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On The Public Investment-Debt-Cash Linkages In The State Government Of Ceará

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Abstract
We exploit a decade of bimonthly fiscal dataset of the state government of Ceará to search for a systemic relationship between public capital, debt and cash. We add to the empirical literature on public investment sustainability by using an integration vector model, an impulse-response and a conditional multivariate wavelet framework. We find a long-term relationship characterized by a negative (positive) reaction of investments to the increase in debt (cash). Debt and cash shocks impact after seven bimesters investments in expected directions, and such responses do not dissipate in four years. We also find anti-phasic co-movements with cycles of investments led by cycles of debt and cash in the years 2014 and 2015, a period of intense fiscal financial crisis in Brazilian states.

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I. Introduction

The role of public capital, measured by its impact on socio-macroeconomic variables, has been theoretically addressed mainly in growth models, especially in the last five decades. Possibly, Arrow and Kurz (1970) are one of the first to discuss the role of public investments, by assuming that public capital, rather than the flow of public services, enters into the production function. We have observed in most theoretical (endogenous or exogenous) growth models that has been common to specify the source of funding for this public spending as a premise of the framework. In this context, we know that low-income countries can finance public investment preferentially with official aid resources, and concessional borrowings. When both sources are not flowing as desired, government can use three sources of funds: accessing domestic borrowing, supplementing with external commercial borrowing, and covering the resulting gap with tax increases and/or spending cuts.

Regarding the collateral of each financing source, there is a literature studying this issue, when the source comes from external aid. For instance, Chatterjee and Turnovsky (2007) and Berg et al. (2010) discuss the effect of external transfers, arguing that this aid is effective depending on the externalities associated with the public good it helps to finance. There is a wider and more specific literature studying the role of public investment, when it is financed with its own resources, through a fiscal effort to generate a flow of primary current surplus. Barro and Sala-i-Martin (1992) show that if the social return on public investment is greater than the private return, it is possible to change the long-term growth trajectory, and therefore it is important to find an optimal tax policy that connects the characteristics of the social service offered. They suggest that investment and growth incentives are high if the tax amount is fixed. Glomm and Ravikumar (1994) also contribute to the discussion about the optimal amount of tax for the sustainability of efficient public investment.

There is also a strand of this literature studying the effects of indebtedness as collateral for issuing bonds or accessing borrowing to finance investments or even current expenditure. One of the pioneers in this theoretical literature may have been Modigliani (1961), Mishan (1963) and Diamond (1965). Until then, one could expect a nonstationary debt-income ratio implied by some models of optimal government finance. However, a high and growing debt-GDP ratio was viewed as worrisome by macroeconomic models with limited taxation (Blanchard, 1984). Moreover, given the worrying debt levels in several countries from the 80's, the concept of solvency of public debt has become more explored in empirical studies, such as Hamilton and Flavin (1986), and Bohn (1998), for example.

Regardless of the source, there seems to be a consensus on the relevance of the sustainability in the sense of continuity of public investments. In this context, Agenor (2010) addresses the growth dependence on public investment, by assuming that government is responsible for investment in infrastructure, which drives the production of commodities and health services, generating greater labor productivity. However, infrastructure efficiency has no linear relationship with investment, and it is necessary to maintain the governance of public investments to shift the trend from low productivity and growth to a steady state of high growth.

There is still another consensus: the importance of this topic to be addressed by international institutions capable of influencing and determining the conduct of public policies in economies. In Allen and Leipziger (2006), Buffie et al. (2012), and Melina, Yang and Zana (2014), International Monetary Fund (IMF) and World Bank (WB) researchers are studying and developing frameworks aiming to monitor debt sustainability, and its relation with public investments, while Blanchard et al. (2021) discuss the system of fiscal rules in the European Union and suggest that country-specific assessments using stochastic debt sustainability analysis is a good normative. Regarding specifically the cash, IMF (2021) is proposing that governments must address the challenge of meeting extended cash needs to finance COVID-19-related emergency spending. IMF researchers argue that a multi-pronged approach to cash management is required to ensure that liquidity is adequate to satisfy the government’s payment obligations.

Specifically on subnational governments, we claim that we need a greater attention, given the scarcity of the related literature. In this specific context, we highlight a work by the World Bank Group (2016), in which researchers argue that subnational debt levels in developing countries are becoming increasingly significant as central governments continue to decentralize spending responsibilities, revenue-raising authority, and borrowing rights to subnational governments. Therefore, the World Bank, in collaboration with other partners, has developed a global knowledge program on subnational fiscal reform and debt management. The program aims to strengthen developing countries’ institutional capacity to maintain subnational fiscal sustainability and prudent debt management alongside a stable macroeconomic framework.

Concerning this literature applied to Brazilian states, Matos and dos Santos (2020) and Matos et al. (2022) find a relevant role for state government capital spending in cross-state growth. Concerning the sustainability, Simonassi et al. (2021) find an inefficient policy in debt management, and an investment cycle that generates higher revenue returns than the cost increments. Bonomo et al. (2021) find that the main determinant of public investment is the fiscal condition of the country and states, and that controlling personnel expenses would be an effective instrument to increase fiscal space for investments. Even more in line with our paper, Matos and Monteiro (2022) find that public investments by state governments have reacted to ensure its sustainability, in response to changes in debt and cash, based on a dynamic balanced panel from 2015 to 2021.

In this paper, we add to this empirical literature on the sustainability of investment by state governments, with an emphasis on the state of Ceará. According to Brazilian Institute of Geography and Statistics (IBGE), this is one of the states with the highest proportion of poor people in the country (40.6% in 2020), and one with the lowest real GDP per capita: R$ 14,935.68 in 2019, (24th among 27 states). If this state were a country, and considering purchasing power parity, the state would rank 99th in the World Bank's GDP per capita ranking, between South Africa and Iraq. On the other hand, this state government has led the national ranking of Investment/Net Current Revenue (NCR) between 2017 and 2020, becoming 6th place in the in 2021.

Observing the scarce literature on public finance applied to a particular Brazilian local government such as Ceará, Santos and Matos (2021a), Matos (2022a), and Uchoa et al. (2022) used to infer on the analysis of the sustainability of investments, based on the solvency of public debt. This is a very useful indirect approach, considering that debt is a collateral of investment financed via credit operations. We add to this debate, by using a direct approach also used in Matos and Monteiro (2022). We follow theoretically the fiscal dynamics proposed by Matos (2022b) aiming to conclude on the sustainability (or not) of the flow of public investment by state government of Ceará, through its reaction in relation to debt and cash (in the short, mid and log run).

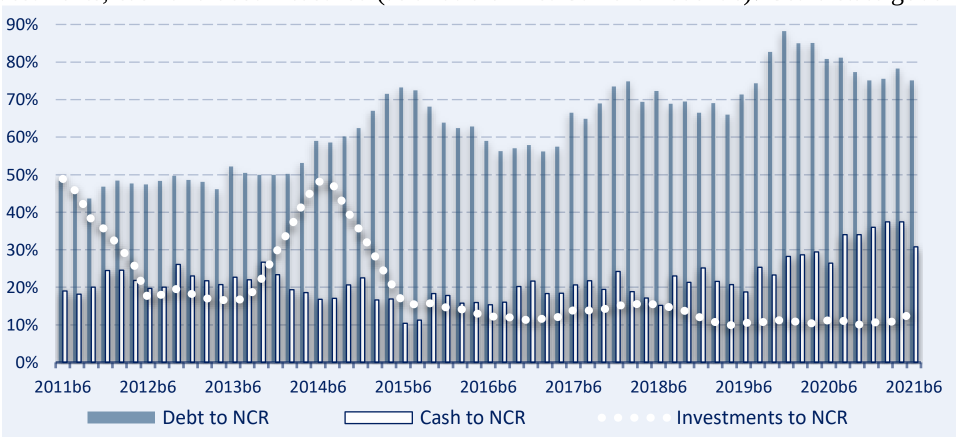
This paper is structured as follows. In Section 2 we discuss recent fiscal situation. Section 3 illustrates the setup of the empirical model, while section 4 reports main findings. Section 5 is devoted to the final discussion.

II. The Fiscal Situation Of Public Investments Of The State Government Of Ceará

When we observe bimonthly series from 2011b6 to 2021b6 of real (R$ December 2021) public investments committed by the state government of Ceará – accumulated values over 6 bimesters –, we identify two atypical periods. At the end of 2011 and 2014, the amounts of real investments committed were R$9.6 billion and R$10.5 billion, respectively. Over the remaining years, the real commitments oscillate around R$ 3 billion. More precisely, they ranged between R$ 2.4 billion (2019b5) and R$ 3.9 billion (2013b2). This visual pattern remains similar when we analyze the series of investments as a ratio of Net Current Revenue (NCR), also accumulated (Figure 1). It is worth noting that both the strong increase in the level of investments throughout 2014, as well as the strong reduction in the following year, both are accompanied by an increase in indebtedness and a reduction in cash.

On consolidated debt, we see a level of approximately 50% of NCR over the first few years, and therefore we find higher levels of indebtedness, which culminated in more than 88% in 2020b3. There is a recent downward trend, and this ratio reached 75% in 2021b6. The cash to NCR ratio, which ranged between 10% and 26% between 2011b6 and 2019b6, has showed a recent upward trend, reaching the highest value of 37% in 2021b5.

Fig. 1. Investments, cash and debt measured (as a ratio of Net Current Revenue): Ceará state government [a, b, c]



Source: Budget Execution Summary Report (RREO), Annexes 1 and 6, available at SICONFI/STN. [a] We use committed investments (nominal accumulated value in 12 months). [b] Cash means the relative balance between financial assets (available cash and other financial assets) and processed remaining payables (except precatory). [c] We use the "gross" consolidated debt of state government.

We know that the sustainability of the flow of this type of capital expenditure is directly related to the sources of financing. Considering the nominally accumulated value (6 bimonths) of domestic credit operations over the analyzed period, we see a heterogeneous behavior in terms of financing of committed investments. From mid-2012 to early 2014, there is a first higher peak of participation, with a representation ranging between 30% and 40% of investments. After that we see a more discreet participation, around 20%, and from 2016 onwards, we show a downward trend in the access to domestic credit by this state government. In 2017, 50% of the accumulated committed investments are financed with domestic credit. Once more, we find a cycle, with low representation in 2019, and a strong representation in 2020, with the peak of domestic financing: more than 70% of investments. At the end of 2021, around 30% of the R$ 3.48 billion of committed investments are being financed by domestic credit.

The external credit (accumulated 6 bimesters) shows stability at the beginning of the period, around 10% of total investment. It is important to note a crowding-out effect in mid-2015, with a reduction in domestic credit and an increase in the paradiplomatic resources. The peak is evident in 2016b5, with almost 50% of investments financed with external credit. Thereafter, there is a slightly negative downward trend. In December 2021, this share of external credit was 10% of the accumulated investments committed by the state government of Ceará.

III. Methodology

Methodological context

The positive macroeconomic impacts of public investments depend on the continuity, robustness, and sustainability of the flow of this type of capital expenditure. This issue is directly related to the sources of financing. In this sense, a subnational government in Brazil can finance public investment with official aid resources. Besides this exogenous source, we also need to assume that concessional borrowings may be available to any government state. However, these funding sources are not flowing as promised or desired. Consequently, Brazilian state governments can try to use three sources of funds: accessing domestic borrowing, supplementing with external commercial borrowing, and covering the resulting gap with tax increases and/or spending cuts. In other words, any state government needs to monitor the level of "stock variables" economically speaking: debt and cash. In this scenario, how are public investments by Ceará state government reacting to the previous change in its own available cash and in its debt? Are these public investments sustainable? Our paper helps answering such questions by using two different frameworks.

In the next subsection, we will briefly describe the most used technique in the case of time series modeling, when the endogenous and exogenous variables present non-stationary behavior. We will therefore comment on the vector error correction model.

In subsection 3.3, we will address the Wavelet transform, a mathematical approach that enables us to investigate the behavior of a parameter over various time scales, besides enables a more flexible approach to deal with high and low frequency components. We also can work with non-stationary data.

Vector error correction

We follow this literature on time series, as Matos and Monteiro (2022), for instance, by estimating a system to assess the role of cash and debt as investment drivers, according to the following regressions:

$$INV_t = c + \alpha INV_{t-1} + \delta DEBT_{t-1} + \gamma CASH_{t-1} + \varepsilon_t \quad (1)$$

$$INV/NCR_t = c + \alpha INV/NCR_{t-1} + \delta DEBT/NCR_{t-1} + \gamma CASH/NCR_{t-1} + \varepsilon_t \quad (2)$$

In both regressions, the subscript t refers to each sample bimester, from 2011b6 to 2021b6. Following this literature, we use all variables in level (December 2021 R$) in equation (1), and as a ratio of the respective Net Current Revenue (NCR), equation (2). As usual, ε refers to the residual.

First, as a type of preliminary test we perform a unit root test to find whether the variables are stationary or not. The results will be reported in subsection 4.2, but we can here anticipate this discussion on the non-stationarity of all variables, aiming to explain how we intend to address this issue.

As usual, the first step in this case is to perform the conditional joint cointegration test, considering, as always, the same vector of instruments. In the scenario where trace test suggests that the null hypothesis that there is no cointegrating vector is rejected, the next step is to estimate the cointegration vector. This vector gives us values about the long-term relationship between the model variables. Next, we estimate the original model, but in difference, considering the cointegrating vector. The last step is to analyzing the response of investments to Cholesky one standard deviation innovations. All these results are reported and discussed in subsection 4.2.

Multivariate wavelet analysis

The continuous wavelet transform

The Fourier analysis can be considered one of the most important bases for the wavelet transform development. The central idea of Fourier analysis it's that any periodic function can be expressed by an infinite sum of trigonometric functions. By defining a basis of sines and cosines of different frequencies, the Fourier Transform capture the relative importance of each frequency on the original signals. This analysis is a powerful tool to modelling time series on frequency domain. The function is reversible, which allow back-and-forth between the original and transformed signals, and it gives an effective localization in frequency. So, we can access the power spectra of the signal, which describe the power distribution on different frequency bands.

Observing the evolution of this literature, the Wavelet transform has some additional features over original Fourier transform, as argued by In and Kim (2012): i) it can decompose the data into several time scales instead of the frequency domain (which allow us to investigate the behavior of a signal over various time scales); ii) it uses local base functions that adjust the window width to deal with different frequencies (this enables a more flexible approach to deal with high and low frequency components) and iii) it allows to work with non-stationary data. The latter feature is especially important to examine financial time series, once that heteroskedasticity, sudden regime shifts, structural breaks at unknown time points are common pattern trough the financial cycle paths. Given a time series $x(t)$, the continuous wavelet transform (CWT) is defined as:

$$W_x(\tau, s) = \int_{-\infty}^{+\infty} x(t)\, \psi^*_{\tau,s}(t) dt \quad (3)$$

where * denotes the complex conjugate, τ determines the position, s is the scaling factor and $\psi_{\tau,s}$ is the basis function suited to scale and shift the original signal, which allows the decomposition of the time series in space and scale. To capture different frequencies of the signal, we use a mother wavelet that is stretched and shifted:

$$\psi_{\tau,s}(t) = \frac{1}{\sqrt{s}} \psi\left(\frac{t-\tau}{s}\right) \quad (4)$$

The factor $1/\sqrt{s}$ is added to guarantee preservation of the unit energy ($\|\psi_{\tau,s}\| = 1$). Low scales are captured rapidly changing detail generating a compressed wavelet ($|s| < 1$), capturing high frequencies movements, and high scales capture slowly changing features ($|s| > 1$), or low frequencies movements (Rua, 2012). So, the CWT can be defined by:

$$W_x(\tau, s) = \int_{-\infty}^{+\infty} x(t) \frac{1}{\sqrt{s}} \psi\left(\frac{t-\tau}{s}\right) dt \quad (5)$$

The basis function $\psi_{\tau,s}$ must obey some criteria, such as: admissibility, similarity, invertibility, regularity, and vanishing moments. There are many options of wavelet mother functions to select. Aguiar-Conraria and Soares (2011) highlight the importance of that choice and suggest picked up an analytic wavelet to study the synchronism between oscillatory signals because its corresponding transform contains information on both amplitude and phase, providing an estimate of the instantaneous amplitude and instantaneous phase of the signal in the neighboring of each time/scale location (τ, s). On subset of analytic wavelet, the Morlet wavelet mother is the most popular alternative because some properties, which is given by:

$$\psi_{\omega_0}(t) = \pi^{-1/4} e^{i\omega_0 t} e^{-t^2/2} \quad (6)$$

where the non-dimensional frequency ω_0 is set $\omega_0 = 6$ to satisfy the admissibility condition. As the wavelet transform decomposes the original signal in a time-scale domain, which put us the necessity to convert scale into frequency. Lilly and Olhede (2009) point that this conversion can be made by associate the wavelet $\psi_{\tau,s}$ with one of three special frequencies (the peak frequency, the energy function, or the central instantaneous frequency), by using the formula $\omega(s) = \frac{\omega_\psi}{s}$, where ω_ψ denotes any of the three angular special frequency.

By the usual "Fourier" frequency f (cycles per unit time) we have that $f(s) = \frac{\omega_\psi}{2\pi s}$. In this sense, the Morlet wavelet is an ideal alternative since it provides us a unique relation between frequency and scale (the peak frequency, the energy frequency and the central instantaneous frequency are all equal) which makes it easier the conversion from scales to frequencies. The choice of $\omega_0 = 6$ give us a conversion ratio equal $f = \frac{6}{2\pi s} \approx \frac{1}{s}$, that direct correspondence between scale and frequency is ideal to simplify an effective interpretation of the results. Since CWT is applied on finite-length time series, border distortions will occur due the fact that values of the transform at the beginning and the end of the sample are imprecisely computed, which involves artificial padding on the extremes of the sample (the most common is set zero to extend the time series). As larger scales decrease the amplitude near the edges as more zeroes enter the analysis, the region that suffers from these edge effects is function of s. The Cone of Influence is the region of the wavelet spectrum in which edge effects become important by a factor of e^{-2}. In the case of the Morlet wavelet this is given by $\sqrt{2s}$.

Wavelet tools

The first wavelet measure that we will present it's the wavelet power spectrum (WPS), which reports the variance distribution of the original time series $x(t)$ around the time-scale (or time-frequency) plane. Following Torrence and Compo (1998) we define the WPS by:

$$WPS_x(\tau, s) = |W_x(\tau, s)|^2 \quad (7)$$

To compare the oscillation in energy among a range of bands (or frequency) we define the Global Wavelet Power Spectrum (GPWS), which takes the average of wavelet power spectrum over all times:

$$GWPS_x(\tau, s) = \int_{-\infty}^{+\infty} |W_x(\tau, s)|^2 d\tau \quad (8)$$

To study the dependencies between two original time series $x(t)$ and $y(t)$ in time-scale/frequency plane, Torrence and Webster (1999) were the first to define the wavelet coherence. The measure that is associated to the cross-wavelet spectrum (XWT), which in turn can be derived by:

$$W_{xy}(\tau, s) = W_x(\tau, s) W_y^*(\tau, s) \quad (9)$$

where $W_x(.)$ and $W_y(.)$ are continuous wavelet transform of $x(t)$ and $y(t)$, respectively, and * denotes the conjugates complex. As the cross-wavelet transform is complex, we can express the XWT as $|W_{x,y}(\tau, s)|$. It computes the local covariance between two signals at each scale. The squared wavelet coherence is given by the squared of the wavelet cross-spectrum normalized by the individual power spectra. Following Torrence and Webster (1999) the squared wavelet coherence is denoted as:

$$R^2(\tau, s) = \frac{\left|S\left(s^{-1} W_{x,y}(\tau, s)\right)\right|^2}{S(s^{-1} W_x(\tau, s)^2) S(s^{-1} W_y(\tau, s)^2)} \quad (10)$$

where $S(.)$ expresses a smoothing operator in both time and scale, s^{-1} is a normalization fator ensuring the conversion to an energy density. Torrence and Webster (1999) note that in numerator of the squared wavelet coherence, both the real and imaginary parts of the cross-wavelet transform are smoothed separately before taking the absolute value, while the smoothing operator is taking on square of the wavelet power spectra in denominator. By these definitions, it's ensured that $0 \leq R^2 \leq 1$.

Hence, the main advantage of the wavelet coherence on XWT is the common measure unit to examine several combinations of signals. Torrence and Compo (1999) reveal that once the wavelet transforms conserves variance, the wavelet coherence is a good representation of the normalized covariance between two-time series, where the closer to zero (one) the coherence, the weaker (stronger) the local correlation between the time-series. The wavelet coherence has not theoretical distribution known, hence we follow the approach of Aguiar-Conraria (2011) deriving the confidence interval using Monte Carlo methods.

Although the wavelet coherence computes the degree of local linear correlation between two signals, it isn't reveals patterns of lead-lag relationship neither if the movements are positives or negatives. To deal with these limitations, the phase-difference is commonly used to examine the delays in the fluctuations between the two time-series. Following Torrence and Webster (1999) we define the phase difference as:

$$\phi_{xy}(\tau,s) = \tan^{-1}\left(\frac{\Im\left\{S\left(s^{-1}W_{x,y}(\tau,s)\right)\right\}}{\Re\left\{S\left(s^{-1}W_{x,y}(\tau,s)\right)\right\}}\right) \tag{11}$$

The smoothed real ($\Re$) and imaginary ($\Im$) parts should already be calculated in the wavelet coherence function. Both $R^2(\tau,s)$ and $\phi_{xy}(\tau,s)$ are functions of the position index (τ) and scale (s). We also need the information on the signs of each part to completely determine the value of $\phi_{xy}\epsilon[-\pi,\pi]$. A phase-difference of zero indicates that the time-series move together at the specified frequency. If $\phi_{xy}\epsilon\left(0,\frac{\pi}{2}\right)$ the series move in phase, but the time-series y leads x, while if $\phi_{xy}\epsilon\left(-\frac{\pi}{2},0\right)$ then it is x that is leading. A phase-difference of $\phi_{xy} = \pm\pi$ indicates an anti-phase relation. Finally, if $\phi_{xy}\epsilon\left(\frac{\pi}{2},\pi\right)$, then x is leading and time-series y is leading if $\phi_{xy}\epsilon\left(-\pi,-\frac{\pi}{2}\right)$.

Finally, aiming to capture the interdependence and/or causality between multiple time series, we follow Aguiar-Conraria and Soares (2014) and Aguiar-Conraria et al. (2018). We use high-order wavelet tools which allow us to investigate the dependency of one time series upon a set of other time series (multiple wavelet coherency) on the time-frequency plane as well to examine the co-movements between two time-series after controlling the oscillations of a subset of time series (partial wavelet coherency, partial phase-difference and partial wavelet gain) on the time-frequency plane. Given a set of p time series $(x_1, x_2, .., x_p)$ with $p > 2$ and $x_i = \{x_{i_n}, n = 0, .., T-1\}$, to compute multiple and partial wavelet measures is necessary to perform a smoothing operation on the cross-spectra. Let S_{ij} be the smoothed version of the cross-wavelet spectrum between the time series x_i and x_j ($S_{ij} = S(W_{ij})$ and $\mathcal{L} = (S_{ij})_{i,j=1}^{p}$ denote the pxp matrix of all the smoothed cross-wavelet spectra. It is important to observe that this matrix is function of specific combination (τ,s) at which the spectra are being computed and it's a Hermitian matrix ($\mathcal{L} = \mathcal{L}^H$) where the symbol H denotes conjugate transpose. So, $S_{ij} = S_{ji}^*$, $\forall\, i \neq j$ and $S_{ii} = S(|W_i|^2)$ is a real (positive) number $\forall i$. The squared multiple wavelet coherency between the series $\boldsymbol{x_1}$ and all the other series $\boldsymbol{x_2}, .., \boldsymbol{x_p}$ is given by:

$$R^2_{1(2..p)} = 1 - \frac{\mathcal{L}^d}{S_{11}\mathcal{L}^d_{11}} \tag{12}$$

where d denotes the determinant of the matrix, and $\mathcal{L}^d_{ij}$ denotes the cofactor of the element in position (i,j), i.e. $\mathcal{L}^d_{ij} = (-1)^{i+j} det\mathcal{L}^j_i$, where $det\mathcal{L}^j_i$ denotes the determinant of the submatrix obtained by crossing out the row i and column j of $\mathcal{L}$ to convert it to $\mathcal{L}_{ij}$. The complex partial wavelet coherency of x_1 and x_j $(2 \leq j \leq p)$ allowing for all the other series will be computed by:

$$\varrho_{1j.q_j} = -\frac{\mathcal{L}^d_{j1}}{\sqrt{\mathcal{L}^d_{11}}\sqrt{\mathcal{L}^d_{jj}}} \tag{13}$$

The partial wavelet coherency of x_1 and x_j allowing for all the other series, is denoted by:

$$R_{1\,j.q_j} = \frac{\left|\mathcal{L}^d_{j1}\right|}{\sqrt{\mathcal{L}^d_{11}}\sqrt{\mathcal{L}^d_{jj}}} \tag{14}$$

The squared partial wavelet coherency of x_1 and x_j is simply the square of $R_{1\,j.q_j}$.

Since that the complex partial wavelet coherency measures the local correlation between the time series $x_1$and x_j after controlling the influence of all the other time series ($x_i; i = 2, .., p$ *and* $i \neq j$), Aguiar-Conraria and Soares (2018) define the partial phase-difference of $x_1$over x_j as:

$$\phi_{1j.q_j} = \tan^{-1}\left(\frac{\Im\left(\varrho_{1j.q_j}\right)}{\Re\left(\varrho_{1j.q_j}\right)}\right) \tag{15}$$

Following them, we define the complex partial wavelet gain of the time-series x_1 over x_j as follow:

$$\mathcal{g}_{1j.q_j} = -\frac{\mathcal{L}^d_{j1}}{\mathcal{L}^d_{11}} \tag{16}$$

The partial wavelet gain is computed by the modulus of the complex partial wavelet *gain.* Then:

$$G_{1j.q_j} = \frac{|\mathcal{L}^d_{j1}|}{\mathcal{L}^d_{11}} \tag{17}$$

Note that the partial wavelet gain provides us a measure similar to the (modulus) coefficient of a multiple linear regression of x_1 against the explanatory variables $x_2, , , x_p$ (for $j = 2, .., p$) at each time and frequency.

These authors also highlight that these measures can be specified by wavelet complex coherencies. Consider a matrix $\mathfrak{L} = \left(\varrho_{ij}\right)_{i,j=1}^{P}$ of all the complex wavelet coherencies ϱ_{ij}. Hence, the multiple wavelet coherency can be computed as:

$$R_{1(2..p)}^2 = 1 - \frac{\mathfrak{L}^d}{\mathfrak{L}_{11}^d} \tag{18}$$

The complex partial wavelet coherency $\varrho_{1j.q_j}$ and the partial wavelet coherency $R_{1j.q_j}$ are given by:

$$\varrho_{1j.q_j} = -\frac{\mathfrak{L}_{j1}^d}{\sqrt{\mathfrak{L}_{11}^d}\sqrt{\mathfrak{L}_{jj}^d}} \quad \text{and} \quad R_{1j.q_j} = \frac{\left|\mathfrak{L}_{j1}^d\right|}{\sqrt{\mathfrak{L}_{11}^d}\sqrt{\mathfrak{L}_{jj}^d}} \tag{19}$$

Finally, the complex partial wavelet gain $g_{1j.q_j}$ and the partial wavelet gain $G_{1j.q_j}$, respectively, by:

$$g_{1j.q_j} = -\frac{\mathfrak{L}_{j1}^d}{\mathfrak{L}_{11}^d}\frac{\sigma_1}{\sigma_j} \quad \text{and} \quad G_{1j.q_j} = \frac{|\mathfrak{L}_{j1}^d|}{\mathfrak{L}_{11}^d}\frac{\sigma_1}{\sigma_j} \tag{20}$$

The results on multivariate analysis will be reported in subsection 4.3.

IV. Empirical Exercise

Database

We report in Table 1 some of the main statistics and the results on the stationarity of the main fiscal variables of state government of Ceará.

Table 1. Summary statistics and unit root test [a, b, c]

	Variables in level (R$ billion dec/2021)			Variables as a ratio of NCR (%)		
	Investments	Cash	Debt	Investments	Cash	Debt
Summary statistics						
Mean	R$4.22	R$4.95	R$14.24	19.40%	21.97%	63.29%
S.D.	R$2.25	R$1.59	R$3.48	11.18%	5.79%	12.01%
Minimum	R$2.40	R$2.15	R$9.15	9.86%	10.40%	43.65%
Maximum	R$10.47	R$9.37	R$20.99	48.89%	37.44%	88.24%
Δ (2011b6 – 2021b6)	–R$6.16	R$4.00	R$9.37	–35.08%	11.77%	26.73%
Unit root test						
ADF test	–1.1914	0.9728	1.6811	–1.3016	0.8840	0.8181
	[0.2110]	[0.9104]	[0.9764]	[0.1762]	[0.8967]	[0.8859]

Notes: [a] Source: Budget Execution Summary Report (RREO), Annexes 1 and 6, available at SICONFI/STN. [b] Augmented Dickey-Fuller unit root test without intercept nor trend over the period from 2011b6 to 2021b6 (H0: unit root). [c] Respective p-values are reported in the brackets. * p-value<0.05. ** p-value<0.01.

Public investments, both in level (R$ dec/2021) and in terms of NCR, show a more volatile behavior than cash and debt, if the average values of the time series are considered. Two aspects can help explain this pattern. Investments are flow-type variables, and are still influenced by the electoral period. According to Matos et al. (2022), the comparison between municipalities and states suggests a cyclical behavior, given that the main component of capital expenditures is investments, and in the years before municipal elections, municipalities commit more to investments than states. In 2019, for example, Brazilian municipal governments invested almost R$ 47 billion, while aggregate state governments invested R$ 34 billion. In the ratio of current revenue, investment since 2015 comes from a more constant trend between 10% and 16%, showing greater resistance to variations. From the peak of investment committed by the state government in 2014, this capital expenditure has remained constant at R$ 3 billion, while as a ratio of NCR, it also remains smooth ranging from 10% to 16%. On the other hand, cash showed a recent trend of real growth, reaching its maximum (R$ 9.37 billion) in 2021b4. The real debt reached its maximum value of almost R$ 21 billion in 2020b5, and since then it has shown a downward trend. It is important to note and record the reduction of more than R$ 2 billion in this debt if the period between the fifth quarter of 2020 and the end of 2021 is considered.

Finally, we perform the unit root test to all variables, in level, and as a ratio of NCR. We need to check whether all variables used are stationary before estimating an Autoregressive Vector model. According to the results on the augmented Dickey and Fuller (1979) test reported in Table 1, we cannot reject the hypothesis of nonstationary behavior of debt, cash, and investments, in level or as a ratio of NCR. We address this issue by applying the Vector Error Correction model (VEC). This nonstationary behavior is not a problem for using multivariate coherency method.

Vector error correction: results

According to the ADF stationarity tests, one cannot reject the null hypothesis that the cash and debt series are non-stationary, in level or as a ratio of the NCR. Therefore, we perform the conditional joint cointegration test, considering the same instruments. For both system specifications, the trace test does not reject by 10% the null hypothesis that there is a cointegrating vector. This finding suggests that investment flow, debt and cash of the Ceará state government may have a long-term structural equilibrium relationship with each other. Next we report the estimation of this cointegrating vector:

$$\underset{}{INV_t} = 7.25X10^{-9} - \underset{(-3.4279)}{0.5624DEBT_t} + \underset{(2.4204)}{0.9831CASH_t} + \varepsilon_t \quad (21)$$

$$INV/NCR_t = 0.4261 - \underset{(-3.2814)}{0.7380DEBT/NCR_t} + \underset{(1.9414)}{1.0432CASH/NCR_t} + \varepsilon_t \quad (22)$$

In both cointegrating vectors, the long-term relationship between investment and debt or cash are significant at 5%, according to respective p-values reported in parenthesis. This finding characterizes the long-term sustainability of Ceará state government investments, given the positive signs of the relationship with cash, and mainly negative signs of the relationship with debt.

We report the results of the estimation of short-term relationships in Table 2. We find robust results for both system specification suggesting that the variation of investments is persistent but nor explosive, and depends on the cointegrating vector, and the lagged cash variation. The value of the explanatory power in both versions of the model (0.70 and 0.63) suggests a reasonable fit for the investments dynamics. We also find that changes in cash and debt seem to have an exogenous behavior, with the exception of the significant dependence of the cash change (as a ratio of NCR) on the second debt lag (as a ratio of NCR).

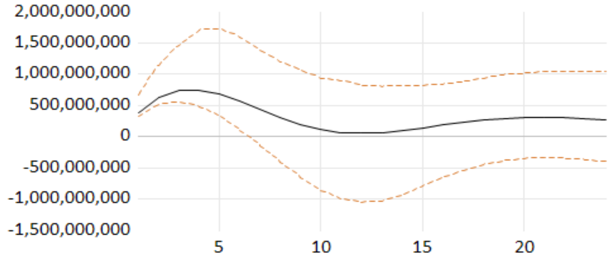
Table 2. Results on investments reaction [a, b]

	Variables in level (R$ billion dec/2021)			Variables as a ratio of NCR (%)		
Error correction	Δ (invest)	Δ (cash)	Δ (debt)	Δ (invest)	Δ (cash)	Δ (debt)
Constant	$2.20x10^7$	$1.28x10^8$	**$1.63x10^8$** *	0.0010	0.0037	0.0053
	(0.3854)	(1.1459)	**(1.7182)**	(0.3352)	(0.7835)	(1.1387)
Cointeg.Equat.	**−0.1135** **	0.0591	−0.0113	**−0.1027** **	0.0550	−0.0168
	(−4.0103)	(1.0618)	(−0.2392)	**(−3.5599)**	(1.1426)	(−0.3565)
Δ (invest(−1))	**0.7270** **	0.1507	−0.0304	**0.6257** **	0.1279	−0.0747
	(5.8259)	(0.6145)	(−0.1468)	**(4.9707)**	(0.6094)	(−0.3624)
Δ (invest(−2))	0.0154	−0.1718	0.1079	0.0598	−0.1746	0.0775
	(0.1210)	(−0.6861)	(0.5093)	(0.4764)	(−0.8338)	(0.3769)
Δ (cash (−1))	**−0.1569** *	−0.1758	−0.0224	**−0.1963** *	−0.1916	−0.0840
	(−2.0160)	(−1.1497)	(−0.1728)	**(−2.2438)**	(−1.3138)	(−0.5862)
Δ (cash (−2))	−0.1017	0.0160	−0.1872	−0.1319	−0.0394	−0.2277
	(−1.3211)	(0.1054)	(−1.4624)	(−1.4958)	(−0.2680)	(−1.5774)
Δ (debt(−1))	0.0102	−0.0795	0.0042	−0.0286	−0.0463	0.0853
	(0.1216)	(−0.4837)	(0.0302)	(−0.3259)	(−0.3168)	(0.5940)
Δ (debt(−2))	0.0441	−0.2609	0.2129	0.0376	**−0.2673** *	0.1506
	(0.5300)	(−1.5944)	(1.5377)	(0.4316)	**(−1.8410)**	(1.0564)
R^2	0.7026	0.1063	0.0994	0.6258	0.1313	0.0778

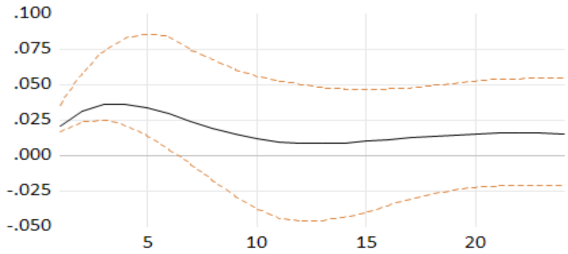
Notes: [a] Estimation of the model in autoregressive vector format with insertion of the error correction vector for the period from the sixth bimester of 2011 to the sixth bimester of 2021. [b] Respective t-statistics are reported in the parenthesis.
* p-value<0.05. ** p-value<0.01.

According to the impulse-response results reported in Figure 2, our robust finding show that investments committed by the state government tend to show persistence in the short term, due to a positive shock, which dissipates in one and a half year (six bimonths). Investments react (significantly) positively to increases in cash and negatively to increases in indebtedness, however only after the seventh or eighth bimester. In both cases, the reactions do not dissipate (even in a four-year horizon) and seem to stabilize after two years.

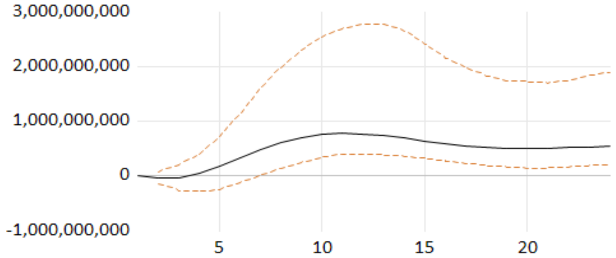
Fig. 2. Impulse-response [a, b]



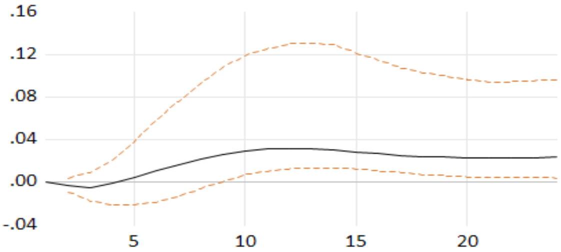
2.a. Response of real investments (in level) to real investments (in level)



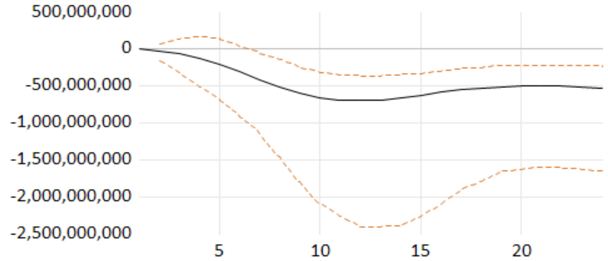
2.d. Response of investments (% of NCR) to investments (% of NCR)



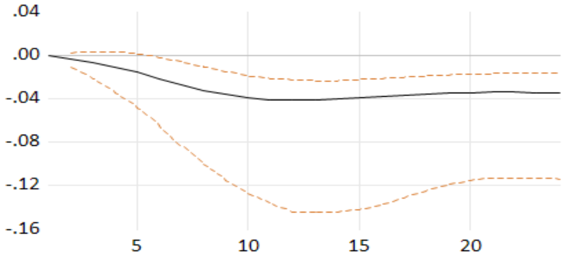
2.b. Response of real investments (in level) to real cash (in level)



2.e. Response of investments (% of NCR) to cash (% of NCR)



2.c. Response of real investments (in level) to real debt (in level)



2.f. Response of investments (% of NCR) to debt (% of NCR)

Notes: [a] Response to Cholesky one standard deviation (d. f. adjusted) innovations for the period from the sixth bimester of 2011 to the sixth bimester of 2021. [b] 95% Confidence interval using Hall´s studentized bootstrap with 999 bootstrap repetitions and 499 double bootstrap reps

Multivariate coherency: results

Aiming to better understand investments-cash-debt linkages, we propose the use of the multiple partial coherence, phase-difference and gain. The first one measures the degree of adjustment of the explanatory variables on the dependent variable in the time-frequency domain. The other measures calculate the relationship between the fluctuations of investments versus debt, controlling the influence of cash on the oscillations in the time-frequency space, as well as the fluctuations of investments versus cash, controlling the influence of debt. According to Figure 3, we do not find a presence of successive areas with significance statistics in the multiple coherence, which denotes we do not have a good overall fit in the model. Next, we analyze whether cash and debt are useful to explain investments in the time-frequency location.

In both analyses, at the level or as a ratio of the NCR, the joint analysis suggests only the same significant area, during a short period between the second half of 2014 and the year 2015, associated with cycles with a frequency between 6 and 9 bimesters. This previous finding – for mid-run frequencies (6 ~9 bimesters), the multiple coherence is continuously significant from 2014 to 2015 – seem to be able to drive our findings on the controlled effects of both debt and cash. We observe that for both models (in level or as a % of NCR) the controlled effect of debt and cash are such that also at mid-run frequency (6~9 bimesters), the partial coherence is significant in a narrow area over the same period (2014 to 2015). For both explanatory variables (cash and debt) the partial phase-difference is set to $(\pi/2, \pi)$ indicating a negative significant co-movement.

Fig. 3. Investments-cash-debt linkages: joint wavelet coherency, partial phase-difference and partial gain. [a,b]

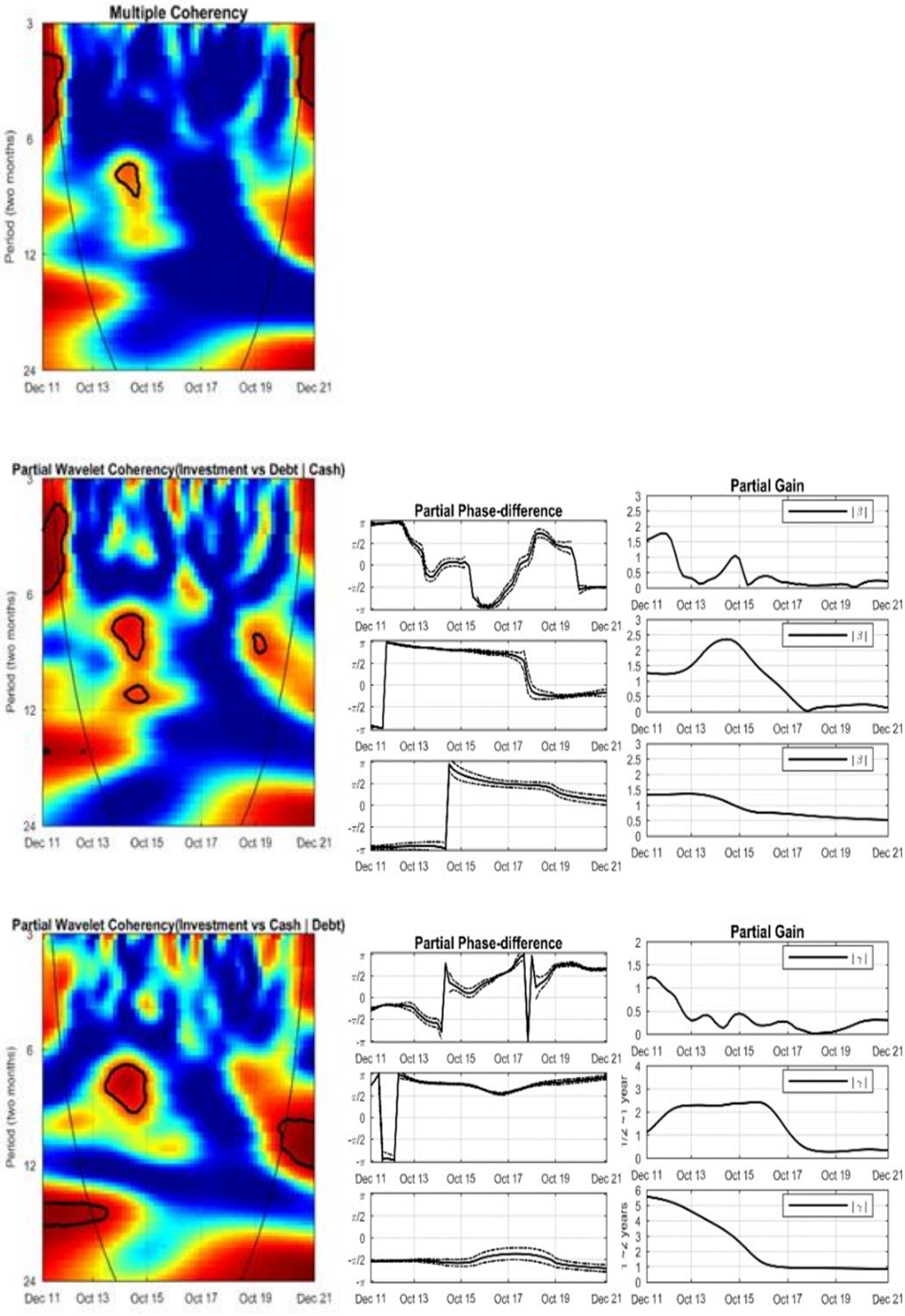


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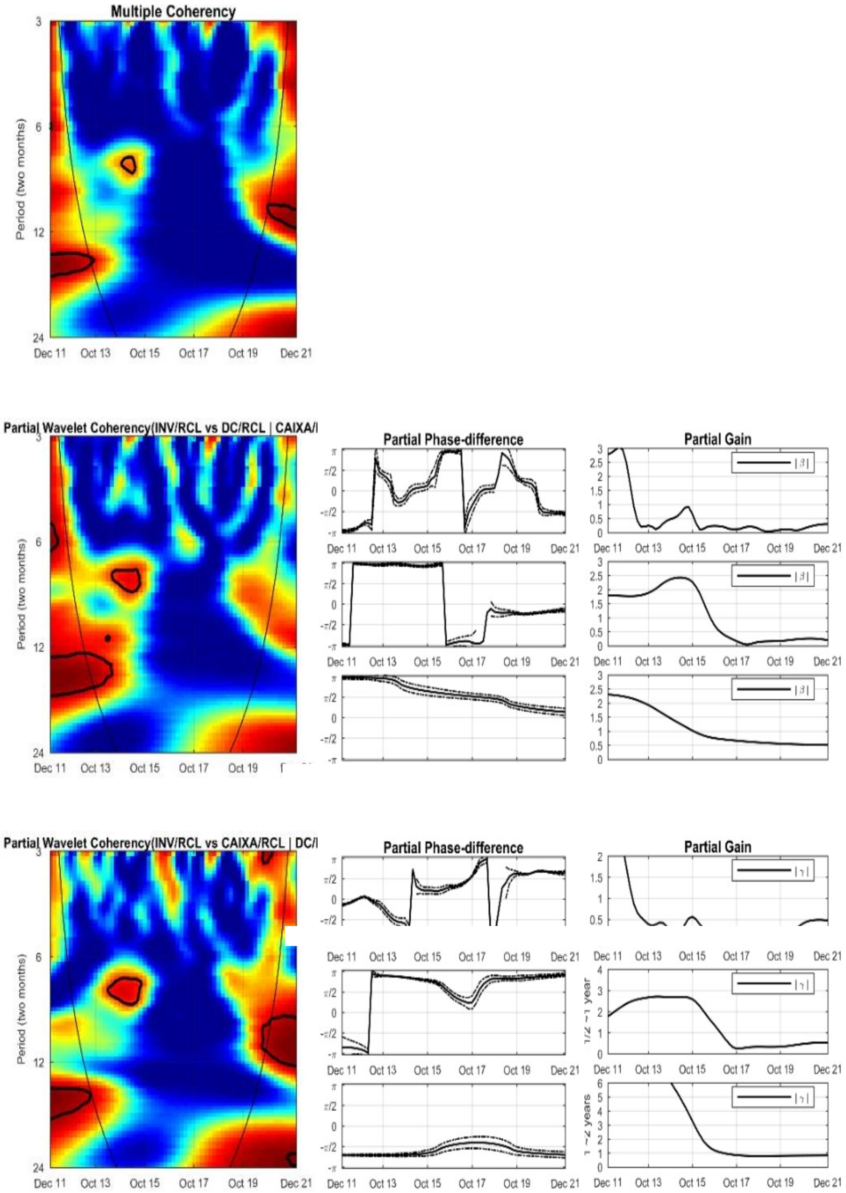
Fig. 3. Investments-cash-debt linkages: joint wavelet coherency, partial phase-difference and partial gain. [a,b]



Notes: [a] The coherency ranges from low (blue) to high (red) values and the respective cone of influence is shown with a black line, designating the 5% significance level. [b] We also plot the phase-difference with plus and minus two standard deviations.

We also find that both cash and debt cycles are able to anticipate fluctuations of investment cycles during this period of fiscal instability for all state governments in Brazil. In both models, the partial gain associated with the leadership of debt cycles oscillates between 1 and 2.5, with an increasing trend throughout 2015. On the other hand, the partial gain associated with cash cycles is more stable over time, ranging between 2 and 3.

Even when we observe an area of significance of the co-movement of investment cycles and high-frequency cycles of debt to level, or low-frequency debt-to-NCR, in the first years of the sample, this co-movement is characterized by the leadership of debt cycles.

V. Conclusion

According to Uchoa et al. (2022), it is possible to infer (based on wavelet) that public investments by the state government of Ceará in equipment have influenced during the years 2006 to 2009 the revenue with the main tax: ICMS. These authors show that the commitment of resources in construction works is capable of influencing economic activity (IBCR-CE) between 2009 and 2012, industrial production between 2009 and 2010, and the volume of retail sales between 2012 and 2013. The authors also show that the investments in equipment between 2007 and 2011 and in construction works (during the period between 2007 and 2013, and during the years 2015 to 2018) were able to impact formal jobs.

This economic impact requires the sustainability of the flow of public investments, and this continuity depends on the sources of resources that finance the investment. This issue can be addressed by a theoretical approach: fiscal dynamics of public investments by Brazilian state governments, reported in Matos (2022b). This dynamic for state governments is specific and different from the dynamics of federal governments that may contract securities debt, as observed by the "Golden Rule" (Art. 167, III, of the Brazilian Federal Constitution, 1988) applied to states in Brazil. Our contribution empirically explores this theoretical dynamic for the state that led the national ranking of public investment in Brazil between 2017 and 2020. In other words, we analyze the relationship between the flow of committed investments and the non-flow variables: available cash and debt. Summarizing our results, we are able to find a long-term relationship suggesting sustainability, as well we find that cash and debt shocks on investment do not dissipate over a 4-year horizon, besides identifying the co-movements of the mid-run cycles of investments led or anticipated by debt and cash cycles, during the 2014 and 2015 fiscal crisis.

Finally, it is essential to make a caveat, because one of the main conceptual hypotheses of our paper is that the policy maker decision – responsive to past fluctuations observed in debt and cash – is the issuance of the purchase commitment note, i.e., the first stage of the execution of the contracting a construction works/installations, and acquisition of permanent material/equipment, for instance. It is important that the literature and the control institutions continue monitoring whether the stages of liquidation and payment of investments are following the amounts committed. In other words, it is reasonable to assume that the decision on whether or not to commit the investment is based on monitoring cash and debt. However, the social and macroeconomic benefits arising from the investment can only be observed and measured, if in fact such investment expenses have been finalized and paid. It is important in this context to question whether the indicators used to rank state governments should be based on committed or paid expenditures, for instance.

Conflict of Interest

The authors declare that he has no conflict of interest.

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