is a constant

$A^2 = 513500$

Say length is divided in equal chords of 5 m length

At the end of 5 m chord $R = A^2/5 = 102700$m

$\varphi_1 = L/2R = 5/(2\times29385)$

Angle with tangent of this 5 m chord $1/3\ \varphi_1 = 1/3\times(5/(2\times102700)$ in radian $= 0.00000811^c$

$\Delta_1 = n_1 - 1/3\varphi_1$

X ordinate of 5 m chord along tangent $= 5 - 5^3/(40\times102700^2) = 5.0$m

Y ordinate of 5 m chord $= 5^2/(6\times102700) - 5^4/(336\times102700^3) = 0.000041$ m

Distance between start point and end of chord is $s_1 = \text{Sqrt}(5.0^2+0.000041^2) = 5.0$m.

Say level at start of transition 100.000m

Ingrade $n_1 = -0.007232381^c$

Level at the end of 5 m chord would be

Level at C = Level at A - $s_1\sin\Delta_1$ = $100-5.0\times\sin(0.007232381-0.00000811) = 99.9638$m.

Similarly, levels along transition curve at certain interval will be calculated measuring the length from start point of transition curve.

Calculating the ordinate of circular curve in a Crest Curve in design of hyperloop:

For calculating the level along the curve, length is divided into a number of equal chords and rest is divided into 2 equal parts and that is applied at start of curve and end of curve and ordinated to the curve calculated at the ends of these chords. Ordinate is calculated in following ways:

Radius of curvature R = 3265m proposed for vertical acceleration of 0.491m/s^2 and speed 40m/s

Length of first chord is $C_1 = 0.880$m measured from start of circular curve

Say level at start of circular curvei.e., level at D = 100.000m

Ingraden$_1 = 0.041095129^c$

$t_1 = 2R\sin\delta_1$

$\delta_1 = C_1/2R$

$\varphi = L/2R$

Length of transition L = 45m

Level at the end of first chord i.e. at C_1 = 0.880 m would be
Level at E = 100+2×Radius×sin (angle subtended by half of first chord at the centre) ×sin (Ingrade-transition angle-angle subtended by half of first chord at the centre)
Level at E = level at D + $t_1$sin (n1- φ - δ_1)= 100+2×3265×sin (0.880/ (2×3265) ×sin (0.041095129-45/ (2×3265)-0.880/ (2×3265)) =100.030m.
Similarly, levels along circular curve at certain interval will be calculated measuring the length from start point of circular curve.
Calculating the ordinate of circular curve in a Sag Curve in design of hyperloop:
For calculating the level along the curve, length is divided into a number of equal chords and rest is divided into 2 equal parts and that is applied at start of curve and end of curve and ordinated to the curve calculated at the ends of these chords. Ordinate is calculated in following ways:
Radius of curvature R = 7900m proposed for vertical acceleration of 0.491m/s^2 and speed 60m/s, however larger radius is provided
Length of first chord is 0.677m measured from start of circular curve
Say level at start of circular curve i.e. level at D = 100.000m
Ingrade n_1 = -0.007232381^c
$t_1 = 2R\sin\delta_1$
$\delta_1 = C_1/2R$
$\varphi = L/2R$
Length of transition L = 65m
Level at the end of first chord i.e. 0.677 m would be
Level at E = 100-2×Radius×sin (angle subtended by half of first chord at the centre) ×sin (+veIngrade-transition angle-angle subtended by half of first chord at the centre)
Level at E = level at D- $t_1$sin (n1- φ - δ_1)= 100-2×7900×sin (0.677/ (2×7900) ×sin (0.007232381-65/ (2×7900)-0.677/ (2×7900)) =99.9979m.
Similarly, levels along circular curve at certain interval will be calculated measuring the length from start point of circular curve.
Sample Calculation of level using the equation of parabola for summit curve for comparison with circular curve:
Let C be the point on the road surface curve at the end of the rth sub chord. Let C1 be the point on the grade line vertically above C. let the Reduced level of the tangent point A i.e.,at start of circular curve be 100.00.
Then R L of C1 = 100 + r×(u×n_1)
R L of C = R L of $C_1 - y_r$ = 100 + rx(u×n_1) - $y_1$xr^2
n_1 = in-grade = 0.034204^c (ingrade calculated for circular curve from ingrade of previous calculation – transition angle of previous transition length) = 0.041095129 – 45/ (2×3265) = 0.034204^c
u = 0.880m
$y_1 = u^2/a$ = u2 /(2L/N)
Here L/N = 3265m, r = 1
y_1 = 0.880^2/ (2×3265) = 0.000186
R L of C = 100 + 1× (0.880×0.034204) –0.0001186×1^2 = 100.030m
Sample Calculation of level using the equation of parabola for summit curve for comparisonwith combined curve (circular with transition curve):
Let C be the point on the road surface curve at the end of the rth sub chord. Let C1 be the point on the grade line vertically above C. let the Reduced level of the tangent point A i.e. at start of circular curve be 100.00.
Then R L of C1 = 100 + r×(u×n_1)
R L of C = R L of $C_1 - y_r$ = 100 + rx(u×n_1) - $y_1$xr^2
n_1 = in-grade = 0.034204^c (ingrade calculated for circular curve from ingrade of previous calculation – transition angle of previous transition length) = 0.041095129 – 45/ (2×3265) = 0.034204^c
u = 5m
$y_1 = u^2/a$ = u2 /(2L/N)
L = 161.761
n_1 = 0.041095129^c
N = -0.03576
Here L/N = 4523.3434m, r = 1
y_1 = 5^2/ (2*4523.3434) = 0.00276
R L of C = 100 + 1×(5×0.041095) – 0.00276×1^2 = 100.2027m
This value 100.2027m could be compared with the value calculated at same point when transition curve is introduced which is 100.2053m as obtained from **Calculating the ordinate of transition curve in a Crest Curve in design of hyperloop**. There is about 2 mm difference in 5 m distance when distance increases it also increases as shown in the Table below.

A comparative designed level for parabolic curve and if parabolic curve is replaced by combined transition and circular curve and if parabolic curve only replaces circular part of combined curve is worked out and a table showing all levels are presented below in **Table A 1 and** presented graphically as shown in Fig. A 3.

Table A 1: comparison of level for various options

Points	**Chainage**	**level while parabolic curve replaced by transition & circular curve**	**Level while parabolic curve replaces only circular part of combined curve**	**Level while profile designed by parabolic curve**
Start	0.000	577.0000		577.0000
TS1	113.603	577.0000		577.0000
	118.603	577.0004		577.0051
	123.603	577.0030		577.0204
	128.603	577.0100		577.0459
	133.603	577.0238		577.0816
	138.603	577.0465		577.1275
	143.603	577.0804		577.1835
SC1	148.603	577.1276	577.1276	577.2498
	148.979	577.1318	577.1318	577.2552
	158.979	577.2747	577.2747	577.4199
centre of circular arcof sag	163.979	577.3697	577.3697	577.5176
	168.979	577.4802	577.4802	577.6256
	178.979	577.7481	577.7482	577.8719
CS1	179.355	577.7600	577.7595	577.8819
	184.355	577.9182		578.0211
	189.355	578.0898		578.1705
	194.355	578.2725		578.3301
	199.355	578.4642		578.4999
	204.355	578.6626		578.6799
	209.355	578.8654		578.8700
ST1	214.355	579.0704		579.0704
TS2	462.173	589.2545		589.2545
	467.173	589.4598		589.4572
	472.173	589.6642		589.6544
	477.173	589.8669		589.8461
	482.173	590.0671		590.0322
	487.173	590.2639		590.2128
	492.173	590.4564		590.3879
	497.173	590.6438		590.5574
	502.173	590.8253		590.7214
SC2	507.173	591.0000	591.0000	590.8799
	508.053	591.0300	591.0300	590.9073
	518.053	591.3539	591.3540	591.2057
	528.053	591.6473	591.6474	591.4821
	538.053	591.9100	591.9102	591.7364

Formatted: Font: (Default) Times New Roman, 11 pt, Not Bold, Italic

Formatted: Font: (Default) Times New Roman, 10 pt

Formatted: Font: Times New Roman, 8 pt, Font color: Auto

Formatted: Font: Times New Roman, 8 pt, Font color: Auto

Formatted: Font: Times New Roman, 8 pt, Font color: Auto

Formatted: Font: Times New Roman, 8 pt, Font color: Auto

Formatted: Font: Times New Roman, 8 pt, Font color: Auto

Formatted: Font: Times New Roman, 8 pt, Font color: Auto

Formatted: Font: Times New Roman, 8 pt, Font color: Auto

Formatted: Font: Times New Roman, 8 pt, Font color: Auto

Formatted ...

Formatted ...

Formatted ...

Formatted ...

Formatted ...

Formatted ...

Formatted ...

Formatted ...

Formatted ...

Formatted ...

Formatted ...

Formatted ...

Formatted ...

Formatted ...

Formatted ...

Formatted ...

Formatted ...

Formatted ...

Formatted ...

Formatted ...

Formatted ...

Formatted ...

Formatted ...

Formatted ...

Formatted ...

Formatted ...

Formatted ...

Formatted ...

Formatted ...

centre of circular arcof crest	543.053	592.0299	592.0301	591.8552
	548.053	592.1422	592.1423	591.9688
	558.053	592.3437	592.3438	592.1788
	568.053	592.5145	592.5147	592.3668
	578.053	592.6548	592.6550	592.5326
CS2	578.933	592.6666	592.6659	592.5461
	583.933	592.7240		592.6198
	588.933	592.7747		592.6879
	593.933	592.8194		592.7505
	598.933	592.8589		592.8076
	603.933	592.8942		592.8591
	608.933	592.9262		592.9051
	613.933	592.9555		592.9456
	618.933	592.9832		592.9806
ST2	623.933	593.0100		593.0100
TS3	1978.804	600.2366		600.2366
	1983.804	600.2633		600.2627
	1988.804	600.2898		600.2877
	1993.804	600.3161		600.3115
	1998.804	600.3419		600.3342
	2003.804	600.3672		600.3558
	2008.804	600.3919		600.3762
	2013.804	600.4157		600.3955
	2018.804	600.4387		600.4136
	2023.804	600.4605		600.4307
	2028.804	600.4812		600.4465
	2033.804	600.5006		600.4613
	2038.804	600.5185		600.4749
SC3	2043.804	600.5348	600.5348	600.4873
	2044.086	600.5356	600.5356	600.4880
	2054.086	600.5629	600.5629	600.5094
	2064.086	600.5833	600.5833	600.5263
centre of circular arcof crest	2074.086	600.5968	600.5968	600.5386
	2084.086	600.6034	600.6034	600.5464
	2094.086	600.6031	600.6031	600.5496
	2104.086	600.5959	600.5959	600.5483
CS3	2104.368	600.5956	600.5956	600.5482
	2109.368	600.5893		600.5457
	2114.368	600.5815		600.5422
	2119.368	600.5722		600.5375
	2124.368	600.5615		600.5316
	2129.368	600.5497		600.5246
	2134.368	600.5368		600.5165

	2139.368	600.5230		600.5073
	2144.368	600.5084		600.4969
	2149.368	600.4931		600.4854
	2154.368	600.4773		600.4727
	2159.368	600.4611		600.4590
	2164.368	600.4446		600.4440
ST3	2169.368	600.4280		600.4280

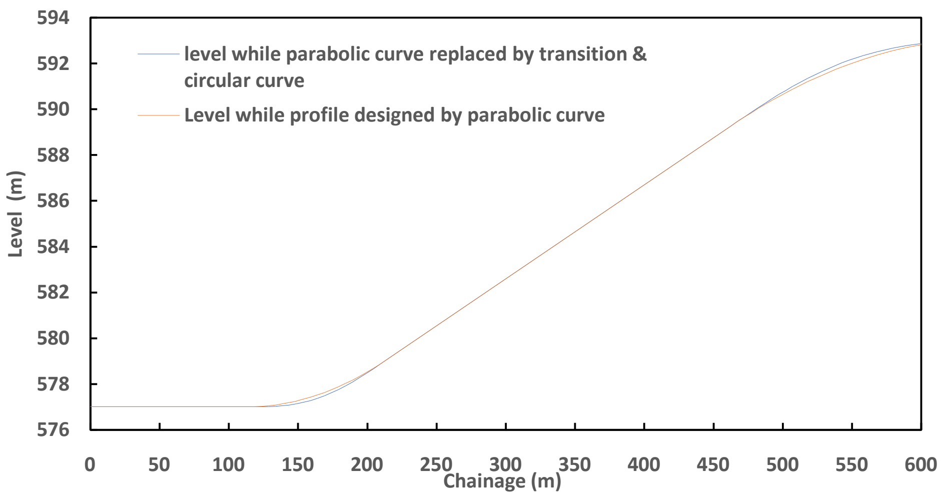

Fig.A 3: Comparison between Circular Curve with Transition & Parabolic Curve

xxxxxxxxxx. "Design of Vertical Curve with Circular and Transition Curve." *IOSR Journal of Computer Engineering (IOSR-JCE)*, 25(1), 2023, pp. 29-42.

Formatted: Font: (Default) Times New Roman, 11 pt, Not Bold, Italic

Formatted: Font: Times New Roman, 8 pt, Font color: Auto

Formatted: Font: Times New Roman, 8 pt, Font color: Auto

Formatted: Font: Times New Roman, 8 pt, Font color: Auto

Formatted: Font: Times New Roman, 8 pt, Font color: Auto

Formatted: Font: Times New Roman, 8 pt, Font color: Auto

Formatted: Font: Times New Roman, 8 pt, Font color: Auto

Formatted: Font: Times New Roman, 8 pt, Font color: Auto

Formatted: Font: (Default) Times New Roman, 10 pt

Formatted: Font: (Default) Times New Roman, 10 pt

Formatted: Font: (Default) Times New Roman, 10 pt

Formatted: Font: (Default) Times New Roman, 10 pt