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The Uncertainty Costs of the Social Discount Rate

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Abstract
It is known that the sampling error diminishes with the sample size. This notion is applied to the case when a government has to select among more than one public project. The uncertainty cost is measured by assuming perpetuities which have a present value that is comparable to the present value of very long term projects.As expected, we find that the uncertainty cost is substantial for individual projects but is quite reduced for a basket of projects. This has important repercussions on macroeconomic risk and volatility.

Key words: *social discount rate, return uncertainty, social cost, simulation, time preference, US.*
JEL Codes*: D61, D15, C15*

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I. Introduction

There are five methods to measure the social discount rate (Zhuang et al., 2007). The first is the social time preference (STP), which assumes that public budgets displace consumption. This is the method that will be studied in this paper. The second is the opportunity cost of capital (OCC) which assumes that public budgets displace and crowd out private investment. The third is the weighted-average, which assumes that public budgets are financed by taxes and consumption eviction, bonds, and foreign borrowings. The fourth is the shadow price of capital (SPC), which takes into consideration reinvestment of benefits at the higher market interest rate. And finally, the fifth is recommended for intergenerational projects that have a useful life longer than 50 years, and invariably this method endorses a time-declining discount rate. The purpose of this paper is to introduce uncertainty in the measurement of the social discount rate. It is true that some authors have proposed a point estimate of 3.5% and lower and upper bounds of 2.5% and 7%, which can be regarded as an uncertainty range, or a probability interval estimate (Moore and Vining, 2018). It is also true that Gollier (2013) has introduced a risk premium in the equation of the social discount rate estimated by $0.5\sigma^2\gamma^2$, where σ^2 is the variance of the rate of growth of aggregate real consumption expenditures per capita, and γ is the coefficient of relative risk aversion. However, these authors do not consider explicitly and they neglect the exact magnitude of the uncertainty cost, and ignore that this uncertainty varies with the chosen number of public projects undertaken. In actual practice a government has a panoply of investment projects, going from spending on infrastructure to environmental regulation, and even subsidizing and partaking a stake in big firms that are near default because of a systemic crisis. The underlying concept is easy to assert: the sampling error variance falls with the size of a sample, and hence a portfolio of projects does not bring about the same uncertainty as a single project. In security portfolio analysis this is known as portfolio diversification which eliminates residual risk but not systematic risk, as long as co-variations of net returns are not perfect. So, even if projects are independent, and have independent costs and benefits, diversification and uncertainty-reduction havestill value.

This note is organized as follows. In the following section the theory is reminded. In section 3 the simulations are carried out, and the results are discussed. The final section concludes.

II. Theory

The theory can be described easily according to a simple two-period model indexed by 1 and 2 (Ramsey, 1928). The representative consumer maximizes the sum of the utilities (U) of present and future consumption levels (C) subject to a wealth constraint (W). The real interest rate is constant at r, the rate of time preference is also constant at ρ, and the utility function is iso-elastic, with a constant coefficient of relative risk aversion (γ). Just for the sake of the analysis, and without a loss of generality, we assume perfect foresight:

$$U(C_1, C_2) = \frac{C_1^{1-\gamma} - 1}{1-\gamma} + \frac{1}{(1+\rho)}\frac{(C_2^{1-\gamma} - 1)}{(1-\gamma)} \quad (1)$$

$$C_1 + \frac{C_2}{(1+r)} = W \qquad (2)$$

The first-order condition for a maximization is (with Δ as the first-difference operator):

$$\frac{\Delta C}{C} \cong \frac{r-\rho}{\gamma} \Rightarrow r \cong \rho + \gamma \frac{\Delta C}{C} = \rho + \gamma g \qquad (3)$$

III. Simulation Results

The rate of time preference (ρ) follows a uniform distribution between 0 and 3%. The coefficient of relative risk aversion (γ) follows a normal distribution with average 1.065542 and standard deviation 3.7654, which are obtained from Azar (2021). The consumption growth rate (g) follows a normal distribution with average 2.0552% and standard deviation 3.7654%, which are the sample characteristics of the logarithmic change in US annual real consumption expenditures per capita, and are obtained from the web site of the Federal Reserve Bank of Saint Louis (FRED). Having specified the probability distributions of the terms on the far right hand side of equations (3), a simulation of 10,000 values for $\rho, \gamma,$and g is conducted, that is repeated 10,000 times. The whole process is carried out by the use of the EViews 11 statistical software. All parameters and variables are normally distributed, as tested by the Jarque-Bera statistic, with the exception of ρ which follows a uniform distribution. See Table 1. The estimates of the social discount rate, which is here given by r, or MEAN, in Table 1, have an average and a median of 3.69%, figures that are extremely reasonable. The uncertainty in the social discount rate is estimated by the average of the standard deviation (STDEV, in Table 1), and which is 4.72%. The standard errors depend on the square root of the sample size. Since there are 10,000 replications then the standard error for the whole simulation is 0.047%, obtained from the average standard deviation of column 5 in Table 1.

Table 2 reports in the first column the sample size selected, from 100 till 10,000. In column 2 is the relative frequency (out of 10,000). In column 3 is the standard error for the given sample size. In columns 4 and 5 are respectively the upper and lower limits of a 95% confidence interval. The 95% interval range is in column 6, and the range in basis points is in column 7. Two uncertainty costs, that are close to each other, are reported in columns 8 and 10.The first cost (column 8, Table 2) is calculated as the 95% range divided by the average rate. The second cost (column 10, Table 2) is the ratio of the present value of the two perpetuities evaluated at the upper and lower rate limits, divided by the present value of the perpetuity at the average rate.The two costs are different because of Jensen's inequality. The uncertainty cost (column 9, Table 2) is the amount by which the present value of a perpetuity of \$ 1 at the lower rate limit exceeds the present value of a perpetuity of \$ 1 at the higher rate limit.This cost is calculated assuming perpetuities, which approximate the present value ofa very long cash flow streams. The present value of a perpetuity of \$ 1, evaluated at the average rate, is \$ 27.10.

Table 2 can portray the effects of the magnitude of investments. Assume the infrastructure can be improved by spending 27.1 million on each investment. Assume there are 10,000 investments possible with a 271 billion total budget. If the government chooses 100 such investments then the 95% range of the ensuing social discount rate is between 3.27% and 5.12%, an uncertainty range of 1.85%. The average remains at 3.69% but the standard error changes. If the government picks 1,000 investments, the 95% range for the uncertainty in the estimate of the social discount rate is between 3.90% and 4.48%, an uncertainty range of 0.58%. If the government selects 5,000 investments the uncertainty range is around 35 basis points (0.35%). Finally, if the total spectrum of investments is adopted, the 271 billion budget, the uncertainty cost is only 0.187%, which is trivial compared with 18.5% for one project. This means that the government can assure a net benefit of around 3.69%, or at the very least 3.60%.

IV. Conclusion

This note has considered the case of a government that has to approve more than one public project at a time. Statistical analysis predicts that the sampling variation falls with the sample size. Hence, and although the average expected return is the same for all sample sizes, the variance of returns is not, and is in fact less as the sample size is bigger. We call this variance a liquidity cost because it resembles the liquidity formula in terms of upper and lower limits or ask and bid prices.Hence this cost diminishes as more projects are initiated. The note presents a measurement of this cost, which is found to be substantial for individual projects, as expected, but not for a reasonable batch of projects. This finding has spillovers on macroeconomic risk and volatility and may affect economic growth and its associated social cost (Aghion and Howitt, 2008).

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Table 1: Summary statistics.

	$\Delta logC$	γ	ρ	MEAN	STDEV
Mean	0.020907	1.068917	0.014853	0.036895	0.047188
Median	0.020605	1.072531	0.014752	0.036898	0.047187
Maximum	0.162540	3.406410	0.030000	0.038889	0.049002
Minimum	-0.125242	-1.246566	9.92E-07	0.034800	0.045320
Std. Dev.	0.037788	0.544241	0.008718	0.000470	0.000467
Skewness	-0.017000	-0.028431	0.024831	-0.034193	0.002092
Kurtosis	2.994192	3.070291	1.786517	3.029754	3.015857
Jarque-Bera	0.495732	3.405878	614.5862	2.317269	0.112044
Probability	0.780464	0.182147	0.000000	0.313915	0.945518

Table 2: The distribution of the standard errors, and related characteristics.

frequency	Relative frequency	Standard error	Upper Limit (1)	Lower Limit (2)	Range (3)	Range in basis points	Liquidity cost 2*(3)/[(1)+(2)]	uncertainty cost (4) 1/(1)-1/(2)	Relative Uncertainty cost (4)*0.0419
100	0.01	0.004719	0.051149	0.032651	0.018498	184.977	0.441	11.076	0.464
200	0.02	0.003337	0.04844	0.03536	0.01308	130.798	0.312	7.636	0.324
300	0.03	0.002724	0.04724	0.03656	0.01068	106.796	0.255	6.184	0.263
400	0.04	0.002359	0.046524	0.037276	0.009249	92.488	0.221	5.333	0.226
500	0.05	0.00211	0.046036	0.037764	0.008272	82.724	0.197	4.758	0.202
600	0.06	0.001926	0.045676	0.038124	0.007552	75.517	0.180	4.337	0.184
700	0.07	0.001784	0.045396	0.038404	0.006991	69.915	0.167	4.010	0.170
800	0.08	0.001668	0.04517	0.03863	0.00654	65.399	0.156	3.748	0.159
900	0.09	0.001573	0.044983	0.038817	0.006166	61.659	0.147	3.531	0.150
1000	0.1	0.001492	0.044825	0.038975	0.005849	58.495	0.140	3.348	0.142
1250	0.125	0.001335	0.044516	0.039284	0.005232	52.319	0.125	2.992	0.127
1500	0.15	0.001218	0.044288	0.039512	0.004776	47.761	0.114	2.729	0.116
1750	0.175	0.001128	0.044111	0.039689	0.004422	44.218	0.106	2.526	0.107
2000	0.2	0.001055	0.043968	0.039832	0.004136	41.362	0.099	2.362	0.100
2250	0.225	0.000995	0.04385	0.03995	0.0039	38.997	0.093	2.226	0.094
2500	0.25	0.000944	0.04375	0.04005	0.0037	36.995	0.088	2.111	0.090
2750	0.275	0.0009	0.043664	0.040136	0.003527	35.274	0.084	2.013	0.085
3000	0.3	0.000862	0.043589	0.040211	0.003377	33.772	0.081	1.927	0.082
3250	0.325	0.000828	0.043522	0.040278	0.003245	32.447	0.077	1.851	0.079
3500	0.35	0.000798	0.043463	0.040337	0.003127	31.267	0.075	1.783	0.076
3750	0.375	0.000771	0.04341	0.04039	0.003021	30.207	0.072	1.723	0.073
4000	0.4	0.000746	0.043362	0.040438	0.002925	29.247	0.070	1.668	0.071
4250	0.425	0.000724	0.043319	0.040481	0.002837	28.374	0.068	1.618	0.069
4500	0.45	0.000703	0.043279	0.040521	0.002757	27.575	0.066	1.572	0.067
4750	0.475	0.000685	0.043242	0.040558	0.002684	26.839	0.064	1.530	0.065
5000	0.5	0.000667	0.043208	0.040592	0.002616	26.160	0.062	1.492	0.063
10000	1	0.000478	0.042837	0.040963	0.001874	18.738	0.045	1.068	0.045