

# Mathematical Model of Foreign Exchange Risk in a Supply Chain with Newsvendor Setting using a Log- Normally Distributed Exchange Rate Error

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**Abstract:** In the international market business between two firms of two different countries when there is a fixed time duration between the payments made while placing the order and the order is realized, risk in the form of exchange rate fluctuation affects the optimal pricing and order quantity decisions. We elaborate the effect of Log- normally distributed exchange rate fluctuation when the retailer or manufacturer undertakes to share the exchange rate risk with additive demand error in newsvendor framework. We have also compared the exchange rate effect of our model with the generalized beta distribution error, normally distributed error and Gamma distribution error carried out by earlier researchers.

**Keywords:** Transaction Exposure, Exchange Rate Error, Newsvendor Problem, Optimal Pricing and Quantity, Log-Normal distribution.

Date of Submission: 01-09-2020

Date of Acceptance: 16-09-2020

## I. Introduction

Suppose two different countries having different currencies are into a business. When the exchange rate between the two currencies gets an exposure to unexpected changes, there exists a financial risk and this risk is known as Foreign Exchange Risk.

A transaction exposure risk is the risk that currency exchange rates will fluctuate after a firm has already undertaken a financial obligation. This is because of the purchase price to buyer/ retailer on the settlement day may differ from that when it was incurred, if the debt is denominated in the manufacturer currency. A mathematical model to find optimum ordering and pricing policies for retailer or manufacturer in news vendor setting is developed by Arcelus, Gor and Srinivasan (2013), when the foreign exchange rate between the two countries doing the business. The complete derivation of optimum policies and expected profit of the foreign exchange model for additive demand error is given in Patel and Gor (2015). Our main contribution is to explain the effect of Log-Normal distribution in the exchange rate error under the linear demand with additive error in news vendor setting.

## II. Literature Review

This paper come after the mathematical model of Arcelus, Gor and Srinivasan (2013). Instances of transaction exposure when a firm has an accounts receivable or payable entitled in a foreign currency has been reported in Goel (2012). Eitemann et al (2010) and Shubita et al (2011) has derived that, The nature of International exchange market is that either the retailer or the manufacturer needs to hold up under transaction exposure risk.

The news vendor framework invented by Petruzzi and Dada (1999) and the price dependent demand forms in the additive and multiplicative error by Mills (1958), Karlin and Carr (1962) have been used. The derivation of the maximum profit and ideal strategies, when demand form is linear are given in Patel and Gor (2015) and for multiplicative demand error in Patel and Gor (2015). They have developed more general hybrid model for additive and multiplicative demand error (2015). Mehta and Gor (2020) have developed a model under Gamma distribution exchange rate error in a Newsvendor framework.

## III. Transaction Exposure Model

Assume, retailer needs to order  $q$  units from a foreign manufacturer of some product. The retailer doesn't have the idea about demand ( $D$ ) of the product, which is undecided. But the demand depends on the price ( $p$ ) and also it is irregular. In this paper, we consider the price dependent demand with additive error which can be given as,  $D(p, \epsilon) = g(p) + \epsilon$ , where  $\epsilon$  is the additive error in the demand and it follows some distribution with mean  $\mu$  in interval  $[A, B]$  and  $g(p) = a - bp$ ,  $a, b > 0$  is the deterministic demand.

Let us denote exchange rate as ‘ $r$ ’ in the retailer currency when the order is placed. Let  $w$  denotes the cost of one unit of the product in the manufacturer currency. If buyer pays on the settlement day, at that point he needs to pay  $W_r$  per unit of the product in his currency. Suppose, there is some time between order is placed and the amount is paid for the product, there exists transaction exposure risk, since the exchange rate may differ. So, the buyer has to pay more or less, depending on the existing rate on the day arrival of the product. We model future exchange rate as, FER= Current exchange rate+ fluctuation in the exchange rate. The difference in the exchange rate is some percentage of  $r$ , so we take FER=  $r + r\epsilon_r = r(1 + \epsilon_r)$ , where  $\epsilon_r$  is a random variable together with the variable  $D$ . We consider  $\epsilon_r$  lies in  $[-a, a]$ . The value of  $\epsilon_r$  is unknown but it depends on distribution  $\psi(\epsilon_r)$ . Our main contribution is to explain the effect of Log- normal distribution in the exchange rate error under the linear demand with additive error in news vendor setting. In this paper, we will discuss two scenarios under additive demand error. In both the cases, the retailer’s optimal policy is to determine the optimum order ( $q$ ) and selling price ( $p$ ) of the product. So, his expected profit is maximum. Also, we will obtain the strategies for manufacturer as well.

We will consider the following assumptions in the foreign exchange transaction exposure model:

- I. The standard newsvendor problem assumptions apply.
- II. The global supply chain consists of single retailer-single manufacturer.
- III. The error in demand is additive.
- IV. Only one of the two-retailer or manufacturer- bears the exchange rate risk.

**The following notations are used in the paper:**

- $q$  = order quantity
- $p$  = selling price per unit
- $D$  = demand of the product= no. of units required
- $\epsilon$  = demand error = randomness in the demand.
- $V$  = salvage value per unit
- $s$  = penalty cost per unit for shortage
- $c$  = cost of manufacturing per unit for manufacturer
- $W_r$  = purchase cost for retailer
- $\epsilon_r$  = the exchange rate fluctuation = exchange rate error = randomness in exchange rate
- $\Pi$  = profit function.

**3.1 Retailer bears the exchange rate risk**

Suppose, we consider that retailer bears the exchange rate risk and manufacturer does not bear. Hence, the producer will get  $w$  per unit at any time and the buyer have to pay according to the existing exchange rate. So, buyer will pay  $wr(1 + \epsilon_r)$  per unit, on the settlement day. This amount in manufacturer currency is  $\frac{wr(1+\epsilon_r)}{r} = w(1 + \epsilon_r) = W_r$ . Hence,  $W_r$  is the purchase cost to buyer in seller’s currency. Now, the retailer will choose the selling price  $p$  & order quantity  $q$ , to maximize his expected profit. The profit function of the exporter is given by,

$$\Pi(p, q) = [\text{revenue from } q \text{ items}] - [\text{expenses for the } q \text{ items}]$$

$$\Pi(p, q) = \begin{cases} [pD + v(q - D)] - [qw_r] & \text{if } D \leq q \text{ (overstocking)} \\ [pq] - [s(D - q) + qw_r] & \text{if } D > q \text{ (shortage)} \end{cases}$$

All the parameters  $p, v, s, w_r$  are taken in manufacturer’s currency and the selvsage value  $v$  is taken as an income from the disposal of each of the  $q - D$  leftovers.

Since, the demand  $D(p, \epsilon) = g(p) + \epsilon$  the exporter’s profit function is given by,

$$\Pi(p, q) = \begin{cases} p(g(p) + \epsilon) + v(q - g(p) - \epsilon) - qw_r & \text{if } D \leq q \\ pq - s(g(p) - q + \epsilon) - qw_r & \text{if } D > q \end{cases} \quad (1)$$

Putting  $g(p) = g$  and define  $z = q - g(p) = q - g$  i. e.  $q = z + g$ , for the additive demand error. Now,  $D \leq q \Leftrightarrow g + \epsilon \leq q \Leftrightarrow \epsilon \leq q - g \Leftrightarrow \epsilon \leq z$  and similarly  $D > q \Leftrightarrow \epsilon > z$

$$\Pi(p, q) = \begin{cases} p(g + \epsilon) + v(z - \epsilon) - w_r(z + g) & \text{if } \epsilon \leq z \\ p(z + g) - s(\epsilon - z) - w_r(z + g) & \text{if } \epsilon > z \end{cases} \quad (2)$$

The equation (2) describes the profit function for the retailer in the manufacturer currency. Now the retailer wants to find the optimal order quantity  $q$  say  $q^*$  and optimal price  $p = p^*$  to maximize his expected profit. In order to do this he must find optimal values of the price  $p$  and the parameter  $z$ , say  $p^*$  and  $z^*$  respectively which maximizes his expected profit so that he can determine the optimal order  $q^* = z^* + g(p^*)$ . The profit  $\Pi$  is a function of the random variable  $\epsilon$  with support  $[A, B]$ . Thus the retailer’s expected profit is given by,

$$E \Pi(z, p) = \int_A^B \Pi(z, p) f(u) du.$$

$$E \Pi(z, p) = \int_A^z p(g + u) + v(z - u) - w_r(z + g)f(u)du. + \int_z^B p(z + g) - s(\epsilon - z) - w_r(z + g)f(u)du.$$

Define  $\Lambda(z) = \int_A^z (z - u)f(u)du$  [expected leftovers] and

$$\Phi(z) = \int_z^B (u - z)f(u)du \text{ [expected shortages]}$$

Then the expected profit of the retailer as a function of  $z$  and  $p$  is given by,

$$E \Pi(z, p) = (p - w_r)(g + \mu) - (w_r - v)\Lambda - (p + s - w_r)\Phi \quad (3) \text{ as derived in Sanjay Patel and Ravi Gor.}$$

Where  $\mu = \int_A^B uf(u)du$  in the equation (3) and it gives the expected value of the randomness  $u$  in the demand  $D$ .

We use whitin's method to maximize the expected profit function. The authors have already derived the optimal policies given below, in Sanjay Patel and Ravi Gor.

$$z^* = F^{-1} \left( \frac{p+s-w_r}{p+s-v} \right) \text{ Where } F(z) = \int_A^z f(u)du \text{ is the CDF.}$$

The retailer's optimal order quantity  $q = q^*$  is given by

$$q^* = g(p^*) + z^* = g(p^*) + F^{-1} \left( \frac{p^*+s-w_r}{p^*+s-v} \right) \quad (6)$$

Also the manufacturer's profit when the buyer bears the risk is

$$[(\text{selling price of seller}) - (\text{cost of purchase to seller})] \times \text{no. of units sold, } \Pi_m = (w - c)q^*$$

### 3.2 Seller bears the exchange rate risk

We assume that the manufacturer bears the exchange rate risk and retailer does not. Thus the retailer pays  $w$  per unit in manufacturer's currency at any point of time and the manufacturer will get according to the existing exchange rate. So the manufacturer will be getting  $\frac{wr}{r(1+\epsilon_r)} = w_m$  per unit on the settlement day in his currency. Now the retailer's profit function, his expected profit and optimal policies to get maximum expected profit can be obtained by replacing  $w_r$  by  $w$  in case-1. So we get the retailer's profit as,

$$\Pi(p, q) = \begin{cases} [pD + v(q - D)] - [qw] \text{ if } D \leq q \text{ (overstocking)} \\ [pq] - [s(D - q) + qw] \text{ if } D > q \text{ (shortage)} \end{cases}$$

And his expected profit as,

$$E \Pi(z, p) = (p - w)(g + \mu) - (w - v)\Lambda - (p + s - w)\Phi$$

The optimal value of  $z$  is given by  $z^* = F^{-1} \left( \frac{p+s-w}{p+s-v} \right)$  and hence the optimum order quantity is,  $q^* = g(p^*) +$

$$z^* = g(p^*) + F^{-1} \left( \frac{p^*+s-w}{p^*+s-v} \right)$$

## IV. Sensitivity Analysis

Here, we have considered linear demand with additive demand error  $u$  which follows the uniform distribution  $f(u)$  with support  $[A, B]$ . We get the ideal strategy and maximum expected profit of the retailer and manufacturer using MAPLE software when either retailer or manufacturer takes the exchange rate risk. We consider Log-normal distribution for exchange rate fluctuation  $\epsilon_r$  with support  $[-0.1, 0.1]$ . The probability density function of lognormal distribution is,

$$f(x) = \frac{1}{x\sigma\sqrt{2\pi}} \exp\left[-\frac{(\ln x - \mu)^2}{2\sigma^2}\right]$$

With mean  $E[X] = e^{\mu + \frac{1}{2}\sigma^2}$ . Here,  $X$  is a log-normally distributed random variable and two parameters  $\mu$  &  $\sigma$  are mean and standard deviation of the variable's natural logarithm respectively.

**We will consider following parameter values:**

Demand support =  $[A, B] = [-3500, 1500]$

Mean demand =  $\mu = \frac{A+B}{2} = -1000$

Linear demand  $g(p) = a - bp, a = 100000, b = 1500$

Salvage value  $v = 10$

Penalty cost  $s = 5$

Cost of producing per unit for producer  $c = 20$

Current exchange rate  $r = 45$

We assumed that the mean and standard deviation are,  $\mu = 0.0001, \sigma = 0.033$

We have observed the optimum values by changing the values of different parameters.

The following results in case-I and case-II we get using MAPLE software.

### 4.1 MAPLE code for Log-normal distribution when Retailer bears the risk:

The expected value of the exchange rate error  $er = erx$  in  $[el, eu]$  using density function of log-normal distribution:

$$erx := eval \left( \int_0^{\infty} (el + (eu - el) \cdot x) \cdot \left( \frac{e^{-\frac{1}{2} \cdot \left( \frac{\ln(x) - \mu}{\sigma} \right)^2}}{\sqrt{2\pi} \cdot \sigma \cdot x} \right) dx, \{ \mu = 0.0001, \sigma = 0.33 \} \right)$$

0.1112130557

The purchase price  $wr$  per unit to retailer in the manufacturer currency on the settlement day:

$$wr = w * (1 + erx)$$

1.111213056w

The expected profit of the retailer for order  $q$  units and selling price  $p$  is given by

$$E(\Pi) = (p - wr) * (g(p) + \mu) - (wr - v)\Lambda - (p + s - wr) * \Phi$$

$$(p - 1.111213056w) (-bp + a + \mu) - (1.111213056w - v) \left( \int_A^{bp - a + q} (bp - a + q - u) f(u) du \right) - (p + s - 1.111213056w) \left( \int_{bp - a + q}^B (-bp + a - q + u) f(u) du \right)$$

Suppose the demand error  $\epsilon = u$  follows the uniform distribution  $f(u) = \frac{1}{B-A}$  over  $[A, B] = [-3500, 1500]$ .

Substituting fixed parameter values and finding the expected profit function  $E(\Pi_r)$  in terms of  $p$  and  $q$ , of the retailer:

$$E(\Pi_r) := eval \left( E(\Pi), \left[ a = 100000, b = 1500, A = -3500, B = 1500, f(u) = \frac{1}{5000}, \mu = -1000, v = 10, s = 5, r = 45 \right] \right)$$

$$(p - 1.111213056w) (-1500p + 99000) - (1.111213056w - 10) \left( \frac{3}{10} p (1500p + q - 96500) + \frac{1}{5000} q (1500p + q - 96500) - \frac{1}{10000} (1500p + q - 100000)^2 + 1931225 - 30000p - 20q \right) - (p + 5 - 1.111213056w) \left( -\frac{3}{10} p (101500 - 1500p - q) - \frac{1}{5000} q (101500 - 1500p - q) + 2030225 - \frac{1}{10000} (1500p + q - 100000)^2 - 30000p - 20q \right)$$

To obtain derivatives of expected profit function  $E(\Pi_r)$  w.r.t.  $p$  and  $q$  for maximizing it using NLPP technique:

$$Dp(E(\Pi_r)) := \frac{\partial}{\partial p} (E(\Pi_r))$$

$$27000p - 1931225 + 1666.819584w - (1.111213056w - 10) \left( 450p + \frac{3}{10} q - 28950 \right) + \frac{3}{10} p (101500 - 1500p - q) + \frac{1}{5000} q (101500 - 1500p - q) + \frac{1}{10000} (1500p + q - 100000)^2 + 20q - (p + 5 - 1.111213056w) \left( -30450 + 450p + \frac{3}{10} q \right)$$

solve for p →

$$\left[ \left[ p = -0.0004444444444 q + 44.55555556 \right. \right. \\ \left. \left. + 0.0002222222222 \sqrt{q^2 - 1.48000 \cdot 10^5 q + 8.146000000 \cdot 10^9} \right], [p = \right. \\ \left. -0.0004444444444 q + 44.55555556 \right. \\ \left. - 0.0002222222222 \sqrt{q^2 - 1.48000 \cdot 10^5 q + 8.146000000 \cdot 10^9} \right] ]$$

$$\begin{aligned} > Dq(E(\Pi r)) := \frac{\partial}{\partial q}(E(\Pi r)) \\ & -(1.111213056 w - 10) \left( \frac{3}{10} p + \frac{1}{5000} q - \frac{193}{10} \right) - (p + 5 - 1.111213056 w) \left( \frac{3}{10} p \right. \\ & \left. - \frac{203}{10} + \frac{1}{5000} q \right) \end{aligned}$$

solve for q

$$\left[ \left[ q = \right. \right. \\ \left. \left. - \frac{1}{-5. + p} (0.001280000000 (1.171875 \cdot 10^6 p^2 - 8.5156250 \cdot 10^7 p + 4.340676 \cdot 10^6 w \right. \right. \\ \left. \left. + 3.57421875 \cdot 10^8)) \right] \right]$$

The manufacturer's expected profit  $E(\Pi m)$  for the order  $q$  of the retailer is:

$$\begin{aligned} E P m & := (w - c) \cdot q \\ (w - c) q \end{aligned}$$

The manufacturer's expected profit  $E(\Pi m)$  for the order  $q$  of the retailer and his purchase cost  $c$  is:

$$\begin{aligned} E(\Pi m) & := eval \left( E P m, \left\{ c = 20, q = - \frac{1}{-5. + p} (0.0001000000000 (1.50000000 \cdot 10^8 p^2 \right. \right. \\ & \left. \left. - 1.090000000 \cdot 10^{10} p + 5.50064471 \cdot 10^8 w + 4.575000000 \cdot 10^{10})) \right\} \right) \\ & - \frac{1}{-5. + p} (0.0001000000000 (w - 20) (1.500000000 \cdot 10^8 p^2 - 1.090000000 \cdot 10^{10} p \\ & + 5.500644710 \cdot 10^8 w + 4.575000000 \cdot 10^{10})) \end{aligned}$$

To determine the maximum expected profit of the manufacturer :

$$\begin{aligned} Optimization[interactive] & \left( E(\Pi m), \left\{ q = - \frac{1}{-5. + p} (0.0001000000000 (1.50000000 \cdot 10^8 p^2 \right. \right. \\ & \left. \left. - 1.090000000 \cdot 10^{10} p + 5.50064471 \cdot 10^8 w + 4.575000000 \cdot 10^{10})) \right\}, p = \right. \\ & \left. -0.0004444444444 q + 44.55555556 \right. \\ & \left. + 0.0002222222222 \sqrt{q^2 - 1.48000 \cdot 10^5 q + 8.146000000 \cdot 10^9} \right) \\ & [3.23631485027937859 \cdot 10^5, [p = 54.2014682413246, q = 16827.9814846762, w \\ & = 39.2317471541456]] \end{aligned}$$

Now the retailer determines his expected profit for the above optimal selling prize  $w$  of the manufacturer:

$$\begin{aligned}
 EPr &:= eval(E(\Pi r), w = 39.2317472343442) \\
 &(p - 43.59482973) (-1500p + 99000) - 10.07844892p (1500p + q - 96500) \\
 &- 0.006718965946q (1500p + q - 96500) + 0.003359482973 (1500p + q - 100000)^2 \\
 &- 6.487917505 10^7 + 1.007844892 10^6 p + 671.8965946q - (p - 38.59482973) \left( \right. \\
 &- \frac{3}{10} p (101500 - 1500p - q) - \frac{1}{5000} q (101500 - 1500p - q) + 2030225 \\
 &\left. - \frac{1}{10000} (1500p + q - 100000)^2 - 30000p - 20q \right)
 \end{aligned}$$

The maximum expected profit of the retailer for the optimal value  $w$  of the manufacturer is :

$$\begin{aligned}
 &Optimization[interactive](EPr, p = 40 ..60, q = 15000 ..30000) \\
 &[1.68381583586378285 10^5, [p = 54.2014682253434, q = 16827.9815119812]]
 \end{aligned}$$

#### 4.2 MAPLE code for Log-normal distribution when Seller bears the risk

The expected value of the exchange rate error  $er = erx$  in  $[el, eu]$  using density function of log- normal distribution:

$$> \text{erx} := eval \left( \int_0^{\infty} (el + (eu - el) \cdot x) \cdot \left( \frac{e^{-\frac{1}{2} \cdot \left( \frac{\ln(x) - \mu}{\sigma} \right)^2}}{\sqrt{2\pi} \cdot \sigma \cdot x}} \right) dx, \{ \mu = 0.0001, \sigma = 0.33 \} \right)$$

0.1001289415

The purchase price  $wr$  per unit to retailer at any point of time, in the manufacturer currency on the settlement day:

$$wr := w$$

$w$

The expected profit of the retailer for order  $q$  units and selling price  $p$  is given by:

$$\begin{aligned}
 E(\Pi) &:= (p - wr) \cdot (g(p) + \mu) - (wr - v) \cdot \Lambda - (p + s - wr) \cdot \Phi \\
 &(p - w) (-bp + a + 0.0001) - (w - v) \left( \int_A^{bp - a + q} (bp - a + q - u) f(u) du \right) - (p + s \\
 &- w) \left( \int_{bp - a + q}^B (-bp + a - q + u) f(u) du \right)
 \end{aligned}$$

Suppose the demand error  $\epsilon = u$  follows the uniform distribution  $f(u) = \frac{1}{B-A}$  over  $[A, B] = [-3500, 1500]$ .

Substituting fixed parameter values and finding the expected profit function  $E(\Pi r)$  in terms of  $p$  and  $q$ , of the retailer:

$$\begin{aligned}
 E(\Pi r) &:= eval \left( E(\Pi), \left[ a = 100000, b = 1500, A = -3500, B = 1500, f(u) = \frac{1}{5000}, \mu = -1000, \right. \right. \\
 &\left. \left. v = 10, s = 5, r = 45 \right] \right)
 \end{aligned}$$

$$\begin{aligned}
 &(p - w) (-1500p + 99000) - (w - 10) \left( \frac{3}{10} p (1500p + q - 96500) + \frac{1}{5000} q (1500p \right. \\
 &\left. + q - 96500) - \frac{1}{10000} (1500p + q - 100000)^2 + 1931225 - 30000p - 20q \right) - (p \\
 &+ 5 - w) \left( -\frac{3}{10} p (101500 - 1500p - q) - \frac{1}{5000} q (101500 - 1500p - q) \right. \\
 &\left. + 2030225 - \frac{1}{10000} (1500p + q - 100000)^2 - 30000p - 20q \right)
 \end{aligned}$$

To obtain derivatives of expected profit function  $E(\Pi r)$  w.r.t.  $p$  and  $q$  for maximizing it using NLPP technique:

$$Dp(E(\Pi_r)) := \frac{\partial}{\partial p} (E(\Pi_r))$$

$$27000p - 1931225 + 1500w - (w - 10) \left( 450p + \frac{3}{10}q - 28950 \right) + \frac{3}{10}p(101500 - 1500p - q) + \frac{1}{5000}q(101500 - 1500p - q) + \frac{1}{10000}(1500p + q - 100000)^2 + 20q - (p + 5 - w) \left( -30450 + 450p + \frac{3}{10}q \right)$$

solve for p

$$\left[ \left[ p = -\frac{1}{2250}q + \frac{401}{9} + \frac{1}{4500} \sqrt{q^2 - 148000q + 8146000000} \right], \left[ p = -\frac{1}{2250}q + \frac{401}{9} - \frac{1}{4500} \sqrt{q^2 - 148000q + 8146000000} \right] \right]$$

$$Dq(E(\Pi_r)) := \frac{\partial}{\partial q} (E(\Pi_r))$$

$$-(w - 10) \left( \frac{3}{10}p + \frac{1}{5000}q - \frac{193}{10} \right) - (p + 5 - w) \left( \frac{3}{10}p - \frac{203}{10} + \frac{1}{5000}q \right)$$

solve for q

$$\left[ \left[ q = -\frac{500(3p^2 - 218p + 10w + 915)}{-5 + p} \right] \right]$$

The manufacturer's expected selling price  $wm$  per unit w.r.t. the future rate  $r(1+erx)$  is:

$$wm := \frac{w}{1 + erx}$$

$$0.9089843579w$$

The manufacturer's expected profit  $E(\Pi_m)$  for the order  $q$  of the retailer is:

$$EPM := (wm - c) \cdot q$$

$$(0.9089843579w - c)q$$

The manufacturer's expected profit  $E(\Pi_m)$  for the order  $q$  of the retailer and his purchase cost  $c$  is:

$$E(\Pi_m) := eval \left( EPM, \left\{ c = 20, q = -\frac{500(3p^2 - 218p + 10w + 915)}{-5 + p} \right\} \right) \\ = \frac{500(0.9089843579w - 20)(3p^2 - 218p + 10w + 915)}{-5 + p}$$

To determine the maximum expected profit of the manufacturer:

$$Optimization[interactive] \left( E(\Pi_m), \left\{ p \geq w, q = -\frac{500(3p^2 - 218p + 10w + 915)}{-5 + p}, p = -\frac{1}{2250}q + \frac{401}{9} + \frac{1}{4500} \sqrt{q^2 - 148000q + 8146000000} \right\}, p = 40..65, q = 10000..25000, w = 30..45 \right)$$

$$\left[ 3.2363148574545164610^5, [p = 54.2014682260209, q = 16827.9815040810, w = 43.1599805141231] \right]$$

Now the retailer maximizes his expected profit for the above optimal selling price  $w$  of the manufacturer:

$$EPr := eval(E(\Pi_r), w = 43.1599805141231)$$

$$\begin{aligned}
 & (p - 43.1599805141231) (-1500p + 99000) - 9.947994153p (1500p + q - 96500) \\
 & - 0.006631996102q (1500p + q - 96500) + 0.003315998051 (1500p + q - 100000)^2 \\
 & - 6.403938336 10^7 + 9.947994153 10^5 p + 663.1996102q - (p - 38.15998051) \left( \right. \\
 & \left. - \frac{3}{10} p (101500 - 1500p - q) - \frac{1}{5000} q (101500 - 1500p - q) + 2030225 \right. \\
 & \left. - \frac{1}{10000} (1500p + q - 100000)^2 - 30000p - 20q \right)
 \end{aligned}$$

The maximum expected profit of the retailer for the optimal value  $w$  of the manufacturer is :

Optimization[interactive](EPr,  $p = 40 \dots 60, q = 15000 \dots 30000$ )

[1.68381583586378285  $10^5, [p = 54.2014682253434, q = 16827.9815119812]$ ]

Table- 1 gives the comparison between generalized beta, normal, gamma and log- normal distribution when retailer bears the risk.

Table- 2 gives the comparison between generalized beta, normal, gamma and log- normal distribution when Manufacturer bears the risk.

Table- 3 gives the observations by taking different salvage values using Log-normal distribution.

Table-4 gives the observations by taking different penalty cost using Log-normal distribution.

Table-5 gives the observations by changing intervals using Log-normal distribution.

**TABLES**

**Table-1 Retailer bears the risk**

| Distribution | Parameters of the Distribution | P*    | q*       | Seller's Selling Price w* | Optimum Expected Profit of Buyer | Optimum Expected Profit of Seller |
|--------------|--------------------------------|-------|----------|---------------------------|----------------------------------|-----------------------------------|
| Beta         | $\alpha=1, \beta=3$            | 53.45 | 18047    | 43.82                     | 195075                           | 429886                            |
| Normal       |                                | 53.7  | 17639    | 42.13                     | 185952                           | 390476                            |
| Gamma        | $k = 1, \theta = 3$            | 56.16 | 13608.81 | 31.48                     | 106590                           | 156236                            |
| Log-normal   |                                | 54.20 | 16827.98 | 39.23                     | 168381                           | 323631                            |
| Beta         | $\alpha=3, \beta=1$            | 53.95 | 17234    | 40.61                     | 177080                           | 355344                            |
| Normal       |                                | 53.7  | 17639    | 42.13                     | 185952                           | 390476                            |
| Gamma        | $k = 3, \theta = 1$            | 56.16 | 13608.81 | 31.48                     | 106590                           | 156236                            |
| Log-normal   |                                | 54.20 | 16827.98 | 39.23                     | 168381                           | 323631                            |
| Beta         | $\alpha=1, \beta=1$            | 53.7  | 17640    | 42.13                     | 185970                           | 390548                            |
| Normal       |                                | 53.7  | 17639    | 42.13                     | 185952                           | 390476                            |
| Gamma        | $k = 1, \theta = 1$            | 54.20 | 16829.02 | 39.23                     | 168403                           | 323708                            |
| Log-normal   |                                | 54.20 | 16827.98 | 39.23                     | 168381                           | 323631                            |
| Beta         | $\alpha=2, \beta=5$            | 53.49 | 17989    | 43.56                     | 193761                           | 423989                            |
| Normal       |                                | 53.7  | 17639    | 42.13                     | 185952                           | 390476                            |
| Gamma        | $k = 2, \theta = 5$            | 63.00 | 2752.99  | 21.14                     | -7189.71                         | 2936.32                           |
| Log-normal   |                                | 54.20 | 16827.98 | 39.23                     | 168381                           | 323631                            |
| Beta         | $\alpha=5, \beta=2$            | 53.91 | 17292    | 40.82                     | 178336                           | 360142                            |
| Normal       |                                | 53.7  | 17639    | 42.13                     | 185952                           | 390476                            |
| Gamma        | $k = 5, \theta = 2$            | 63.00 | 2572.94  | 21.14                     | -7189.71                         | 2936.32                           |
| Log-normal   |                                | 54.20 | 16827.98 | 39.23                     | 168381                           | 323631                            |

**Table-2 Manufacturer bears the risk**

| Distribution | Parameters of the Distribution | P*    | q*       | Seller's Selling Price w* | Optimum Expected Profit of Buyer | Optimum Expected Profit of Seller |
|--------------|--------------------------------|-------|----------|---------------------------|----------------------------------|-----------------------------------|
| Beta         | $\alpha=1, \beta=3$            | 53.45 | 18047    | 41.62                     | 195075                           | 429886                            |
| Normal       |                                | 53.7  | 17639    | 42.14                     | 185952                           | 390476                            |
| Gamma        | $k = 1, \theta = 3$            | 56.16 | 13608.81 | 47.22                     | 106590                           | 156236                            |
| Log-normal   |                                | 54.20 | 16827.98 | 43.15                     | 168381                           | 323631                            |
| Beta         | $\alpha=3, \beta=1$            | 53.95 | 17234    | 42.63                     | 177080                           | 355344                            |
| Normal       |                                | 53.7  | 17639    | 42.14                     | 185952                           | 390476                            |
| Gamma        | $k = 3, \theta = 1$            | 56.16 | 13608.81 | 47.22                     | 106590                           | 156236                            |



|                   |                     |       |          |       |          |         |
|-------------------|---------------------|-------|----------|-------|----------|---------|
| <b>Log-normal</b> |                     | 54.20 | 16827.98 | 43.15 | 168381   | 323631  |
| <b>Beta</b>       | $\alpha=1, \beta=1$ | 53.7  | 17640    | 42.13 | 185970   | 390548  |
| <b>Normal</b>     |                     | 53.7  | 17639    | 42.14 | 185952   | 390476  |
| <b>Gamma</b>      | $k = 1, \theta = 1$ | 54.20 | 16829.02 | 43.15 | 168403   | 323708  |
| <b>Log-normal</b> |                     | 54.20 | 16827.98 | 43.15 | 168381   | 323631  |
| <b>Beta</b>       | $\alpha=2, \beta=5$ | 53.49 | 17489    | 41.7  | 193761   | 423989  |
| <b>Normal</b>     |                     | 53.7  | 17639    | 42.14 | 185952   | 390476  |
| <b>Gamma</b>      | $k = 2, \theta = 5$ | 63.00 | 2572.94  | 61.30 | -7189.72 | 2936.32 |
| <b>Log-normal</b> |                     | 54.20 | 16827.98 | 43.15 | 168381   | 323631  |
| <b>Beta</b>       | $\alpha=5, \beta=2$ | 53.91 | 17292    | 42.56 | 178336   | 360142  |
| <b>Normal</b>     |                     | 53.7  | 17639    | 42.14 | 185952   | 390476  |
| <b>Gamma</b>      | $k = 5, \theta = 2$ | 63.00 | 2572.94  | 61.30 | -7189.72 | 2936.32 |
| <b>Log-normal</b> |                     | 54.20 | 16827.98 | 43.15 | 168381   | 323631  |

**Table-3 Different Salvage value**

| Distribution                 | Parameters of the distribution | P*    | q*        | Seller's selling price w* | Optimum expected profit of buyer | Optimum expected profit of seller |
|------------------------------|--------------------------------|-------|-----------|---------------------------|----------------------------------|-----------------------------------|
| <b>Buyer bears the risk</b>  |                                |       |           |                           |                                  |                                   |
| Log-normal                   | $v=10, s=5$                    | 54.20 | 16827.98  | 39.23                     | 168381                           | 323631                            |
|                              | $v=15, s=5$                    | 54.28 | 16884.638 | 38.91                     | 168403                           | 319424                            |
|                              | $v=20, s=5$                    | 54.31 | 17080.82  | 38.88                     | 170967                           | 322493                            |
|                              | $v=25, s=5$                    | 53.77 | 18324.88  | 41.97                     | 195344                           | 402618                            |
| <b>Seller bears the risk</b> |                                |       |           |                           |                                  |                                   |
| Log-normal                   | $v=10, s=5$                    | 54.20 | 16827.98  | 43.15                     | 168381                           | 323631                            |
|                              | $v=15, s=5$                    | 54.28 | 16884.63  | 43.24                     | 168403                           | 319428                            |
|                              | $v=20, s=5$                    | 54.31 | 17080.82  | 43.20                     | 170967                           | 322493                            |
|                              | $v=25, s=5$                    | 54.33 | 17355.65  | 43.13                     | 174561                           | 326614                            |

**Table-4 Different penalty cost**

| Distribution                 | Parameters of the distribution | P*    | q*       | Seller's selling price w* | Optimum expected profit of buyer | Optimum expected profit of seller |
|------------------------------|--------------------------------|-------|----------|---------------------------|----------------------------------|-----------------------------------|
| <b>Buyer bears the risk</b>  |                                |       |          |                           |                                  |                                   |
| Log-normal                   | $v=10, s=4$                    | 54.22 | 16715.34 | 38.91                     | 168155                           | 316168                            |
|                              | $v=10, s=5$                    | 54.20 | 16827.98 | 39.23                     | 168381                           | 323631                            |
|                              | $v=10, s=6$                    | 53.73 | 17661.85 | 42.16                     | 184373                           | 391553                            |
|                              | $v=10, s=8$                    | 53.79 | 17701.93 | 42.22                     | 181330                           | 393473                            |
| <b>Seller bears the risk</b> |                                |       |          |                           |                                  |                                   |
| Log-normal                   | $v=10, s=4$                    | 54.22 | 16715.34 | 43.24                     | 168155                           | 316168                            |
|                              | $v=10, s=5$                    | 54.20 | 16827.98 | 43.15                     | 168381                           | 323631                            |
|                              | $v=10, s=6$                    | 54.28 | 16760.10 | 43.30                     | 164873                           | 317915                            |
|                              | $v=10, s=8$                    | 54.34 | 16801.16 | 43.35                     | 161800                           | 319562                            |

**Table-5 Different Intervals**

| Distribution                 | Parameters of the distribution | P*       | q*       | Seller's selling price w* | Optimum expected profit of buyer | Optimum expected profit of seller |
|------------------------------|--------------------------------|----------|----------|---------------------------|----------------------------------|-----------------------------------|
| <b>Buyer bears the risk</b>  |                                |          |          |                           |                                  |                                   |
| Log-normal                   | [-2000,1500]                   | 54.74    | 17001.89 | 39.47                     | 170429                           | 331118                            |
|                              | [-2500,1500]                   | 54.02    | 17804.72 | 42.52                     | 189053                           | 401051                            |
|                              | [-3000,1500]                   | 53.86    | 17722.23 | 42.32                     | 187892                           | 395656                            |
|                              | [-3500,1500]                   | 54.20    | 16827.98 | 39.23                     | 168381                           | 323631                            |
|                              | [-4000,1500]                   | 53.54    | 17559.22 | 41.96                     | 183290                           | 385716                            |
|                              | [-3500,1000]                   | 54.26    | 16579.07 | 43.13                     | 173927                           | 312001                            |
|                              | [-3500,1200]                   | 53.70    | 17542.26 | 42.05                     | 190609                           | 386933                            |
|                              | [-3500,1800]                   | 53.70    | 17738.44 | 43.36                     | 161688                           | 320197                            |
| [-3500,2000]                 | 53.70                          | 17803.68 | 42.28    | 177664                    | 396779                           |                                   |
| <b>Seller bears the risk</b> |                                |          |          |                           |                                  |                                   |
|                              | [-2000,1500]                   | 54.74    | 17001.89 | 43.86                     | 170429                           | 331118                            |
|                              | [-2500,1500]                   | 54.02    | 17804.72 | 42.52                     | 189053                           | 401051                            |
|                              | [-3000,1500]                   | 54.41    | 16824.96 | 43.45                     | 168547                           | 321483                            |

|                   |              |       |          |       |        |        |
|-------------------|--------------|-------|----------|-------|--------|--------|
| <b>Log-normal</b> | [-3500,1500] | 54.20 | 16827.98 | 43.15 | 168381 | 323631 |
|                   | [-4000,1500] | 54.09 | 16652.47 | 43.10 | 163685 | 312873 |
|                   | [-3500,1000] | 54.26 | 16579.07 | 43.13 | 173927 | 312001 |
|                   | [-3500,1200] | 54.26 | 16642.71 | 43.18 | 171035 | 313995 |
|                   | [-3500,1800] | 54.25 | 16833.74 | 43.36 | 161688 | 320197 |
|                   | [-3500,2000] | 54.24 | 16897.44 | 43.42 | 158350 | 322339 |

### V. Conclusion

We elaborate log-normally distributed exchange rate fluctuation when the retailer or manufacturer undertakes to share the exchange rate risk and the demand error is modeled in the additive form in the news vendor framework. We have compared our model with the exchange rate effect with the generalized beta distribution error given in Arcelus, Gor and Srinivasan (2013), Normally distributed exchange rate error given in Patel and Gor (2016) and model under Gamma distribution exchange rate error given in Mehta and Gor (2020). We have also observed our model by changing the values of parameters.

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