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A Study Of Proposed Algebra Problem-Analysis

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Abstract
The National Mathematics Advisory Panel (2008) states that algebra is a gateway to high school graduation and college success. While existing research emphasizes the importance of quality algebra instruction, the current body of research on algebra problem-analysis for struggling secondary students is small. This paper proposes a problem-solving model to help support those students struggling with algebra. The model integrates the recommendations from math policy boards and research. It is composed of five core sections, each focusing on a specific critical component of school algebra. The study examines the relationship between the five skills within the model and an established measure of algebra, as well as the validity of the measures being used to assess the different skill areas. The results indicate that there is a significant relationship between the five sections of the model and algebra proficiency and that the model is able to identify non-proficiency students with a high degree of accuracy.

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I. Introduction

Mathematics is a fundamental skill that is required to successfully maneuver through adult life. The expectations regarding the type of mathematics skills that students are expected to possess by the time they leave school are changing and are very different from just a few decades ago: students are expected to not only be able to compute numbers but also reason and think mathematically (National Research Council, 2001). The different areas of math vary, but they are all important. Recently, much focus has been shifted towards algebra, requiring students to have some level of proficiency prior to graduation (National Mathematics Advisory Panel, 2008).

Statement of the Problem

The National Mathematics Advisory Panel (2008) states that algebra is a gateway to later achievement and that there is a strong correlation between the completion of algebra II and the likelihood of college success and college graduation. Students who complete algebra II are also more likely to have higher college GPAs, graduate from college, and have higher earnings in later life (Gaertner, Kim, DesJardins, & McClarty, 2014). There is a call for algebra proficiency at the high school level as well. At the present time, a number of states require that students take and pass at least one algebra course while in high school (American Diploma Project Network, 2009), making algebra a gateway to not just college but to high school graduation (Stein, Kaufman, Sherman, & Hillen, 2011). To meet this demand, schools are requiring that students take algebra in eighth grade or earlier. Earlier exposure to content can be beneficial to some students (Rickles, 2013), but for others it may prove to be harmful (Stein, Kaufman, Sherman, & Hillen, 2011). Requiring students to take higher-level math courses without adequate preparation can result in higher failure rates and lower grades in both their current class and future classes (Allensworth, Nomi, Montgomery, & Lee, 2009; Domina, 2014; Clotfelter, 2012). Putting struggling students in algebra classes with higher-performing students might impact the performance of both higher- and lower-performing students (Nomi, 2012).

The educational and professional systems in the U.S.A. . are designed in such a way that they require all students to possess an operating knowledge of algebra, but present-day academic tendencies indicate that this goal continues to be a ways off. On the national evaluation of tutorial progress, an assessment supposed to represent the average U.S. scholar's success in one-of-a-kind educational areas, the average rating for a twelfth grade pupil on the mathematics test's algebra scale became a hundred and fifty-five, which falls substantially below the anticipated talent score of 176 and suggests the most effective basic understanding of algebra and its packages and processes (countrywide center for schooling facts [NCES], 2014). There are many potential causes of the gaps in fulfillment mentioned above; however, what remains clear is that a massive number of college students are struggling with algebra.

Cutting-edge algebra intervention techniques are reflective of traditional algebra strategies. training; the interventions are oriented towards the student's modern-day curriculum and classwide getting to know desires and recognition on strategies to teach new and hard principles, however, does no longer comprise a proof-based problem-solving framework for talent acquisition and studying. Before identifying the troubles with the interventions focusing basically on present-day curriculum, it has to be noted that, at the present time, there is not a complete review of curriculum-primarily based as opposed to hassle-solving-based algebra interventions, so an evaluation of the effectiveness between the two is not viable. With that in mind, this paper posits that there are three primary issues with focusing totally on algebra curriculum coaching strategies.

Study Purpose

The need for algebra research is paramount because achievement data indicates our students continue struggling with algebra, but there is not a well-developed problem-analysis model for intervening with struggling students. Direct instructional activities and guidelines exist for a number of the content areas within algebra (Foegen, 2008; NMAP, 2008), but academic interventions are more successful if they directly address the student deficit (Burns, VanDerHeyden, & Boice, 2008). As of now, there are no clear guidelines about how to assess for specific skill and knowledge-based deficits and then select appropriate, evidence-based interventions.

II. Materiel and Method

The study was conducted between 2022 and 2024 at the Department of Mathematics at Malwanchal University, Indore, MP. The rationale behind this study is to propose a model that utilizes an evidence-based problem-analysis model to identify specific skill and knowledge deficits related to algebra and to develop an assessment that effectively assesses a student's performance in each of the target areas.

Symbols and Expressions

The first major topic of school algebra is that of symbols and expressions. Symbols are items or signs that represent something else (i.e., a value or a process), and they are used to make statements about things. Symbols include basic expressions like + to mean addition and - to mean subtraction, as well as more complex symbols like Σ, representing sigma and indicating a summation of all the values within a given series (Van De Walle, Karp, & Bay-Williams, 2013). An expression refers to a mathematical phrase made up of a finite number of symbols that can include constants, variables, operations, functions, or a number of other symbols (Kaufman & Schwitters, 2004). The National Research Council (2008) identifies three different skills under the heading of symbols and expressions that students should know: polynomial expressions, rational expressions, and arithmetic and finite geometric sequences. Polynomial expressions include real numbers and variables; more than one term; can only include addition, subtraction, and multiplication; and include exponents. Rational expressions (or fractional expressions) occur when one polynomial is divided by another polynomial. Finally, in an arithmetic sequence, a number is found by adding a constant from one term to the next; in a geometric sequence, each number is found by multiplying a constant by the previous term (NRC, 2008).

Statistical Analyses

Question 1: Relationship between the proposed model and algebra. The data for question four was analyzed using correlations between each skill, subskill, and the MAP-M algebra RIT scores. The internal consistency of the measures was all reported.

Question 2: Proposed five-factor structure. The data for question three was analyzed using confirmatory factor analysis to assess the theoretical nature of the algebra-learning construct. The factor analysis will help confirm the proposed factor dimensions of the model. Chi-square, Root Mean Square Error of Approximation (RMSEA), Comparative Fit Index (CFI), Tucker-Lewis Index (TLI), and Standard Root Square Mean Residuals (SRMR) were used to calculate goodness of fit. The reliability of the assessments will be measured using internal consistency.

Question 3: Diagnostic accuracy. The diagnostic accuracy was assessed by measuring the sensitivity and specificity of each skill against the MAP-Algebra Proficiency Score of 181. The sensitivity was measured by identifying students who scored below the proficiency MAP-Algebra score, and identifying the students above the proficiency score was used to measure the specificity. The diagnostic accuracy was calculated by measuring the agreement versus the disagreement between the cut score and diagnostic criteria (Stage & Jacobsen, 2003).

III. Results

Table 1: ***Characteristics of Subskill Assessments***		
	Number of Questions	**Time Limit**
Integers Ordering	6	1 minute
Rational Number Ordering	9	2 minutes
Integers Calculation	100	2 minutes
Rational number calculation	30	3 minutes
Integer Word Problems	6	3 minutes
Rational Number Word Problems	6	3 minutes
Patterns	9	3 minutes
Arithmetic-to-Algebra	24	3 minutes
Generalization	6	3 minutes
Proportional Reasoning	6	3 minutes
Vocabulary	9	3 minutes
Conceptual Understanding	6	3 minutes
Problem Solving	10	4 minutes
Authentic application survey	20	Unlimited

A total of 373 students participated in the testing, and 327 students were included in the final study. Of those participants, 42% were male and 58% were female. Less than 1% identify as American Indian or Alaskan Native, 11% identify as Asian or Pacific Islander, 16% identify as Hispanic, 23% identify as Black, and 50% identify as White. Of the total number of participants, 10% were receiving special education services, and 6% were receiving educational support through a 504 plan. The average age of the participants was 14.1, and the average grade was 8.5. The distribution of students among grades is as follows: 62 students (19%) in sixth grade, 59 students (18%) in seventh grade, 50 students (15%) in eighth grade, 51 students (16%) in ninth grade, 40 students (12%) in 10th grade, 39 students (12%) in 11th grade, and 26 students (8%) in 12th grade.

Table 2: Descriptive Table of Subskill Measures		
	Mean	**Standard Deviation**
Integers Ordering	5.65	0.94
Rational Number Ordering	4.41	2.12
Integers Calculation	40.81	17.19
Rational number calculation	8.78	3.71
Integer Word Problems	4.52	1.59
Rational Number Word Problems	2.54	1.38
Patterns	6.84	1.99
Arithmetic-to-Algebra	18.42	5.19
Generalization	3.35	2.06
Proportional Reasoning	2.93	1.79
Vocabulary	6.32	1.99
Conceptual Understanding	3.86	1.65
Problem Solving	5.20	2.34

Table 3:Descriptive Table of Skill Measures

Skill	**Mean**	**Standard Deviation**
Basic Skills	66.70	22.94
Algebraic Thinking	31.53	9.45
Content Knowledge	15.38	4.67

Table 4: ***Pearson Correlations Among the Basic Skills Subsections***

Integer			**Rational**		
Ordering	**Calculation**	**Word Problem**	**Ordering**	**Calculation**	**Word Problem**
(Order-I)	(Calc-I)	WP- I	Order-R	Calc-R	WP-R
Order: I 1.00*	.32*	.31*	.33*	.35*	.22*
Calc: I.32*	1.00*	.55*	.52*	.57*	.46*
WP-I.31*	.55*	1.00*	.61*	.50*	.56*
Order: R.33*	.52*	.61*	1.00*	.51*	.59*
Calc: R.35*	.57*	.50*	.51*	1.00*	.46*
WP-R.22*	.46*	.56*	.59*	.46*	1.00*
*Note: *p* <.01					

Table 5

Pearson Correlations Among the Algebraic Thinking Subsections

	Patterns	**Arithmetic to Algebra**	**Generalization**	**Proportional Reasoning**
Patterns	1.00*	.65*	.58*	.55*
Arithmetic to Algebra	.65*	1.00*	.62*	.60*
Generalization	.58*	.62*	1.00*	.62*
Proportional Reasoning	.55*	.60*	.62*	1.00*
*Note: *p* <.01				

Table 6: ***Pearson Correlations Among the Content Knowledge Subsections***

Vocabulary	**Conceptual**	**Problem Solving**
Vocabulary 1.00*	.40*	.43*
Conceptual .40*	1.00*	.38*
Problem Solving: 43*	.38*	1.00*
***Note: *p* <.01**		

Table 7:*Pearson Correlations Among the Sections*

Basic Skills	**Algebraic Thinking**	**Content Knowledge**
Basic Skills 1.00*	.76*	.54*
Algebraic Thinking (76*)	1.00*	.62*
Content Knowledge: 54	.62*	1.00*
*Note: *p* <.01		

MAPM criterion. Pearson correlations were calculated for the sections and the composite score of the test in relation to the Measures of Academic Progress (MAP) assessment. The correlations for the three sections and MAP test were as follows: Basic Skills r (325) =.71, p <.001; Algebraic Thinking r (325) =.76, p <.001; and Content Knowledge r (325) =.68, p <.001. The correlation between the MAP and the composite score was r (325) =.79, p <.001.

The correlation between the Engagement and Authentic Application sections, along with the total composite score and the MAP scores, can be found in Table 8. The correlations between the Authentic Application and Engagement sections are generally weaker than those found in the academic skills sections. The correlations within these sections range from weak to very strong. However, the correlation between the total sum score of the assessment and the MAP math test is very strong at.79.

Table 8: Pearson Correlations Among the Authentic Application, Engagement, MAP, and Total Scores

	Positive	**Negative**	**C**	**A**	**BC**	**BEP**	**D**	**MAP**	**Total**
Positive	1.00*	-0.78*	0.37*	0.48*	0.38*	0.42*	-0.31*	0.15*	0.24*
Negative	-0.78*	1.00*	-0.33*	-0.50*	-0.35*	-0.41*	0.35*	-0.18*	-0.23*

C	0.37*	-0.33*	1.00*	0.43*	0.51*	0.53*	-0.29*	0.09	0.13
A	0.48*	-0.50*	0.43*	1.00*	0.43*	0.60*	-0.39*	0.06	0.12
BC	0.38*	-0.35*	0.51*	0.43*	1.00*	0.55*	-0.42*	0.24*	0.22*
BEP	0.42*	-0.41*	0.53*	0.60*	0.55*	1.00*	-0.30*	0.12	0.17*
D	-0.31*	0.35*	-0.29*	-0.39*	-0.42*	-0.30*	1.00*	-0.04	-0.08
MAP	0.15*	-0.18*	0.09	0.06	0.24*	0.12	-0.04	1.00*	0.79*
Total	0.24*	-0.23*	0.13	0.12	0.22*	0.17*	-0.08	0.79*	1.00*

Note. Positive = Positive Mindset; Negative = Negative Mindset; C = Cognitive Engagement; A = Affective Engagement; BC = Behavioral Engagement Compliance; BEP = Behavioral Engagement Participation; D = Disengagement; MAP = Measures of Academic Progress Mathematics RIT Score; Total = Total Score for combined Basic Skills, Algebraic Thought, and Content Knowledge sections. $^*p < .01$

IV. Discussion

The purpose of the study was to examine evidence for the validity of a proposed problem-solving model for identifying skill deficits in students struggling with algebra. Three research questions guided the study. First, what is the relationship between each of the five sections within the problem-analysis model and an established measure of algebra? Second, to what extent does assessment data support the proposed five-factor structure? Third, to what extent do the five sections and each subsection within the problem-analysis model accurately identify the level of a student's difficulty with algebra as measured by a criterion.

The problem-analysis model has five skill sections, and within each skill section are different subskills. To review, the skills are Basic Skills, Algebraic Thinking, Content Knowledge, Engagement, and Authentic Application. The Basic Skills section contains the following subskills: comparing and ordering, calculation, and word problems. The Algebraic Thinking Skills section contains the following subskills: patterns, arithmetic to algebra, proportional reasoning, and generalization. The Content Knowledge Skills section contains the following subskills: problem solving, vocabulary, and conceptual understanding. The Authentic Application section contains the following subskills: positive mindset and negative mindset. The Engagement Skills section contains the following subskills: cognitive engagement, affective engagement, behavioral engagement, behavioral engagement participation, and disengagement.

These findings were consistent with the culminating literature on the relationship between prerequisite skills required for algebra proficiency (NMAP, 2008). Having an understanding of integer magnitude is the basis for calculation and word problems (Zaslavsky, 2001; Hallet, Nunes, Bryant, & Thorpe, 2012; NRC, 2001; Siegler et al., 2012). Understanding fraction magnitude is especially important, as it not only relates to calculation and word problems but also algebraic thinking and algebraic problem solving (Bailey, Hoard, Nugent, & Geary, 2012; Booth & Newton, 2012; Siegler, Thompson, & Schneider, 2011). Proficiency with word problems and calculation supports flexible application and algebraic thinking (O'Loughlin, 2007; Fuchs et al., 2012; NRC, 2001).

Algebraic thinking serves as the bridge between arithmetic and algebra problem solving (Warren & Cooper, 2008; Cai & Moyer, 2008; Ferrucci, Kaur, Carter, & Yeap, 2008). The understanding of variables and equivalence assists with the flexible use of arithmetical strategies (Stacey & MacGregor, 2000), which can be applied to solve patterns and functions (Kaput, Carraher, & Blanton, 2008; Kieran, 2004) and proportional reasoning (Özgün-Koca & Altay, 2009). Understanding generalization and equivalence promotes flexibility and generalization, which are critical to algebra problem solving (Banchoff, 2008; Kinach, 2014; Johanning, 2004).

Directions for Future Research

Future research should focus on targeted interventions for the different related skills and subskills required for proficiency, with the goal of generalizing skills across algebra. There is a substantial body of research on improving the basic arithmetic skills of struggling students through intervention, but there is substantially less literature on delivering interventions for improving students' algebraic thinking and content knowledge.

Implications for Practice

While evaluating the use of the model in a problem-analysis context, the current research did not directly apply the model in an educational setting. Therefore, it is important that the practice implications be interpreted with caution. In addition, more research needs to be conducted before specific instructional and intervention recommendations can be made through the use of the model. In practice, the model may help in identifying students who are struggling with algebra. The model can also assist practitioners in identifying specific areas where a student may have deficits. However, the data gathered through applying the model should not be the only evidence used in making the claims.

The model can also be used by teachers to analyze why a student may be continuing to struggle with algebra content despite having received quality core instruction in addition to more intensive content support. Future research should search for optimal methods for this instruction support and intervention delivery. However, it is important to note that with a focus on required skills, if a student is lacking certain skills, that student will not be retained or prevented from advancement but rather concurrently provided with intervention support. One avenue for future research is finding the optimal balance between algebraic intervention and instruction.

V. Conclusions

This research adds to the literature supporting the skills required for algebra. It also adds to the sparse literature on problem analysis for students struggling with algebra. It provides a systematic way of identifying skill deficits, which can be used to deliver targeted interventions.

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