

## **Speed control of Two Phase Induction Motor using Sliding Mode Controller**

<sup>1</sup>Nidhin V E, <sup>2</sup>Aswin R B

*Mar Baselios College of Engineering and Technology Thiruvananthapuram, Kerala, India*  
*Electrical and Electronics Engineering Mar Baselios College of Engineering and Technology*  
*Thiruvananthapuram, Kerala, India*

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**Abstract:** *The induction motors are characterized by complex, non-linear and time-varying dynamics, and hence their speed control is always a challenging problem. There are lot of speed control techniques are available. Out of all that techniques sliding mode control is an efficient one for the speed control due to its robustness and insensitivity to the parameter variation. In this paper, a sliding mode controller is designed for the speed control of Two Phase Induction Motor (TPIM) and TPIM is the viable replacement for the Single Phase Induction Motor (SPIM)*

**Index Terms:** *Sliding mode control, single phase induction motor, two phase induction motor, chattering phenomenon*

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### **I. Introduction**

Single-phase induction motors (SPIMs) are employed widely in household applications where a three-phase ac electrical supply is not available. SPIMs are used typically to maintain a constant speed, for example, in fans and vacuum cleaners. SPIMs normally require auxiliary winding and main winding as well as a capacitor to produce the starting torque.

Several studies have made on vector control strategies for SPIMs [1]–[5] because they are used widely. Based on that studies, the SPIMs are classified as unsymmetrical two-phase induction motors (TPIMs) because the parameters of the main and auxiliary windings are not identical. Therefore, when the SPIM is operated by the vector control strategy, significant problems are generated due to unbalanced operation. These problems associated with vector control of SPIM can be overcome by replacing single phase induction motor with symmetrical two phase induction motor which is the practical replacement for the vector controlled SPIM. The implementation of Two phase induction motor (TPIM) vector control is simpler and more accurate than vector controlled SPIM and the vector control strategy is derived from the three phase induction motor vector control strategy [6]–[7].

The speed control of induction motor drives are more complex due to its high nonlinear behavior in the dynamics. These highly nonlinear nature is because of the rapid parameter variations in the dynamics. These parameter variation issues can be solved by advanced control techniques such as self-tuning regulators, and SMC [8]–[9].

Sliding Mode Control (SMC) is a powerful robust control technique to control nonlinear systems with uncertainty. There are so many advantages for the sliding mode control technique such as insensitivity to parameter variations, external disturbance rejection, and fast dynamic response. These advantages of SMC have been employed in the speed control of induction motor drives. The major drawback of the Sliding mode control is the chattering phenomenon. To reducing this undesirable phenomenon, we have so many methods such as State dependent gain method, Describing function analysis method etc [10].

### **II. Mathematical Modelling Of TPIM**

The mathematical modelling of TPIM consist of derivation of the basic stator and rotor voltage equations and also the flux and torque equations. Fig.1 shows the schematic diagram of TPIM

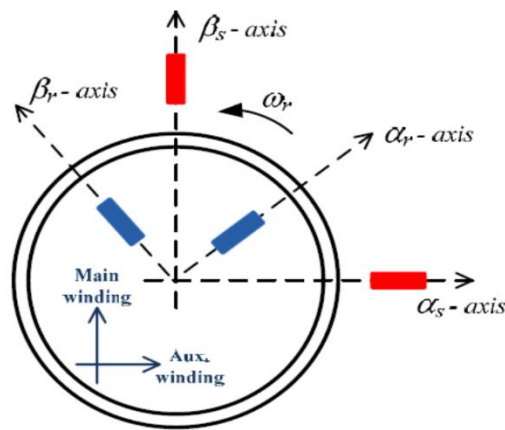


Fig 1.Schematic diagram of Two Phase Induction Motor

TPIM consist of two windings which are main winding and auxiliary winding and these windings are placed at an angle 90. These windings are fed from separate ac supply

Fig.2 indicates the equivalent circuit of an asymmetrical TPIM in the stationary reference (αβ) frame

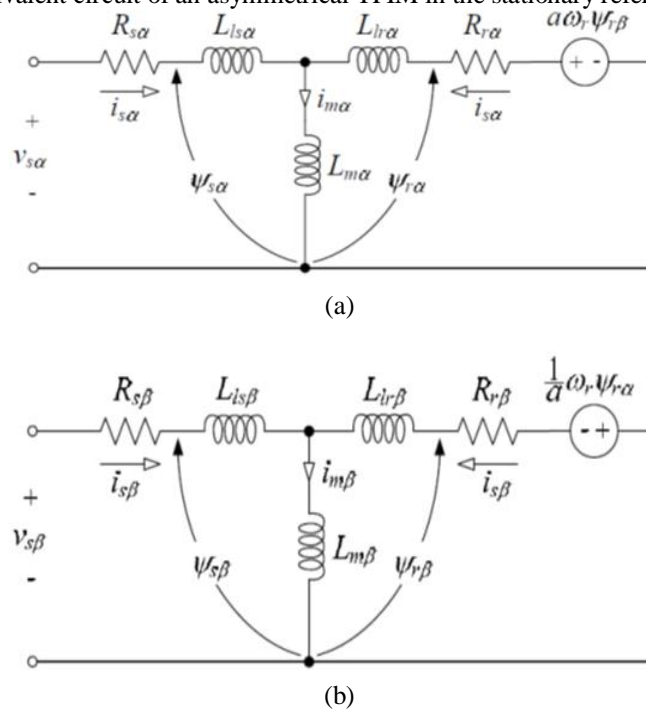


Fig 2. Equivalent circuit of an asymmetrical TPIM in stationary reference (αβ) frame (a). auxiliary winding in α-axis (b) main winding in β-axis

The basic stator and rotor voltage equations of the TPIM can be express as follows: equations (1)-(4)

$$v_{s\alpha} = R_{s\alpha} i_{s\alpha} + \frac{d}{dt} \psi_{s\alpha} \quad (1)$$

$$v_{s\beta} = R_{s\beta} i_{s\beta} + \frac{d}{dt} \psi_{s\beta} \quad (2)$$

$$v_{r\alpha} = 0 = R_{r\alpha} i_{r\alpha} + \frac{d}{dt} \psi_{r\alpha} + a \omega_r \psi_{r\beta} \quad (3)$$

$$v_{r\beta} = 0 = R_{r\beta} i_{r\beta} + \frac{d}{dt} \psi_{r\beta} - \frac{1}{a} \omega_r \psi_{r\alpha} \quad (4)$$

Where  $v_{s\alpha}, v_{s\beta}$  are the stator voltages in  $\alpha$ & $\beta$  frames  $v_{r\alpha}, v_{r\beta}$  are the rotor voltages in  $\alpha$ & $\beta$  frames.  $\psi_{s\alpha}, \psi_{s\beta}, \psi_{r\alpha}, \psi_{r\beta}$  are the stator and rotor fluxes in  $\alpha$ & $\beta$  frames.  $i_{s\alpha}, i_{s\beta}, i_{r\alpha}, i_{r\beta}$  are the stator and rotor currents in  $\alpha$ - $\beta$  frames. 'a' is  $\alpha$  to  $\beta$  frame turns ratio and here it is taken as  $a=1$ .

The stator and rotor flux component equations can be expressed as follows:

$$\psi_{s\alpha} = L_{s\alpha} i_{s\alpha} + L_{m\alpha} i_{r\alpha} \quad (5)$$

$$\psi_{s\beta} = L_{s\beta} i_{s\beta} + L_{m\beta} i_{r\beta} \quad (6)$$

$$\psi_{r\alpha} = L_{m\alpha} i_{s\alpha} + L_{r\alpha} i_{r\alpha} \quad (7)$$

$$\psi_{r\beta} = L_{m\beta} i_{s\beta} + L_{r\beta} i_{r\beta} \quad (8)$$

where  $\psi_{s\alpha}, \psi_{s\beta}, \psi_{r\alpha}, \psi_{r\beta}$  are the stator and rotor fluxes in  $\alpha$ & $\beta$  frames.  $L_{s\alpha}, L_{s\beta}, L_{r\alpha}, L_{r\beta}$  are the inductance of the stator and rotor in  $\alpha$ - $\beta$  frames.  $L_{m\alpha}, L_{m\beta}$  are the mutual inductance of stator and rotor in  $\alpha\beta$  frame

The electro-magnetic torque equations can be expressed as equation (9) and also mechanical dynamics can be expressed as equation (10)

$$T_e = p_p (L_{m\beta} i_{s\beta} i_{r\alpha} - L_{m\alpha} i_{s\alpha} i_{r\beta}) \quad (9)$$

$$J \frac{d}{dt} \omega_r = T_e - T_L \quad (10)$$

where  $\omega_r$  is the machine speed, 'J' is the moment of inertia  $p_p$  is the machine pole pair,  $T_e, T_L$  are the electromagnetic and load torque

### III. Sliding Mode Control

Sliding mode control [11] is a popular approach to the robust control of uncertain systems. The principal goal of the sliding mode control technique is to force a system state to a certain prescribed surface known as the sliding surface. Once the manifold is reached, the system is forced to remain on it thereafter. The main drawback of the sliding mode control is the requirement of a discontinuous control law across the sliding manifold and this will lead to an undesirable phenomenon called chattering. Chattering can be reduced by various methods.

#### A. Sliding mode controller

In the controller design we have to find the feedback control law to verify the sliding condition and also for the desired tracking performance a discontinuous control law has to be found. The design of sliding mode controller for the stability and desired performance consists of three major steps.

1. Choice of Sliding surface
2. The establishment of the existence of convergence conditions.
3. The determination of the control law

#### 1. Choice of Sliding surface

The choice of the sliding surface includes not only the necessary number of these surfaces but also their shape, depending on the application and purpose. Generally, for a system defined by the following state equation

$$\begin{aligned} \dot{x}(t) &= f(x, t) + g(x, t)u(t) \\ y &= Cx \end{aligned} \quad (11)$$

We have to choose "m" sliding surfaces for a vector y of dimension "m". As regarding the shape of the surface, there are two possibilities; either in the phase plane or in the state space. In this case, we find the so-called "switching law by against state reaction. JJSlotine offers a form of following general equation [12] to determine the surface slip which ensures the convergence of a variable to the desired value

$$S(x) = \left( \frac{\partial}{\partial t} + \lambda_x \right)^{r-1} e(x) \quad (12)$$

Where  $e(x)$  is the error signal which shows the speed difference between the actual speed and the desired speed,  $\lambda_x$  is a positive constant that interprets the bandwidth of the desired control, 'r' is the relative degree equal to the number of times it derives the output to display the order.

### 2. Establishment of convergent condition

Convergence conditions allow the system dynamics converge to sliding surfaces. For that we have direct function switching approach which is proposed and studied by Utkin. This approach is to give the surface a convergent dynamics towards zero. It is given by [11]:

$$\dot{s}(x) \cdot s(x) < 0 \quad (13)$$

We also have Lyapunov function approach and which is typically used to ensure the stability of non linear system.

By defining the the Lyapunov function by

$$v(x) = \frac{1}{2} s^2(x) \quad (14)$$

And its derivative with:

$$\dot{v}(x) = s(x) \dot{s}(x) \quad (15)$$

When the Lyapunov function decreases which simply ensure that its derivative is negative. This is verified if

$$\dot{s}(x) s(x) < 0 \quad (16)$$

This equation shows that the square of the distance to the surface, measured by  $s^2(x)$  decreases all the time, forcing the trajectory of the system to move towards the surface of both sides. This condition assumes an ideal sliding mode.

### 3. Determination of control law.

The control law determination procedure mainly consist of two steps. First, a feedback control law 'u' is selected to verify sliding condition. The central assumption in the design of variable structure systems controlled by the sliding mode is that the command should switch between  $u_{max}$  and  $u_{min}$  instantaneously, depending on the sign of the sliding surface. The imperfections in the command leads to an undesirable phenomenon called 'chattering'. Thus, in a second step, the discontinuous control law 'u' is suitably smoothed to achieve an optimal trade-off between control bandwidth and tracking precision. The first step achieves robustness for parametric uncertainty, the second step achieves robustness to high frequency unmodeled dynamics. Therefore, the structure of a controller has two parts; a first on the exact linearization and a second stabilizing last. Thus the control law can be represented as follows:

$$u(t) = u_{eq}(t) + u_N \quad (17)$$

$u_{eq}(t)$  is the equivalent command proposed by Filipov and Utkin [11]. This is considered as the most direct and simplest. It is calculated recognizing that the behavior of the system during the drag mode is described by

$$\dot{s}(x) = 0$$

' $u_N$ ' is the term introduced to satisfy the convergence condition  $s(x)\dot{s}(x) < 0$ . It determines the dynamic behavior of the system during the convergence mode, so to ensure the attractiveness of the variable to be controlled to the sliding surface and is given by:  $\dot{s}(x) = u_N$ .

#### 4. Chattering Reduction

For reducing the undesirable phenomenon called chattering we have an effective method called 'State Dependent Gain method' and which is used here for the chattering reduction. This is an alternative way to suppress chattering without designing an additional dynamic subsystem. The magnitude of chattering is proportional to the switching gain 'M'. Thus, the idea is to reduce the value of M to decrease the amplitude of chattering preserving the existence of sliding mode. To support this idea, a state-dependent gain method is proposed.

The proposed sliding mode controller with a state dependent gain ' $M(x)$ ' for the system described in equation (11) is as follows:

$$u = -M \text{sign}(\mu)$$

$$M = M(x_1) = M_0(|x_1| + \mu) \quad (18)$$

Where ' $M_0$ ' is the positive constant value, and ' $\mu$ ' is sufficiently small, positive constant. Note that the gain M is not fixed value but a function of the state  $x_1$ . The constant ' $M_0$ ' should be selected to force a sliding mode to occur along the switching surface.

The State dependent gain method for the tracking performance, following controller may be suggested using the state dependent gain method to substitute a conventional sliding mode controller as follows:

$$e = x_1 - x_d$$

$$\mu = \chi e + \dot{e} \quad (19)$$

$$u = -\{M_0(|e| + \varepsilon) + |a_1 x_d|\} \text{sign}(\mu)$$

where ' $x_d$ ' is the desired constant value of ' $x$ ' which is speed here, and ' $\varepsilon$ ' is sufficiently small. ' $M_0$ ' is a constant, which should be large enough to force a sliding mode, and here the gain ' $M$ ' depends on the error ' $e$ '

### IV. Simulink Modelling

SIMULINK modelling for the speed control of TPIM using Sliding Mode Controller consist of modelling of TPIM and Modelling of Sliding Mode Controller

#### B. Modelling Of TPIM

In the modelling of TPIM all the electrical quantities are expressed in a fixed reference frame linked to the stator called ' $\alpha\beta$ ' model and is given by

$$\dot{x} = f(x) + g(x)u(t) \quad (20)$$

With

$$u = \begin{bmatrix} v_{s\alpha} & v_{s\beta} \end{bmatrix}^T$$

$$x = \begin{bmatrix} i_{s\alpha} & i_{s\beta} & \psi_{r\alpha} & \psi_{r\beta} & \Omega_r \end{bmatrix} \quad (21)$$

$$= \begin{bmatrix} x_1 & x_2 & x_3 & x_4 & x_5 \end{bmatrix}$$

The variables ' $x$ ' are consist of two electrical states ( $i_{s\alpha}$  &  $i_{s\beta}$ ) and two magnetic states ( $\psi_{r\alpha}$  &  $\psi_{r\beta}$ ) and a state representing rotational speed of the rotor ' $\Omega_r$ '

' $f$  &  $g$ ' are the functions of ' $x$ '

$$\begin{aligned} \dot{x}_1 &= -\lambda x_1 + \frac{\Gamma}{T_r} x_3 + p\Gamma x_4 x_5 + \delta v_{s\alpha} \\ \dot{x}_2 &= -\lambda x_2 + \frac{\Gamma}{T_r} x_4 - p\Gamma x_5 x_3 + \delta v_{s\beta} \quad (22) \\ \dot{x}_3 &= \frac{M}{T_r} x_1 - \frac{1}{T_r} x_3 - p x_4 x_5 \\ \dot{x}_4 &= \frac{M}{T_r} x_2 - \frac{1}{T_r} x_4 + p x_3 x_5 \\ \dot{x}_5 &= \eta (x_2 x_3 - x_1 x_4) - \frac{C_r}{J} \end{aligned}$$

With

$$\begin{aligned} \lambda &= \frac{R_s}{\sigma L_s} + \frac{1}{T_r} \left( \frac{1-\sigma}{\sigma} \right) \\ \Gamma &= \frac{1-\sigma}{\sigma} \cdot \frac{1}{M} \quad (23) \\ \delta &= \frac{1}{\sigma L_s} \\ \eta &= \frac{pM}{J L_r} \end{aligned}$$

### C. Modelling Of Controller

Sliding mode controller modelling consist of 3 steps

#### 1. Sliding Surface design

The choice of sliding surface can be done in following way

**Speed  $\Omega_r$**

$$S_1 = K_1 \cdot (\Omega_r - \Omega_{rref}) + \left( \dot{\Omega}_r - \dot{\Omega}_{rref} \right) \quad (24)$$

If we introduce the velocity error,

$$\varepsilon_\Omega = \Omega_r - \Omega_{rref}$$

Then equation (24) can be written as

$$S_1 = K_1 \cdot \varepsilon_\Omega + \dot{\varepsilon}_\Omega \quad (25)$$

**Flux  $\psi_r$**

$$S_2 = K_2 \cdot (\psi_r - \psi_{rref}) + \left( \dot{\psi}_r - \dot{\psi}_{rref} \right) \quad (26)$$

With an error

$$\varepsilon_{\psi_r} = \psi_r - \psi_{rref}$$

Then the equation (25) can be written as

$$S_2 = K_2 \cdot \varepsilon_{\psi_r} + \dot{\varepsilon}_{\psi_r} \quad (27)$$

#### 2. Convergent condition

For selected variables converge to their reference values, both sliding surface must be zero

$$S_1 = \frac{d}{dt} (\Omega_r - \Omega_{rref}) + K_1 (\Omega_r - \Omega_{rref}) = 0$$

$$S_2 = \frac{d}{dt}(\psi_r - \psi_{rref}) + K_2(\psi_r - \psi_{rref}) = 0 \quad (28)$$

### 3. Sliding mode control law

Our goal is to generate a control law  $[V_{s\alpha}, V_{s\beta}]$  using the theory sliding mode control. The states considered for induction motor control are: Speed  $(\Omega_r)$  and rotor flux  $(\psi_r)$ .

For the convenience we have

$$\Psi_r = \Psi_{r\alpha}^2 + \Psi_{r\beta}^2 = x_3^2 + x_4^2 \quad (29)$$

The corresponding derivatives are

$$\dot{\Psi}_r = 2 \cdot x_3 \cdot \dot{x}_3 + 2 \cdot x_4 \cdot \dot{x}_4 \quad (30)$$

Sliding surface will be

$$S_1 = K_1 \cdot (x_5 - \Omega_{rref}) + (\dot{x}_5 - \dot{\Omega}_{rref}) \quad (31)$$

$$S_2 = K_2 \cdot (\Psi_r - \Psi_{rref}) + (\dot{\Psi}_r - \dot{\Psi}_{rref}) \quad (32)$$

The successive derivatives of the  $S_1$  and  $S_2$  will be

$$\begin{aligned} \dot{S}_1 &= K_1 \cdot (\dot{x}_5 - \dot{\Omega}_{rref}) + (\ddot{x}_5 - \ddot{\Omega}_{rref}) \\ \dot{S}_2 &= K_2 \cdot (\dot{\Psi}_r - \dot{\Psi}_{rref}) + (\ddot{\Psi}_r - \ddot{\Psi}_{rref}) \end{aligned} \quad (33)$$

By finding the derivatives of the equation (22) and substitute those values in equation (33) and find the control law for the speed control of induction motor.

## V. Simulation And Results

To verify the effectiveness and the speed control capability of the designed sliding mode controller, numerical simulation are conducted with step time of 1.5 sec.

The parameters and the data sheet of TPIM are given in Table 1

| PARAMETERS OF TPIM |                        |       |             |
|--------------------|------------------------|-------|-------------|
| H.P                | 0.25                   | $V_s$ | 110V        |
| Frequency (f)      | 60Hz                   |       |             |
| $R_r$              | 6.3 $\Omega$           | $R_s$ | 10 $\Omega$ |
| $L_s$              | 0.4612 H               | $L_r$ | 0.4612 H    |
| $M$                | 0.4212H                | $L_M$ | 0.4212 H    |
| $J$                | 0.02 Kg.m <sup>2</sup> | $p_p$ | 2           |

Table 1. TPIM Parameters

The controller gains are taken as  $K_1=300$  &  $K_2=600$ . For the simulation process the initial condition is taken as zero and the tracking performances are verified.

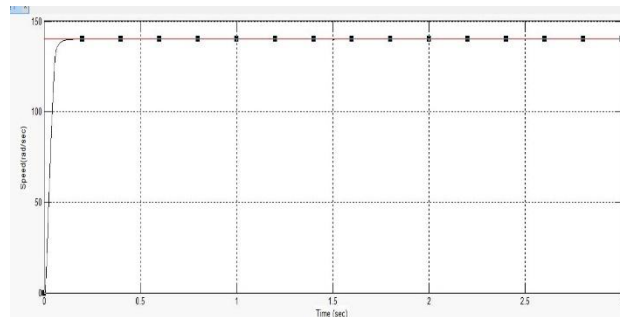


Fig 3 . Speed Graph

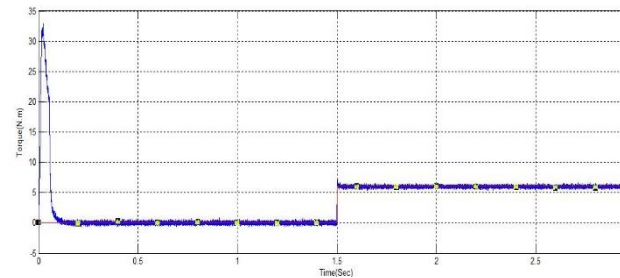


Fig 4 .Electromagnetic Torque

The simulation results shows that, by using the sliding mode controller the speed response shows the perfect tracking for different speed. Here the speed response shows the perfect tracking at the set speed of 140 rad/sec. The speed response follows the set speed at the time of 0.14 sec.

## VI. Conclusions

In order to get the perfect speed control for the induction motor, the sliding mode controller can be use as perfect robust control technique. By proper designing of the controller the speed control can be done effectively with in minimum time.

The simulation results show a robust performance of the designed controller with respect to the disturbances caused by the load torque. Moreover, the proposed controller ensures the perfect speed tracking performance.

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