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Further Discussion for Chainwise Paired Comparisons

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Abstract: *We study the paper of Ra that was published in Decision Sciences. He tried to develop a new method to simplify the paired comparison for Analytic Hierarchy Process. We show that for a three by three comparison matrix, the normalized relative weights derived by the chainwise paired comparison proposed by Ra is the same as the normalized results of the row geometric mean method. Our results provide a framework for further analytical explanation for the chainwise paired comparisons. On the other hand, we provide an example in which Ra's method comes up with questionable outcomes. Researchers who use Ra's method should be aware of this discrepancy.*

Key Word*: Multicriteria Decision Making; AHP; Measure of Consistence.*

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I. Introduction

The Analytical Hierarchy Process (AHP) was developed by Saaty [17] as a decision making aid. It has been successfully applied in many fields. Zanakis et al. [22] examined over 100 applications of AHP within the service and government sectors, some researchers still dispute for its suitability and completeness. For example, just to cite a few, Apostolou and Hassell [2] studied whether comparison matrices with consistent ratio (CR) >0.1 could be accepted. Bernhard and Canada [4] recommended that the incremental benefit/cost ratios should be compared with a cutoff ratio instead of the benefit/cost ratios of Saaty [18, 19]. Finan and Hurley [11] developed a diagonal procedure to build a rank-order consistent matrix. On the other hand, some researchers have tried to revise those improvements. For example, Chu and Liu [10] explained problems in Apostolou and Hassell [2]. Yang et al. [21] showed that the method of Bernhard and Canada [4] was deficient and then revised it. Chao et al. [6] reported that the diagonal procedure of Finan and Hurley [11] did not pass the consistent test of Saaty [18]. Following this trend, we consider the paper of Ra [16] to create a new approach to simplify the computation in AHP.

AHP is particularly appropriate for complex judgments, which contain the comparison of decision alternatives that are difficult to quantify. Hence, it is built on the ground that when faced with a complicated decision problem the natural human response is to gather the decision alternatives according to their common features. It includes construction of a hierarchy of decision criteria and then making comparisons between each possible pair with respect to criteria to derive relative weight for alternatives. Pairwise comparison is commonly used to estimate preference values of finite alternatives with respect to a given criterion.

There are $n(n-1)/2$ comparisons which need to be decided by a decision maker for n alternatives. Ra [16] considered that that might be a time consuming procedure for problems with large numbers of decisions. For example, in Ra's paper, an AHP problem with a hierarchy consisting of five levels, where each level except the top has 10 elements (criteria and sub-criteria for those median level, and alternatives for the bottom level) and the top level has one element (goal) requires a total of 1395 paired comparisons for a decision maker to derive the final results. There are several authors who tried to simplify the pairwise comparison procedure. For example, Macharis et al. [15] considered using the first row of a comparison matrix to develop a consistent matrix with the desired values for several specified entries. However, Jung et al. [12] pointed out that the procedure of Macharis et al. [15] contained questionable results. Ra [16] suggested a simplified procedure, with his notation:*chainwise comparisons*, such that there are only n comparisons that need to be decided by a decision maker for n alternatives. Hence, the 1395-paired comparisons in the previous example would reduce to 310-paired comparisons.

Ra [16] used D_i and I_i to decide whether ordinal consistency is violated or not. He used the upper bound of ordinally inconsistency sets as the lower bound for the ordinally consistency and the lower bound of ordinally consistency sets as the upper bound for the ordinally inconsistency. He tested the all-possible sixty cases for the Saaty's example of nation's wealth (Saaty [18], p.40) to show that all the eigenvector solutions for the sixty chain orientations implied identical weights obtained through the chainwise method. Hence, Ra [16] claimed that chainwise comparisons may take the place of complete pairwise comparisons in AHP, to significantly reduce the number of paired comparisons. Up to now, there have been six papers, namely Sirola [20], Chen and Lin [8], Choo and Wedley [9], Kuchanov et al. [14], Chen [7], and Kim et al. [13] that have cited Ra [16] in their references. Owing to this high citation rate, it may be worthwhile to provide a deep examination of his new method. Sirola [20] used conceptual decision model in a case study to utilize rule-based procedures, numerical algorithms and procedures, statistical methodologies and visual support. Chen and Lin [8] used analytic hierarchy process to derive the multifunctional knowledge and the teamwork capability of team members and then applied Myers-Briggs type indicator to assess the working relationship model. Choo and Wedley [9] discussed 18 estimating methods for deriving preference values from pairwise judgment matrices under a common framework of distance minimization and correctness in error free cases. They recommend the simple geometric mean method and the simple normalized column sum method to have closed-form formulas for easy calculation and good performance. Kuchanov et al. [14] discussed the theory of poly-condensation. They considered various models and the methods of their solution for the calculation of the statistical characteristics of the chemical structure of polymers to analysis them in detail. Chen [7] developed an integrated methodological framework for project task coordination and team organization from the concurrent engineering perspective in order to assign the right team members to the right tasks. Kim et al. [13] proposed a customer-oriented evaluating method for rating in house of quality. The 1-9 scaling pairwise comparison (Saaty [18]) is not suitable for customers such as non-experts, housewives, or even children such that they avoided the phenomenon of inconsistency. However, none of these six papers provided further discussions for the groundwork for the theoretical development of chainwise paired comparisons approach proposed by Ra [16]. Hence, in this note, we will first provide an explanation for Ra’s method for three by three comparison matrix to show that his result will coincide with the geometric mean method (Barzilai et al. [3], Brugha [5], and Aguarón and Moreno-Jiménez [1]).

II. Review Of The ChainwisePaired Comparison

We review the chainwise paired comparison proposed by Ra [16]. First, he defined the direct comparison, D_i, as the relative weight for alternative i to alternative $i+1$ for $i=1,2,\cdots,n-1$ and D_n as the relative weight for alternative n to alternative 1. This means that for a $n\times n$ comparison matrix, say $(a_{ij})_{n\times n}$, Ra [16] only constructs $a_{i,i+1}$ for $i=1,2,\cdots,n-1$ and $a_{n,1}$ where $D_i=a_{i,i+1}$ for $i=1,2,\cdots,n-1$ and $D_n=a_{n,1}$. Secondly, he defined the indirect comparison, I_i, as the value computed from the other values, that is, the reciprocal of the product of other $D_i s$. For example, $I_1=1/(D_2D_3\cdots D_n)$ or we may rewrite $I_1=D_1\Big/\prod_{i=1}^{n}D_i$. Hence, Ra [16] assumed that $I_i=D_i\Big/\prod_{i=1}^{n}D_i$ for $i=1,2,\cdots,n$. Thirdly, to derive the best estimation, denoted by $\tilde{R}_i$ from two different values of D_i and I_i, he proposed the weighted geometric mean $\tilde{R}_i=D_i^{(n-1)/n}I_i^{1/n}=D_i\Bigg/\left(\prod_{j=1}^{n}D_j\right)^{1/n}$ for $i=1,2,\cdots,n$.

We may say that $\tilde{R}_i$ is the normalization of D_i by multiplication operation. Fourthly, for the relative weights, he assumed that $M_n=1$ and the other elements are computed in descending order from $(n-1)$ to the first element by $M_i=\tilde{R}_iM_{i+1}=\prod_{j=i}^{n-1}\tilde{R}_j$ for $i=n-1,\cdots,1$. We may say that M_i is the relative weight between alternative i and alternative n through R_i, $R_{i+1,}$ $\cdots$, and R_{n-1}. Fifthly, he defined the normalized relative weights, V_i, with $V_i=M_i\Big/\sum_{j=1}^{n}M_j$. Ra [16] did not offer additional clarification why he constructed V_i by the previous five steps. In the consequent six papers as we mentioned before, no one provided further explanations.

III. Our Explanation For The ChainwisePaired Comparisons

In this section, we will show that for a three by three comparison matrix, say $\left(a_{i\,j}\right)_{3\times 3}$, then the normalized relative weights, V_i, correspond to the relative weights derived by the row geometric mean method. First, we list the results from the row geometric mean method, say $\left(B_1, B_2, B_3\right)^T$ with $B_1 = \left(a_{12}a_{13}\right)^{1/3}$, $B_2 = \left(a_{21}a_{23}\right)^{1/3}$ and $B_3 = \left(a_{31}a_{32}\right)^{1/3}$. Next, we follow the chainwise paired comparison proposed by Ra [16] to find that $D_1 = a_{12}$, $D_2 = a_{23}$ and $D_3 = a_{31}$.

To simplify the expression, we assume that $\Delta = \left(\prod_{j=1}^{3} D_j\right)^{1/3}$ and $\Omega = \left(a_{13}a_{23}\right)^{1/3}$. This yields that $\tilde{R}_i = \dfrac{D_i}{\Delta}$ for $i = 1, 2, 3$, and then $M_3 = 1$, $M_2 = \tilde{R}_2$ and $M_1 = \tilde{R}_1\tilde{R}_2$.

We obtain that

$$M_1 = \frac{a_{12}a_{23}}{\Delta^2} = \left(a_{12}a_{23}a_{13}^2\right)^{1/3} = \Omega B_1, \qquad (1)$$

$$M_2 = \frac{a_{23}}{\Delta} = \left(a_{21}a_{13}a_{23}^2\right)^{1/3} = \Omega B_2, \qquad (2)$$

and

$$M_3 = 1 = \Omega B_3. \qquad (3)$$

This implies that the weights derived by the row geometric mean method, say $\left(B_1, B_2, B_3\right)^T$ and the relative weights by the chainwise paired comparison proposed by Ra [16], say $\left(M_1, M_2, M_3\right)^T$ become by Equations (1), (2) and (3)

$$\left(M_1, M_2, M_3\right)^T = \Omega\left(B_1, B_2, B_3\right)^T. \qquad (4)$$

It follows that the normalized relative weights, V_i, satisfies

$$V_i = B_i \Big/ \sum_{j=1}^{3} B_j \qquad (5)$$

for $i = 1, 2, 3$. We summarize our findings in the next theorem.

Theorem1. For a three by three comparison matrix, the normalized relative weights derived by the chainwise paired comparison proposed by Ra [16] is the normalized results of the row geometric mean method.

IV. Inherent ProblemsOf Ra's Method

We will provide an example to explain inherent problems in Ra's method. First, we recall the six nation's wealth (Saaty [18], p. 40). By Saaty's eigenvector method or the geometric mean method, the US has the highest weight. By his method, Ra computed all sixty possible arrangements ($\dfrac{6!}{6\cdot 2} = 60$, since $6!$ for all permutation; divide by 6 for circular permutation, for example $(1,2,3,4,5,6)$ and $(2,3,4,5,6,1)$; divide by 2 for flipped over, for example $(1,2,3,4,5,6)$ and $(6,5,4,3,2,1)$. In 56 of the 60, the US still has the highest weight, but in the other 4 cases, the USSR has the highest weight.

This reveals two important features of Ra's method:

(a) Sometimes, by Ra's method, the alternative with the highest weight does not coincide with the Saaty's eigenvector method.
(b) We may need to check all possible arrangements to find the alternative with the highest weight.

Here, we assume a four by four comparison matrix, say A, for alternatives in the following order A_1, A_2, A_3 and A_4, with

$$A=\begin{bmatrix} 1 & 2 & 1/5 & 1/9 \\ 1/2 & 1 & 3 & 1/7 \\ 5 & 1/3 & 1 & 4 \\ 9 & 7 & 1/4 & 1 \end{bmatrix}, \tag{6}$$

such that by Saaty's eigenvector method, $\lambda_{\max}=6.833$ and the normalized relative weight is $(0.085, 0.194, 0.343, 0.378)$ so A_4 has the highest weight.

There are 3 different cases by Ra's method, since $\dfrac{4!}{4\cdot 2}=3$ as we explained early. In the original order, by Ra's method, we imply that

$$(M_1, M_2, M_3, M_4)=(0.426, 0.816, 1.043, 1). \tag{7}$$

By Equation (7), in the arrangement of A_1, A_2, A_3 and A_4, Ra's method implies that A_3 has the highest weight.

Next, we consider the arrangement of A_1, A_2, A_4 and A_3, which means the comparison, say B, becomes

$$B=\begin{bmatrix} 1 & 2 & 1/9 & 1/5 \\ 1/2 & 1 & 1/7 & 3 \\ 9 & 7 & 1 & 1/4 \\ 5 & 1/3 & 4 & 1 \end{bmatrix} \tag{8}$$

which means that in B (a) the third and the fourth rows of A are interchanged and (b) then the third and the fourth columns of the result of (a). By Ra's method, we derive that

$$(M_1, M_2, M_3, M_4)=(0.155, 0.060, 0.323, 1). \tag{9}$$

By Equation (9), in the arrangement of A_1, A_2, A_4 and A_3, Ra's method implies that A_3 still has the highest weight.

Lastly, we examine the arrangement of A_1, A_3, A_2 and A_4, which means the comparison, say C, becomes

$$C=\begin{bmatrix} 1 & 1/5 & 2 & 1/9 \\ 5 & 1 & 1/3 & 4 \\ 1/2 & 3 & 1 & 1/7 \\ 9 & 1/4 & 7 & 1 \end{bmatrix} \tag{10}$$

which means that in C (a) the second and the third rows of A are interchanged and (b) then the second and the third columns of the result of (a). By Ra's method, we obtain that

$$(M_1, M_2, M_3, M_4)=(0.060, 0.163, 0.264, 1). \tag{11}$$

By Equation (11), in the arrangement of A_1, A_3, A_2 and A_4, Ra's method implies that A_4 has the highest weight.

If we compute all possible arrangements by Ra's method, then it yields that for 67%, A_3 has the highest weight and for 33%, A_4 has the highest weight.

Hence, by Ra's method we summarize our findings in the next observations.

(i) If we only apply Ra's method once, then for comparison matrices A and B, we cannot find A_4 has the highest weight.

(ii) If we compute all possible cases, then the percentages for A_3 and A_4 having the highest weight are 67% and 33%, respectively. Hence, most researchers will accept that A_3 has the highest weight.

Here, we point out inherent problems by Ra's method.

(a) If we only consider some possible arrangements, then we may not derive the best alternative as proposed by Saaty's eigenvector method.

(b) If we evaluate all possible arrangements, then the alternative with the most percentage may not be the best alternative.

Owing to these two inherent problems by Ra's method, we may advise researchers do not apply this new algorithm to avoid questionable results.

V. Conclusion

Our findings provide a theoretical ground for the chainwise paired comparison of Ra [16] so that more researchers can confidently use this new technique to avoid the time consuming procedure required to construct the complete set of comparison matrices. However, our example reveals two inherent problems of Ra's method which seems to indicate that this new approach sometimes cannot decide the best alternative as proposed by Saaty's eigenvector method.

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