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Some Relations ON Fuzzy Pre-Open Set IN Fuzzy Topological Space

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The aim of this paper is to introduce and study the notion of a fuzzy pre-open set, fuzzy θ- open set , fuzzy δ-open , fuzzy $\boldsymbol{\gamma}$- open set , fuzzy M-open set , fuzzy Z-open set , fuzzy Z*-open set , fuzzy e-open set , fuzzy e*-open set and some properties , remarks related to them .

I. Introduction

The concept of fuzzy set was introduced by Zadeh (1965) in classical paper. Chang (1968) introduced the notion of a fuzzy topology. Also in (1987) MaximlianGanster appearedin:Kyungpook Math. Introduced pre-open sets and resolvable spaces *Also in (1992) chakraborty M.K. and T.M.G Ahsanullah introduced "fuzzytopology on fuzzy sets and tolerance topology"fuzzy sets and systems.Also, in 2011 A.I.EL-Magharabi and A. M. Mubarki introduced the Z-open sets and fuzzy continuity in topological spaces, in 2013 Ahmed I. EL. Magharabi, Mohammed A. AL –Juhani introduced the new types of functions by M-open sets,θ-open set ,pre-open set, e- open set ,e*-open set $\boldsymbol{\gamma}$-open set , and in 2013 A.M. Mubarkiali M.M AL-Rshudi M.A.AL-Juhani introduced β* -open sets , pre-open set, e-open set , Z*-open set , Z-open set , e*-open set , $\boldsymbol{\gamma}$-open set and β*-continuity in topological spaces.

II. Fuzzy Topological Space On Fuzzy Set

In this we introduced the definition fuzzy topological space on fuzzy set and study some properties and some remarks of this subject.

2.2. Definition

Let $\boldsymbol{X}$ be anon empty set, a fuzzy set $\tilde{A}$ in $\boldsymbol{X}$ is characterized by function

$\mu_{\tilde{A}} : \boldsymbol{X} \longrightarrow I$ where $I= [0, 1]$ which is written as $\tilde{A} = \{(x, \mu_{\tilde{A}}):0\leq \mu_{\tilde{A}}(x) \leq 1\}$

The collection of all fuzzy sets in $\boldsymbol{X}$will denoted by I^X that is

$I^X = \{ \tilde{A} : \tilde{A}$ is a fuzzy sets in $\boldsymbol{X}\}$ where $\mu_{\tilde{A}}$ is called the membership function.

2.3. Definition

A collection $\tilde{T}$ of fuzzy subsets $\tilde{A}$ such that $\tilde{T} \subset p(\tilde{A})$ is said to be fuzzy topology on $\tilde{A}$ if it satisfies the following conditions:

1- $\tilde{\emptyset}, \tilde{A} \in \tilde{T}$

2- If $\tilde{G}, \tilde{H} \in \tilde{T}$ Then $\tilde{G} \cap \tilde{H}\tilde{T} \in \tilde{T}$

3-If $\tilde{G}_i \in \tilde{T}$ then $\cup \tilde{G}_i$ $i \in \lambda$

The order pair $(\tilde{A}, \tilde{T})$ is said to be the fuzzy topological space and every member of $\tilde{T}$, is called fuzzy open ($\tilde{T}$-open) set in $\tilde{A}$ and the complement is called fuzzy closed ($\tilde{T}$ –closed) set .

[2.3]Definition

If $(\tilde{A},\tilde{T})$ be a fuzzy topological space and be $\tilde{B}$ be a fuzzy set in $\tilde{A}$ Then the closure and interior of $\tilde{B}$ is defined by.

CL $(\tilde{B}) = \cap\{ \tilde{F} : \tilde{F}$ is a fuzzy closed set in $\tilde{A}, \mu_{\tilde{B}}(x) \leq \mu_{\tilde{F}}(x)\}$

Int $(\tilde{B}) = \cup \{ \tilde{G} : \tilde{G}$ is a fuzzy open set in$\tilde{A}, \mu_{\tilde{G}}(x) \leq \mu_{\tilde{B}}(x)\}$

2.4. Definition

Let $\tilde{B}$ a fuzzy set of a fuzzy topological space $(\tilde{A}, \tilde{T})$ then $\tilde{B}$ is said to be fuzzy pre- open set if $\mu_{\tilde{B}}(x) \leq \mu_{int(cl(\tilde{B}))}(x)$

2.5. Definition

Let ($\tilde{A}$,$\tilde{T}$) be a fuzzy topological spaces then fuzzy pre- closure (cl_p)

And fuzzy pre - interior (int_p) of a fuzzy set $\tilde{A}$ are a defined by as follows:

1. p-cl ($\tilde{B}$) = ⋂{ $\tilde{F}$: $\tilde{F}$ is a fuzzy p- closed set in $\tilde{A}$, $\mu_{\tilde{B}}$(x) ≤ $\mu_{\tilde{F}}$ (x)}

2. p-int ($\tilde{B}$) = ⋃ { $\tilde{G}$: $\tilde{G}$ is a fuzzy p - open set in $\tilde{A}$, $\mu_{\tilde{G}}$ (x) ≤ $\mu_{\tilde{B}}$ (x)}

2.6. Definition

A fuzzy set $\tilde{B}$ of a topological space ($\tilde{A}$,$\tilde{T}$) is said to be

1-fuzzy θ-open set if provided that $\mu_{\tilde{B}}$ (x) = $\mu_{int\theta(x)}$ and fuzzy θ- closed set in $\tilde{A}$ if $\mu_{\tilde{B}}$ (x) = $\mu_{cl\theta}$(x)

2- Fuzzy δ -open set if $\mu_{int(cl(\tilde{B})}$(x) ≤ $\mu_{\tilde{B}}$ (x) and fuzzy δ- closed set in $\tilde{A}$ if $\mu_{\tilde{B}}$(x) ≤ $\mu_{cl(int(\tilde{B})}$(x)

3-fuzzy M-open set if $\mu_{\tilde{B}}$(x) ≤ max {$\mu_{cl(int_{\theta(\tilde{B})}}$ (x), $\mu_{int(cl_{\delta(\tilde{B})})}$(x)}

And fuzzy M- closed set in $\tilde{A}$ if min { $\mu_{int_{\theta}(cl(\tilde{B})}$(x) , $\mu_{cl_{\delta}(int(cl(\tilde{B})}$(x) } ≤ $\mu_{\tilde{B}}$ (x)

4- Fuzzy $\boldsymbol{\gamma}$-open set if $\mu_{\tilde{B}}$(x) ≤ max { $\mu_{cl(int(\tilde{B}))}$(x) ,$\mu_{int(cl(\tilde{B}))}$(x)} and fuzzy $\boldsymbol{\gamma}$- closed set in $\tilde{A}$ if

min {$\mu_{int(cl(\tilde{B})}$(x), $\mu_{cl(int(\tilde{B})}$(x)} ≤ $\mu_{\tilde{B}}$(x)

5- Fuzzy e-open set if $\mu_{\tilde{B}}$(x) ≤ max {$\mu_{cl(int_{\delta(\tilde{B})}}$(x) , $\mu_{int(cl_{\delta(\tilde{B})}}$(x) } and fuzzy e-closed set in $\tilde{A}$ if

min {$\mu_{int_{\delta}(cl(\tilde{B})}$(x) , $\mu_{cl_{\delta}(int(cl(\tilde{B})}$(x)} ≤ $\mu_{\tilde{B}}$(x)

6-fuzzy e*-open set if $\mu_{\tilde{B}}$(x) ≤ $\mu_{cl(int(cl_{\delta(\tilde{B})}}$(x) and fuzzy e* –closed set in $\tilde{A}$ if $\mu_{cl_{\delta}(int(cl(\tilde{B})}$(x) ≤ $\mu_{\tilde{B}}$(x)

7- Fuzzy Z- open set if $\mu_{\tilde{B}}$(x) ≤ max { $\mu_{cl(int_{\delta(\tilde{B})}}$(x) , $\mu_{int(cl(\tilde{B}))}$(x) } And fuzzy Z-closed set in $\tilde{A}$ if $\mu_{cl(int(\tilde{B})}$(x) ≤ $\mu_{\tilde{B}}$(x)

8-fuzzy Z*-open set if $\mu_{\tilde{B}}$(x) ≤ max {$\mu_{cl(int(\tilde{B}))}$(x) , $\mu_{int(cl_{\delta(\tilde{B})})}$(x) } and fuzzy Z*-closed set in $\tilde{A}$ if

min { $\mu_{int(cl(\tilde{B})}$(x), $\mu_{cl_{\delta}(int(\tilde{B})}$(x) } ≤ $\mu_{\tilde{B}}$ (x)

2.7. Propositions:-

1- Every fuzzy pre-open set is fuzzy M-open set
2- Every fuzzy θ-open set is fuzzy pre- open set.
3- Every fuzzy pre- open set is fuzzyγ- open set.
4- Every fuzzy pre- open set is fuzzy e- open set.
5- Every fuzzy pre- open set is fuzzy e* - open set.
6- Every fuzzy pre-open set is fuzzy Z-open set.
7- Every fuzzy pre-open set is fuzzy Z*-open set.
8- Every fuzzy Z-open set is fuzzy Z*-open set.

2.8. Remark :-

The convers of propositions is not true in general as show the following.

[1]Example

$\tilde{A}$= {(a, o.9) ,(b,0.7),(c,0.8)}

$\tilde{B}$ = {(a, 0.5), (b, 0.4),(c,0.4)}

$\tilde{C}$= {(a, 0.4),(b,0.2),(c,0.1)}

$\tilde{D}$= {(a, 0.6),(b,0.5),(c,0.8)}

Let $\tilde{T}$= { $\tilde{\emptyset}$, $\tilde{A}$, $\tilde{B}$, $\tilde{C}$ } be a fuzzy topological space .

On $\tilde{A}$ then $\tilde{D}$ is fuzzy M- open set but not fuzzy pre-open set

[3]Example:-
X ={ a ,b ,c }

$\tilde{A}$={(a,0.7),(b,0.8) ,(c,0.9)}

$\tilde{B}$={(a,0.6),(b,0.6),(c,0.5)}

$\tilde{C}$={(a,0.1),(b,0.3),(c,0.2)}

$\tilde{D}$={(a,0.5),(b,0.5),(c,0.5)}

Let $\tilde{T}$= { $\tilde{\emptyset}$, $\tilde{A}$, $\tilde{B}$, $\tilde{C}$ } be a fuzzy topological space let

on $\tilde{A}$ then $\tilde{D}$ is fuzzy γ- open set but not fuzzy pre- open

[6]Example:-
X={ a , b , c }

$\tilde{A}$={(a,0.7),(b,0.8),(c,0.9)}

$\tilde{B}$={(a,0.4),(b,0.3),(c,0.3)}

$\tilde{C}$={(a,0.0),(b,0.2),(c,0.3)}

$\tilde{D}$={(a,0.3),(b,0.4),(c,0.5)}

Let $\tilde{T}$= { $\tilde{\emptyset}$, $\tilde{A}$, $\tilde{B}$, $\tilde{C}$ } be a fuzzy topological space on $\tilde{A}$ then $\tilde{D}$ is fuzzy Z- open set but not fuzzy pre- open set

[8]Example:-
X={a ,b ,c}

$\tilde{A}$={(a,0.8),(b,0.9),(c,0.9)}

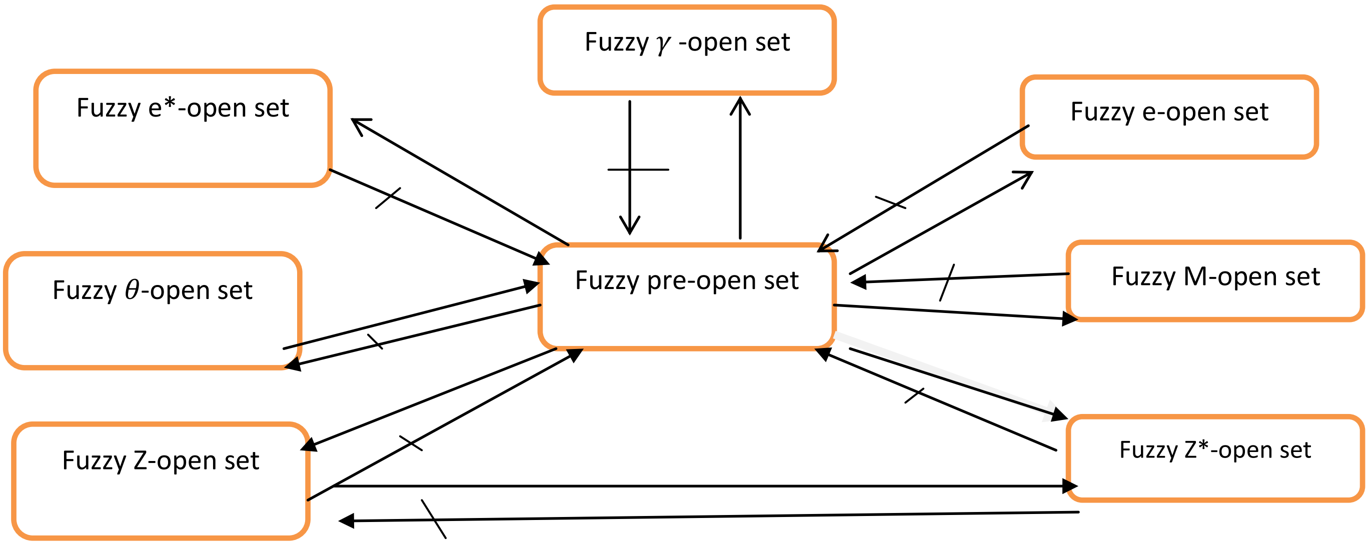
$\tilde{B}$={(a,0.4),(b,0.4),(c,0.5)}

$\tilde{C}$={(a,0.3),(b,0.2),(c,0.2)}

$\tilde{D}$={(a,0.3),(b,0.4),(c,0.6)}

Let $\tilde{T}$= { $\tilde{\emptyset}$, $\tilde{A}$, $\tilde{B}$, $\tilde{C}$ } be a fuzzy topological space On $\tilde{A}$ then $\tilde{D}$ is fuzzy Z*- open set but not fuzzy Z- open set

2.9. Remark
This diagram holds for a set $\tilde{B}$ of a fuzzy topological space ($\tilde{A}$, $\tilde{T}$)



(1) Diagram

III. Propositions

1- Every fuzzy θ-open set is fuzzy open set.
2- Every fuzzy open set is fuzzy pre- open set.
3- Every fuzzy θ-open set is fuzzy δ -open set.
4- Every fuzzy open set is fuzzy δ -open set.
5- Every fuzzy open set is fuzzy γ-open set.
6- Every fuzzy δ-open set is fuzzy γ-open set.
7- Every fuzzy θ -open set is fuzzy M -open set.
8- Every fuzzy M -open set is fuzzy e-open set.
9- Every fuzzy Z*-open set is fuzzy e*-open set.
10- Every fuzzy Z*-open set is fuzzy e –open set
11- Every fuzzy γ -open set is fuzzy e* –open set.

3.1. Remark
The convers of propositions is not true in general as show the following.

[1]Example:
X={ a , b ,c }

$\tilde{A}$={(a,0.6),(b,0.7),(c,0.9)}

$\tilde{B}$={(a,0.5),(b,0.5) ,(c,0.6)}

$\tilde{C}$={(a,0.4),(b,0.3) (c,0.5)}

$\tilde{D}$= {(a, 0.3), (b, 0.2), (c, 0.4)}

Let $\tilde{T}$= { $\tilde{\emptyset}$, $\tilde{A}$,$\tilde{B}$, $\tilde{C}$,$\tilde{D}$ } be a fuzzy topological space then $\tilde{D}$

Is fuzzy open but not fuzzy θ - open

[2]Example:-
X={a ,b ,c}

$\tilde{A}$={(a,0.7),(b,0.6),(c,0.6)}

$\tilde{B}$={(a,0.6),(b,0.6),(c,0.4)}

$\tilde{C}$={(a,0.4),(b,0.3),(c,0.3)}

$\tilde{D}$={(a,0.6),(b,0.4),(c,0.3)}

Let $\tilde{T}$= { $\tilde{\emptyset}$, $\tilde{A}$, $\tilde{B}$, $\tilde{C}$ } be a fuzzy topological space

On $\tilde{A}$ then $\tilde{D}$ is fuzzy pre- open set but not fuzzy open

[3]Example:-
X={a ,b ,c}

$\tilde{A}$={(a,0.6),(b,0.7),(c,0.9)}

$\tilde{B}$={(a,0.5),(b,0.5),(c,0.6)}

$\tilde{C}$={(a,0.2),(b,0.1),(c,0.0)}

$\tilde{D}$={(a,0.3),(b,0.2),(c,0.4)}

Let $\tilde{T}$= { $\tilde{\emptyset}$, $\tilde{A}$, $\tilde{B}$, $\tilde{C}$ } be a fuzzy topological space

on $\tilde{A}$ then $\tilde{D}$ is fuzzy δ- open set but not fuzzy θ- open

[4]Example
$\tilde{A}$={(a,0.6),(b,0.7),(c,0.5)}

$\tilde{B}$={(a,0.5),(b,0.5),(c,0.3)}

$\tilde{C}$ ={(a,0.3),(b,0.2),(c,0.2)}

$\tilde{D}$= {(a,0.5),(b,0.3),(c,0.2)}

Let $\tilde{T}$= { $\tilde{\emptyset}$, $\tilde{A}$, $\tilde{B}$, $\tilde{C}$, $\tilde{D}$ } be a fuzzy topological space

on $\tilde{A}$ then $\tilde{D}$ fuzzy open but not fuzzy δ –open set

[5]Example:
X={a ,b ,c}

$\tilde{A}$={(a,0.7),(b,0.8),(c,0.9)}

$\tilde{B}$={(a,0.6),(b,0.5),(c,0.5)}

$\tilde{C}$ ={(a,0.0),(b,0.2),(c,0.3)}

$\tilde{D}$={(a,0.2),(b,0.4),(c,0.5)}

Let $\tilde{T}$= { $\tilde{\emptyset}$, $\tilde{A}$, $\tilde{B}$, $\tilde{C}$ } be a fuzzy topological space

on $\tilde{A}$ then $\tilde{D}$ is fuzzy γ–open set but not fuzzy open

[7]Example
X={ a , b , c }

$\tilde{A}$={(a,0.6),(b,0.7),(c,0.9)}

$\tilde{B}$={(a,0.5),(b,0.5),(c,0.6)}

$\tilde{C}$={(a,0.4),(b,0.3),(c,0.5)}

$\tilde{D}$={(a,0.3),(b,0.2),(c,0.4)}

Let $\tilde{T}$= { $\tilde{\emptyset}$, $\tilde{A}$, $\tilde{B}$, $\tilde{C}$ } be a fuzzy topological space

on $\tilde{A}$ then $\tilde{D}$ is fuzzy M- open set but not fuzzy θ-open set

[8]Example:-
X={a ,b ,c}

$\tilde{A}$={(a,o.9),(b,0.9),(c,0.9)}

$\tilde{B}$ ={(a,0.5),(b,0.5),(c,0.5)}

$\tilde{C}$={(a,0.3),(b,0.3),(c,0.3)}

$\tilde{D}$={(a,0.7),(b,0.6),(c,0.5)}

Let $\tilde{T}$= { $\tilde{\emptyset}$, $\tilde{A}$, $\tilde{B}$, $\tilde{C}$ } be a fuzzy topological space

on $\tilde{A}$ then $\tilde{D}$ is fuzzy e- open set but not fuzzy M-open set

[9]Example :-
X={ a ,b , c }

$\tilde{A}$={(a,0,8),(b,0.7),(c,0.9)}

$\tilde{B}$={(a,0.5),(b,0.4),(c,0.6)}

$\tilde{C}$={(a,0.3),(b,0.2),(c,0.4)}

$\tilde{D}$={(a,0.2),(b,0.6),(c,0.1)}

Let $\tilde{T}$= { $\tilde{\emptyset}$, $\tilde{A}$, $\tilde{B}$, $\tilde{C}$ } be a fuzzy topological space

on $\tilde{A}$ then $\tilde{D}$ is fuzzy e* open set but not fuzzy Z*- open set

[10]Example :-
X= {a, b, c}

$\tilde{A}$= {(a, o.9) ,(b,0.9), (c,0.9)}

$\tilde{B}$ = {(a, 0.5), (b, 0.5),(c,0.4)}

$\tilde{C}$= {(a, 0.6),(b,0.5),(c,0.4)}

$\tilde{D}$= {(a,0.7),(b,0.6),(c,0.5)}

Let $\tilde{T}$= { $\tilde{\emptyset}$, $\tilde{A}$, $\tilde{B}$, $\tilde{C}$ } be a fuzzy topological space

on $\tilde{A}$ then $\tilde{D}$ is fuzzy e- open set but not fuzzy Z*-open set

[11]Example :-
X={a ,b ,c}

$\hat{A}$={(a,o.9),(b,0.9),(c,0.9)}

$\tilde{B}$ ={(a,0.5),(b,0.5),(c,0.4)}

$\tilde{C}$={(a,0.3),(b,0.2),(c,0.1)}

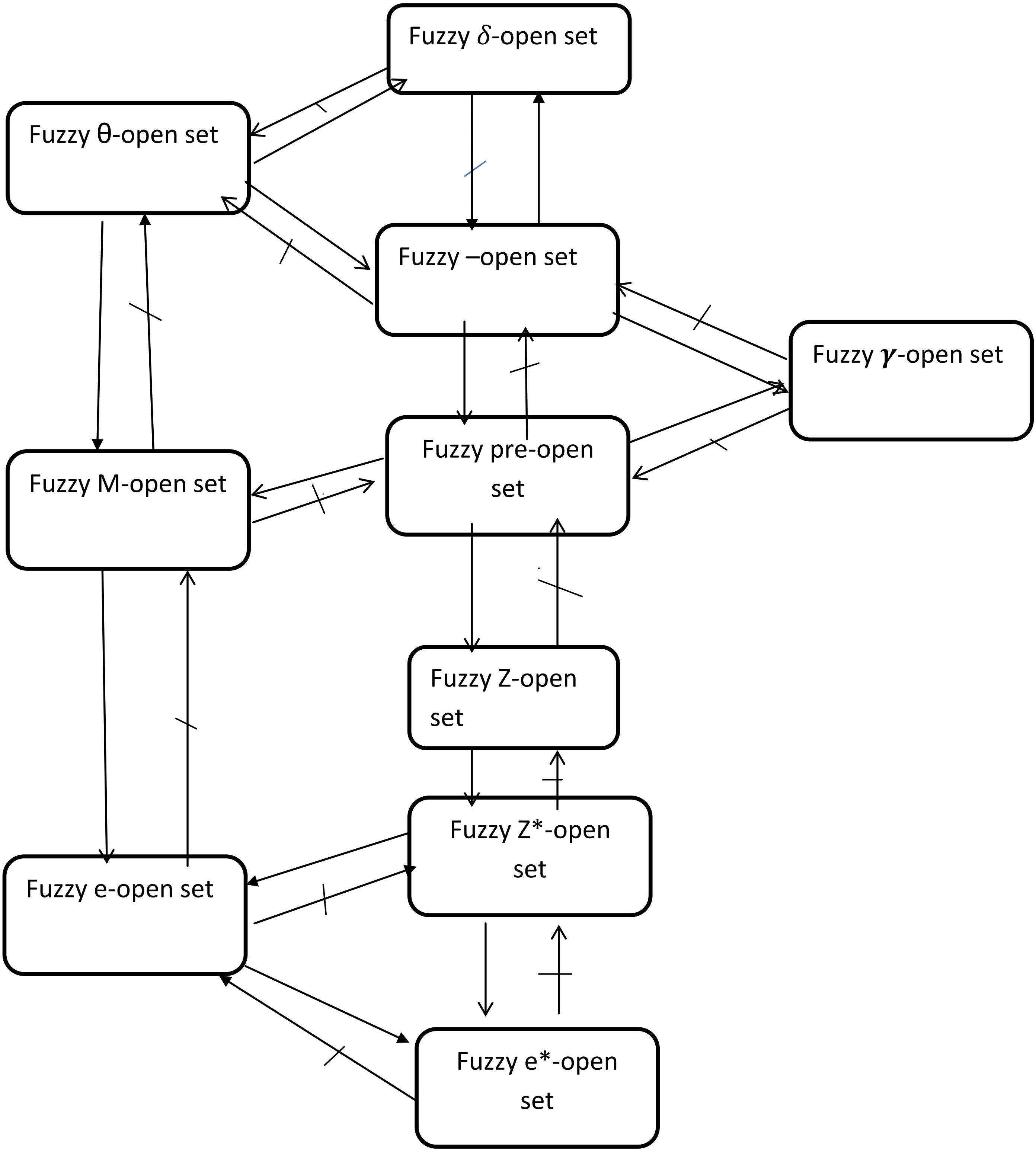
$\tilde{D}$={(a,0.6),(b,0.5),(c,0.4)}

Let $\tilde{T}$= { $\tilde{\emptyset}$, $\tilde{A}$, $\tilde{B}$, $\tilde{C}$ } be a fuzzy topological space

on $\tilde{A}$ then $\tilde{D}$ is fuzzy e*- open set but not fuzzy γ-open set

[3.2]Remark

This diagram holds for a set $\tilde{B}$ of a fuzzy topological space $(\tilde{A}, \tilde{T})$



(2)Diagram

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