IOSR Journal of Computer Engineering (IOSR-JCE)
e-ISSN: 2278-0661,p-ISSN: 2278-8727, Volume 25, Issue 5, Ser. 2 (Sept. – October. 2023), PP 01-10
www.iosrjournals.org

Characteristics Of Explosive Burst Synchronization In Small World Neural Network

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Abstract—*Inspired by the theory of 'Sandpile Model', the concept of 'Self-Organized Criticality' is transferred to this paper. Aiming at the realization of explosive burst synchronization, from two aspects: some continuously (and randomly) distributed neurons are excited by equal-amplitude currents (can be called excitation mode 1, 2 respectively) and the influence of network scale, the occurrence conditions, change characteristics and effect factors of the explosive burst synchronization under Newman-Watts (NW) small world neural network based on Hindmarsh-Rose (HR) model neurons is studied in deep. The relevant numerical analysis shows that, under different neuron node excitation coverage, relationship curve between the average number of neurons in burst synchronization and the connection probability, can be divided into Monotonic Silence, Oscillatory Silence and Oscillatory Explosion, and their qualitative explanations are given. This study will help to establish a reliable trigger and inhibition strategy for explosive burst synchronization in complex neural networks, and provide a possible theoretical basis for deepening the understanding of the formation of neural function at the mesoscopic level.*

Index Terms— *NW small world neural network, HR model neuron, explosive burst synchronization, excitation mode, network scale influence*

Date of Submission: 01-09-2023 Date of Acceptance: 13-09-2023

I. Introduction

The 'Sandpile Model' test of Bak et al.[1] shows that, with the increase of the size and height of the sandpile, its slope will reach a critical value at a certain time. Under this condition, the addition of even one grain of sand (representing small interference from the outside) may cause sand avalanche as small as one or several grains of sand, and as large as all grains of sand on the surface of the whole sandpile. At this time, the sandpile is in a 'Self-Organized Critical' state, the emergence and maintenance of this kind of situation is essentially determined by the structure and parameters of the constituent elements of the sandpile, which has nothing to do with external factors. In this case, the system is in a very sensitive state, and small local changes can be continuously amplified and extended to a certain area or even the whole system.

Relevant studies have shown that, complex neuronal circuits obtained from the brain learning patterns are also prone to burst synchronization with different scope and scale similar to 'sand avalanche' [2,3], which can be called explosive burst synchronization. This shows that the explosive burst synchronization of neurons in the brain world plays an important role in learning, memory and thinking. At the same time, some latest research results also show that Epilepsy and Parkinson's disease are transient brain dysfunction caused by sudden abnormal discharge and excessive super-synchronization of brain neurons [4-7]. When these diseases attack, the working modes of single neurons are also distributed in bursts. Burst discharge shows the mutual transformation between resting state and repeated discharge state. It is the most common multi-scale fast-slow dynamic phenomenon unique to the nervous system. There are a large number of complex oscillation modes. It undertakes the core mission of neural information transmission and plays a key role in neural information coding [8-11]. Due to the importance of burst discharge, burst synchronization has attracted extensive attention in recent years. For example, lvanchenko et al.[12,13] gave the chaotic phase synchronization condition of coupled map neuron burst discharge, and found that the phase synchronization is not only related to the coupling strength. In addition to the burst synchronization of electrically coupled chaotic neural networks, the references [14,15]

also disclose its mechanism. Shi[16] studied the burst synchronization properties of two chemical synaptic coupled neurons system. It is found that excitatory coupling can make two neurons produce in-phase burst synchronization, while inhibitory coupling can make them produce anti-phase one. Moreover, on account of the characteristics of burst dynamics for a single neuron, a measurement criterion named ‘Width Factor’ is introduced. Based on this, burst discharge are roughly divided into short burst and long burst, and further studies show that excitatory coupling is more conducive to the establishment of burst discharge, and the short burst is more stable. However, the study of discharge mechanism based on complex neural network may be more valuable. Therefore, owning the research results of synchronous mode decision and transition mechanism of electrically coupled small world neural networks, Wang et al.[17] found that the synchronization state of the system is completely determined by its own external excitation, which has nothing to do with the coupling strength and connection probability; under the excitation of two amplitude currents, only almost complete synchronization can be achieved, and the firing patterns of the synchronized NW small-world neuronal network only distributes the interval with two endpoints, which just correspond to the two modes for a single model neuron with these two amplitude stimuli current, the specific almost complete synchronization discharge mode is related to the proportion of the number of neurons excited by the two currents. Encouraged by this, Wang et al.[18,19] and Fan et al.[20] also carried out research on the synchronous target mode decision mechanism of electrically coupled neuron system under asymmetric excitation, some qualitative and quantitative conclusions that are easier to be understood and grasped have been obtained. However, explosive burst synchronization is obviously a very important discharge state of the neural system, and its in-depth relationship with network topology, scale and excitation characteristics has not been reported, which is one of the important connecting links between micro neuronal cells and mesoscopic neural tissue. Consequently, research on explosive burst synchronization and regulation mechanism of complex neural networks, will greatly promote the disclosure of the mathematical mechanism of advanced brain function and its pathological effect.

II. Model

The neural network coupling connection topology proposed by Newman and watts[21], which is now widely accepted, is specifically described as follows: In a regular ring network, without destroying any connection of the original ring structure, gradually increase the connection between the neurons that have not established the connection yet. Moreover, the two neuron nodes can only be connected once, and the neuron cells represented by each node have no self-feedback. In this system, for the connection probability (also known as edging probability) $p=0$, it means that all the neuron members is in the initial ring regular connection; for $p=1.0$, it means that the cluster is in the global coupling state; for $0<p<1.0$, it is the concrete embodiment of the characteristics of the small world, in this case, the formed network is neither regular network nor random network, but between the two, the system is called NW small world network.

In fact, even in a single tissue (such as the visual nerve cell network of the fundus), in many cases, only some neurons can obtain excitation, while others cannot. Therefore, combined with the basic framework of NW small world neural network, this study proposes two representative excitation strategies: 1) Some continuously distributed neurons are excited by equal-amplitude currents (excitation mode 1, see Fig. 1 (a)); 2) Some randomly distributed neurons are excited by equal-amplitude current (excitation mode 2, see Fig. 1 (b)). Of course, in order to facilitate the research and make the synchronized small world neural network stable in burst state, the excitation intensity used in this paper is all 2.75.

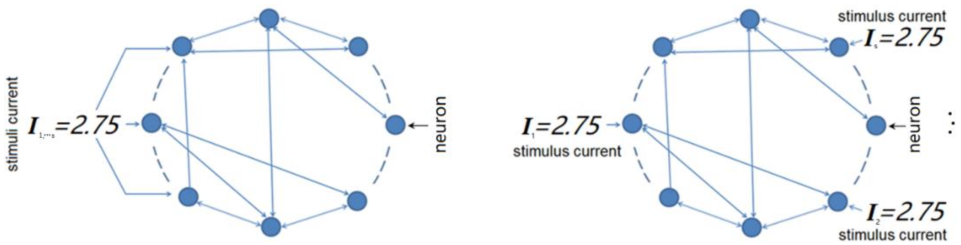

Fig. 1 The NW small world network and its excitation framework in this study

(a) Some continuously distributed neurons are excited by equal-amplitude currents (excitation mode 1)

(b) Some randomly distributed neurons are excited by equal-amplitude current (excitation mode 2)

The small world neural network composed of N HR neurons can be expressed by the set of differential equations shown in model (1) [21,22]:

$$\begin{cases} \dot{x}_i = y_i - ax_i^3 + bx_i^2 - z_i + I_i + C\sum_{j=1}^{N} a_{ij}(x_j - x_i) \\ \dot{y}_i = c - dx_i^2 - y_i \\ \dot{z}_i = r[s(x_i - \chi) - z_i] \end{cases} \tag{1}$$

where, i, j = 1, 2, . . . , N, xi represents the neuronal membrane potential, yi is the fast recovery variable (which is related to Na+ and K+ currents), and zi represents the K+ current variable activated by Ca2+. Besides, I is the external excitation current, C is the coupling strength; $\left(a_{ij}\right)_{N\times N}$ is the coupling relationship matrix, and its elements are only 0 and 1, if the i-th neuron is connected with the j-th neuron, there is $a_{ij} = a_{ji} = 1$, otherwise, $a_{ij} = a_{ji} = 0$, in addition, $a_{ii} = 0$ must be met, i.e., there is no self-coupling for any individual neuron. Other parameters are: a=1.0, b=3.0, c=1.0, d=5.0, $\chi = -1.6$, r=0.006, s=4.0.

III. Influence of network excitation modes

Based on the NW small world network and its excitation framework shown in Fig. 1 and equation (1), in order to facilitate research and save time consumption, in addition to the above-mentioned parameters, the network scale is taken as N=100, the system coupling strength is C=0.5 (According to reference [17], the electrical coupling strength does not affect the stable synchronous mode of system (1). For this study, the coupling strength C=0.5 is strong enough, so as, the synchronization state can be stabilized), and the HR neuron node excitation coverage on the basic ring of the small world network is measured by P1 (0< P1<1.000).

In the study on the influence of excitation modes 1 and 2 to explosive burst synchronization, the effects of neuron node excitation coverage P1 and the connection probability p need to be considered at the same time. The specific methods are as follows: Under a certain P1, due to the different connection probability p, it will inevitably lead to the difference in the number of neurons under explosive burst synchronization; moreover, even for the same p, the uncertainty of the specific connection, even the uncertainty of the initial value of each variable, is bound to lead to the uncertainty of the neuron member in explosive burst synchronization. Therefore, only the average number of neurons (m) in explosive burst synchronization is suitable to describe the influence of network excitation mode under a certain P1 and different p. The number of tests used to obtain each m is 20. The test results are shown in Fig. 2.

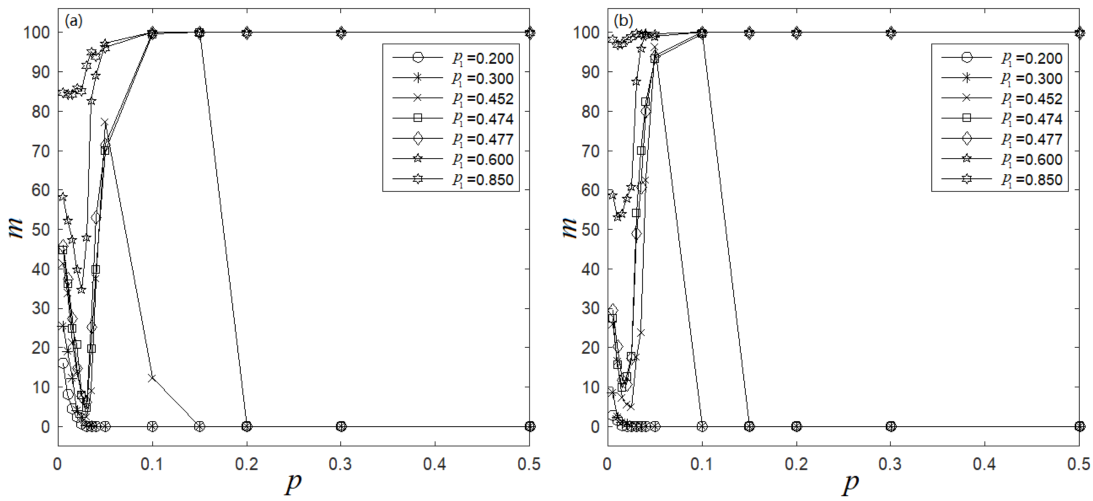

Fig. 2 Scale change diagram of system (1) explosive burst synchronization under *N*=100, *I*=2.75, *C*=0.5
(a) Excitation mode 1 (b) Excitation mode 2

It can be found from Fig. 2 that, for the corresponding excitation modes 1 and 2, the common feature is that: The relationship curve between m and p under different P1, can be divided into three categories, namely Monotonic Silence, Oscillatory Silence and Oscillatory Explosion. For example, the curves corresponding to P1=0.200 and 0.300 in the figure belong to Monotonic Silence class. Because along with the increase of p, m decreases sharply, and this is due to the limited excitation cannot meet the increasing connection. However, the curves corresponding to P1=0.452 and 0.474 belong to Oscillatory Silence. Corresponding to these curves, at the beginning, m decreases sharply with the increase of p, but it will not touch the bottom. Then, with the increase of p, m will experience a violent rise and fall, finally, it touch the bottom and remain at zero, thus showing a complete oscillation. The decrease of m in the initial stage is due to the rapid increase of coupled neurons with the increase of p, while the stimulated neurons remain unchanged, this is still a situation of insufficient excitation. With the further increase of p, the number of stimulated neurons has increased significantly, and the rapid increase of effective coupling greatly increases the probability of obtaining effective excitation indirectly. Therefore, m will rise rapidly after stopping the decline. However, with the further increase of p, the rise of coupling connections will be faster than the acquisition opportunity of new effective excitation for a single neuron, and the dilemma of limited excitation is staged again. Consequently, the whole system quickly enters the global substandard oscillation. Next, for P1=0.477, 0.600 and 0.850 in Fig. 2, with the increase of p, the number of m will first decrease to different degrees and then stop falling, and then quickly pull up to enter the global burst synchronization. These curves belong to Oscillatory Explosion. The reason for the initial m decline is similar to that of Monotonic Silence, but it can quickly enter the global burst synchronization after stopping the decline. It is because that, for the network scale of N=100, even if the global coupling connection occurs, for all neurons, the excitation obtained at this time is neither very large enough to induce the physiological inactivation, nor small enough to make the nervous system unable to vibrate.

Further, according to Fig. 2, compared with excitation mode 1, for excitation mode 2, we have: 1) the number of m corresponding to the same p is small; 2) the critical connection probability p required to enter the global substandard oscillation under the same conditions is small. For conclusion 1), just because stimulated nodes of excitation mode 2 are randomly distributed on the basic ring of NW small world network, especially when the connection probability p is relatively small, it is difficult to make the m-number neurons corresponding to excitation mode 1 oscillate. Therefore, there is a congenital defect in the ability to stimulate explosive burst synchronization under excitation mode 2. And for conclusion 2), in this case, the rapidly increasing connection cannot easily obtain more effective excitation due to the aggregation effect as in excitation mode 1, so it will enter the global substandard oscillation earlier.

It should be clear that, the nerve pulse is the carrier of nerve information, and the pulse peak value large enough is necessary. This paper requires that membrane potential peak must meet xi·max≥1.0, otherwise it will be regarded as substandard oscillation. Of course, the specific implementation process of explosive burst synchronization described in this paper is also the focus of research. The relevant results are shown in Fig.3 and 4. Fig. 3 (a) and (b) show the results of P1=0.200 and p=0.020 in excitation mode 1. Under this condition, only a small number of neurons meet the basic conditions of explosive burst synchronization. It can be seen that the membrane potentials x1 and x9 are actually phase synchronized, but it is obvious that the pulse peak of x1 does not meet the standard and is determined as substandard oscillation; x9 and x19 have sufficient amplitude, which obviously belongs to the in-phase synchronization of periodic-1 burst. Fig. 3 (c) ~ (f) are all standard periodic-1 burst synchronization, but x1 and x10 shown in Fig. 3 (c) and (e) are almost completely synchronized, while Fig. 3 (d) and (f) are only in-phase burst synchronization. Therefore, both excitation modes 1 and 2 have the opportunity to produce the same type of discharge. It should be emphasized that, according to references[17,19], the synchronization between any neuronal membrane potentials shown in Fig. 3 and 4 cannot be complete synchronization, but can only be almost complete synchronization at most.

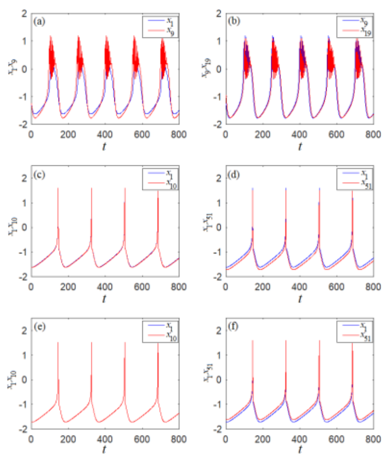

Fig. 3 Response change of system (1) to excitation mode under condition N=100, I=2.75, C=0.5

(a),(b) Excitation mode 1, P_1=0.200, p=0.020

(c),(d) Excitation mode 1, P_1=0.477, p=0.500
(e),(f) Excitation mode 2, P_1=0.477, p=0.500

Although it is one matter to produce a certain scale of explosive burst synchronization, and the stable firing mode after synchronization is another important matter. Since system (1) is based on the linear combination of single HR model neurons, according to the relevant conclusions of references [17,19], the stable firing mode after synchronization should be closely related to the relevant excitation response of the original single HR neuron. Based on the relevant parameters of system (1), the discharge time evolution course of membrane potential of a single HR neuron under external excitation current I=2.75 is shown in Fig. 4 (a). Here only take excitation mode 1 as an example, and excitation mode 2 also has similar conclusions. The stable periodic-1 burst synchronization can be seen in Fig.3; for periodic-2 burst synchronization, see Fig. 4 (b) (P1=0.600, p=0.500); periodic-3 burst synchronization can be found in Fig.4 (c) (P1=0.800, p=0.500); and periodic-4 burst synchronization is plotted in Fig.4 (d) (P1=0.900, p=0.500). Thereafter, even if P1 is increased to 1.000, there will be no new firing modes, and the periodic-4 burst synchronization will still be maintained, but the performance of the specific interval-spike intervals (ISIs) will be exactly the same as that calculated from the relevant data in Fig. 4 (a). The above performance fully shows that, the conclusions obtained in literature[17,19] can be verified again in this paper. However, it must be pointed out that, on the one hand, literature[17]does not give a quantitative conclusion on the determination of ISI position. Although literature[19] gives a specific formula for the determination of ISI position, it is only for the electrically coupled two neuron coupled system, that is, it belongs to a deterministic connected system in spatial topology, which is significantly different from the system (1) concerned in this study. On the other hand, although the firing mode shown in Fig. 3 (b) belong to the periodic-1 in phase burst synchronization, if compared with Fig. 1 in the literature[19], it will be found that the corresponding ISIs distribution characteristics do not exist in the figure, which means that the system (1) has generated new discharge modes, its synchronous burst firing modes are more diverse, and correspondingly, the information code that can be transmitted must be richer. In addition, through comprehensive analysis of Fig. 2 ~ 4 and the system simulation process, it can be found that under the same P1, even if the connection probability p changes continuously, it can only affect the specific scale of burst synchronization as well as the synchronized forms, and cannot fundamentally affect its stable synchronization pattern. This conclusion does not contradict the relevant results in reference[17,19], although the virtual equivalent symmetrical excitation current I'_{Cal} corresponding to its ISI position cannot be accurately determined as what in document[19] in this study, it still has a guiding role in controlling its stable synchronous mode according to the calculated value.

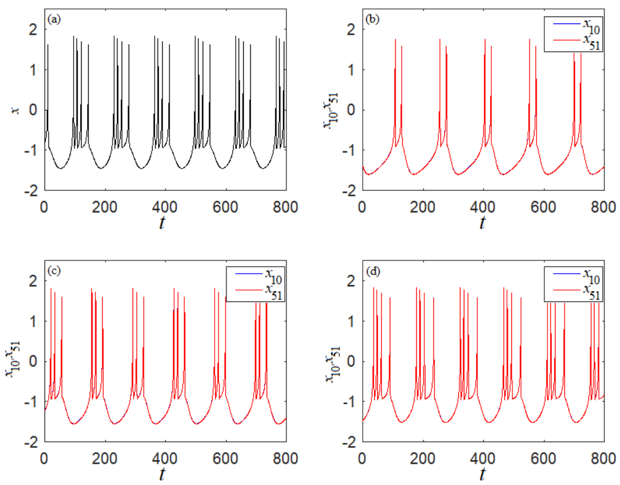

Fig. 4 The relationship between excitation mode and stable synchronous mode under

N=100, I=2.75, C=0.5 and excitation mode 1 in system (1)
(a) Membrane potential of single HR neuron at I=2.75 (b) P_1=0.600, p=0.500
(c) P_1=0.800, p=0.500 (d) P_1=0.950, p=0.500

IV. Impact of network scale

In addition to excitation mode, the impact of network scale on explosive burst synchronization should also be paid attention to. Here, there is no need to make a comprehensive investigation, just consider 2~3 typical P1 values under excitation modes 1 and 2 respectively, the global change trend can be mastered as a whole. Therefore, the test design and relevant result of this section are shown in Fig. 5. In this figure, M is a defined ratio expressed as a percentage, which is determined by dividing the number of neurons determined to have undergone explosive burst synchronization by the actual network scale. This index is conducive to more directly to compare the relative size of explosive burst synchronization under different network scales.

According to Fig. 5 (a) and (c), compared with Fig. 2, under the condition of lower P1 (e.g. P1=0.200), the figure M of different network scale also decreases sharply, finally, M will touch the bottom and remain at zero. However, under the same p, the lower network size can induce a larger number of explosive burst synchronization M, that is, the smaller network scale has a stronger ability to generate explosive burst synchronization under the same conditions. This also lies in that the larger the network, when the connection probability p increases by the same amplitude, the number of neurons in the connected state increases geometrically, while the increase of the original excitation can only be linear, so the excitation is obviously insufficient, and the dilemma that more neurons are difficult to be stimulated will be more significant. However, for medium and high P1 conditions (such as P1=0.477 corresponding to Fig. 5 (b) and (d)), in beginning, i.e., p is still relatively small, the trend and its theory of curve M is similar to that under low P1 conditions (See Fig. 5 (a) and (c)), except that the specific M is relatively large and the decline speed of the curve is significantly lower. But then M will stop falling, rebound and quickly pull up, and the larger the network scale is, the faster the pull up speed is. For networks beyond a specific scale, after M reaching 100% and continuing a certain p interval, it will quickly drop to zero and remain on the bottom, but the M of networks below this scale can always maintain the distribution trend of the global burst synchronization. The mechanism of this process is that, due to the expansion of the network scale and the fact that P1 has also crossed the required minimum critical point, the number of neurons in the connected state can be balanced with the increase of the original excitation under a smaller p than the bottom connection probability corresponding to Fig. 5 (a) and (c), and the M curve will not continue to decline. Then, with the increase of p, there will be more opportunities to connect the neurons with the existing excited members, so as to form a fierce positive feedback and make the curve rise rapidly. But the problem is that the larger the scale of the network, the greater the connection probability p, the more stimuli will be collected on a single neuron, and soon some neurons (and finally all neurons) will be inactivated. Therefore, the larger the network scale, the earlier the deactivation effect will occur. However, for the system with the network scale below a certain critical point, because the minimum connection probability p of turning the curve to the bottom needs will beyond 1.000, which is absolutely impossible, so, explosive burst synchronization can always cover the global range stably, as shown in the N=100 curve in Fig. 5 (b) and (d). Therefore, in practice, if the coupling strength of neuronal functional organizations is low, they should have more opportunities to realize more neuronal connections, and those with high coupling strength can only be on a smaller scale. Otherwise, a wider and stronger excitation will often make them fall into global violent excitation and show some pathological behavior.

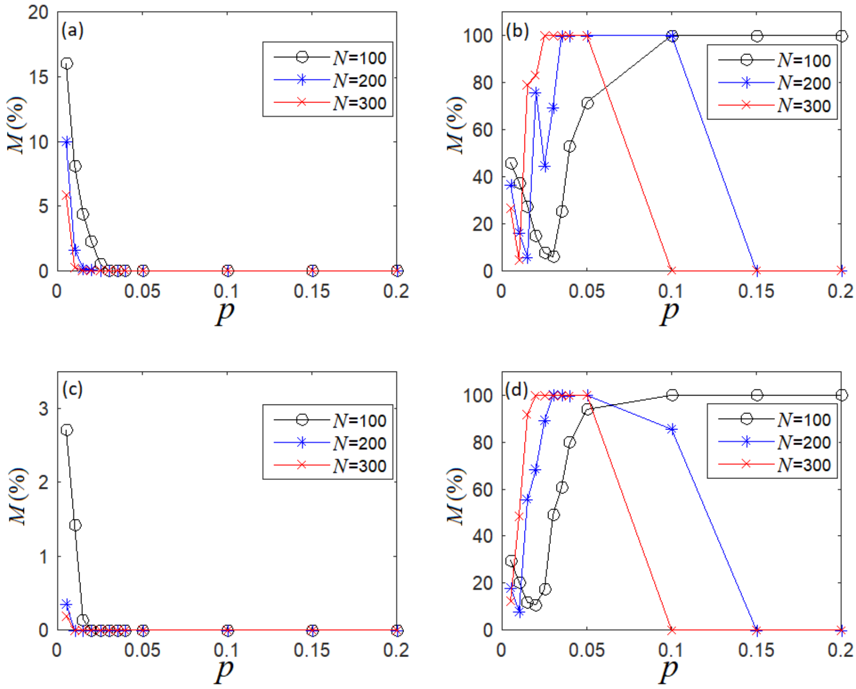

Fig. 5 The relationship between network scale, excitation modes and explosive burst synchronization scale under condition I=2.75, C=0.5 in system (1)

(a) Excitation mode 1, P_1=0.200 (b) Excitation mode 1, P_1=0.477

(c) Excitation mode 2, P_1=0.200 (d) Excitation mode 2, P_1=0.477

V. Conclusions

Based on HR model neuron and electrically coupled NW small world neural network, the explosive burst synchronization characteristics of complex neural network are investigated. The main conclusions are as follows:

1) According to the relationship curves between the average number of synchronized explosive burst neurons and connection probability under different neuron node excitation coverage, these curves can be divided into three subcategories: Monotonic Silence, Oscillatory Silence and Oscillatory Explosion. The rapid monotonic decrease of these curves under small connection probability, is due to the fact that the speed of excitation increase is far less than that of connection. However, when the neuron node excitation coverage is in the medium and high stage, with the rise of connection probability, the sharp boost of effective connection is superimposed with the effect of the growth of stimulated neurons, so that the probability of indirectly obtaining effective excitation can be increased at least in a certain connection probability section, and the number of explosive burst synchronized neurons can rise rapidly. If the neuron node excitation coverage is high enough, this situation can continue until the global coupling. Otherwise, the number of explosive burst synchronized neurons will reach a certain peak value before the global synchronization. Then, the speed of adding effective excitation to a single neuron will be lower than the increasing number of coupling connections again, so that the whole system will soon fall into substandard oscillation state due to overall insufficient excitation.

2) Compared with excitation mode 1, the global explosive burst synchronization under excitation mode 2 requires a higher connection probability, but for entering the global substandard oscillation is lower.

3) The impact of network scale on explosive burst synchronization is reflected in: when the connection probability is small, similarly, because the increase of coupling connection is much faster than the increase of excitation, the larger the network scale is, the more prominent the contradiction of insufficient incentive is, which will lead to the weaker explosive burst synchronization ability with larger network scale. However, under the

medium and high neuron node excitation coverage, when the connection probability increases slightly, the non-excited neurons will have a greater probability to connect to the direct or indirect effective excitation, and the larger the network scale, the smaller connection probability to get to global burst synchronization. At this time, for the network lower than a specific scale, the excitation obtained by a single neuron still cannot reach the intensity of deactivation, this kind of global burst synchronization can exist continuously, but the network beyond this specific scale, with the increase of connection probability, due to the collection of more feedback incentives, many neurons will be overexcited, the membrane potential amplitude will be more and more limited, and soon enter the global substandard oscillation state.

Data Availability Statements

The data used to support the findings of this study are available from the corresponding author on reasonable request.

acknowledgment

We are grateful for the support of the National Natural Science Foundation of China under Grant No. 11762002 and the Fuxi Talent Project of Gansu Agricultural University (GAUFX-02J01).

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