Deconvolution and Interpretation of Well Test Data 'Masked' By Wellbore Storage in A Build Up Test

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Abstract*: When a well test contains a series of different flow rates, or a continuously varying flow rate, the combination of the pressure transients due to varying flow rate is called convolution. while deconvolution means removing a distorting effect upon the variable of interest. This paper is on the study of an analytical technique that can be used to explicitly deconvolve wellbore storage distorted well test data using pressure data and the flow rate. Then to determine the reservoir properties from this deconvolved well test data by using the conventional well test interpretation methods. Also the comparison of the material balance deconvolution method results with the β-deconvlolution method result were carried out and then used to determine which method was a better deconvolution tool. The results showed that the material balance deconvolution technique performed very well with minor discrepancies and gave better estimation of the reservoir parameters.*

Keywords: *Buildup Test, β-deconvlolution, Wellbore Storage Effect, Material Balance deconvolution,*

I. Introduction:

Well testing can be said to be the only technique that examines a significant portion of the reservoir under dynamic conditions in order to determine its production capability and reservoir properties. It has long been recognized that wellbore storage (after flow) can impede pressure transient test analysis thus several methods have been suggested for determining the effects of afterflow when well known semi-logarithmic techniques cannot be used for transient test analysis. Often times, during well testing, the test may not be carried out for a very long time so as to acquire sufficient information that can be used to interpret the result in the usual conventional method available in literatures, hence, the need to make use of the early time region (ETR) data, for the interpretation. In such situation, we then have to try to make the data as reliable as possible by eliminating wellbore storage effect from the data.

Ramey H.T(1970) concluded that annulus unloading and wellbore storage are important physical effect that often controls the behavior of early pressure data taken during a well test. Van Everdingen and Hurst.. (1953) reported that wellbore storage effects include a "skin effect" or a region of altered permeability adjacent to the wellbore and that in many cases the production flow rate can be approximated using equation. Kuchuk F.J, (1985) applied "β deconvolution" for the analysis of wellbore storage distorted pressure transient data and formulated the β-deconvolution equation that helps to computes the undistorted pressure drop function directly from the wellbore storage affected data. Bourdet et al (1989) showed that the most recently documented pressure derivative approach has combined the most powerful aspects of the two previously distinct methods into a single stage interpretative plot Roumboustsos and Stewart (1985) developed convolution and deconvolution methods based on the ideas proposed by kuchuk. Kuchuk presented a generalized rate-convolution and deconvolution methods. He obtained deconvolved pressure values from the Riemann sum and from exponential wellbore flow-rate case. Igbokoyi, A.O (2007) used the deconvolution approach and the resulting Duhamels integral formulation to develop a model that successfully interpreted short-time pressure data distorted by wellbore storage and skin in a buildup test.

II. Material Balance Deconvolution

Material balance deconvolution is a practical approach for the analysis of pressure transient data distorted by wellbore storage effects, The general form of material balance deconvolution provide for the pressure drawdown case in terms of the material balance time function and the rate normalized pressure drop function. The material balance time function and the rate-normalized pressure drop function is given by the equations 1and 2

$$t_{mb} = \frac{N_P}{q} \text{_____}(1)$$

$$\frac{\Delta p}{q} = \frac{(p_i - p_{wf})}{q} \text{_____}(2)$$

From the first principle, applying material balance to a well with wellbore storage, the following equations are stated,

$$q_{sf} = q + \frac{24C_s}{B}\frac{dP_w}{dt}$$ _____ (3)

For buildup, the flow rate at the surface q = 0, so we have:

$$q_{sf} = \frac{24C_s}{B}\frac{dP_w}{dt}$$ _____ (4)

Then for a normalization of the sandface flow rate, q_{ref}

$$\frac{q_{sf}}{q_{ref}} = \frac{24C_s}{q_{ref}B}\frac{dP_w}{dt}$$ _____ (5)

But for this case, we can say that $\dfrac{q_{sf}}{q_{ref}} = q_{wbs}$. _____ (6)

Then the equation 5 becomes:

$$\frac{dp_w}{dt} = q_{wbs} * \frac{q_{ref}B}{24C_s}$$ _____ (7)

But $\quad C_s = C_{wb} * V_{wb}$ _____ (7a)

From equation (7),

A plot of $\dfrac{dp_w}{dt}$ against q_{wbs} gives the slope m_{wbs} ,

which can be express as:

$$m_{wbs} = \frac{qB}{24C_s}$$ _____ (8)

Substituting equation (8) into equation (6)

$$q_{wbs} = \frac{1}{m_{wbs}}\frac{d}{d\Delta t}(\Delta p_{ws})$$ _____ (9)

For a buildup test, the pressure drop is measured against pressure at time t =0, thus the pressure drop is given as:

$$\Delta P_{ws} = p_{ws} - p_{wf}(\Delta t = 0)$$ _____ (10)

NOTE:

$$\left(\frac{d(\Delta P_{ws})}{d(\Delta t)}\right)_n = \frac{(\Delta P_{ws})_{\Delta t_{n+1}} - (\Delta P_{ws})_{\Delta t_{n-1}}}{\Delta t_{n+1} - \Delta t_{n-1}}$$ _____ (11)

Integrating the equation :

$$N_p = \int q_{sf}dt = \frac{24C_s}{B}\int \frac{dp_w}{dt}dt$$ _____ (12)

$$N_p = \frac{24C_s}{B} * P_w$$ _____ (13)

Now, to normalize the above equation, we divide all through by reference rate q_{ref}.

$$\frac{N_p}{q_{ref}} = \frac{24C_s}{q_{ref}B} * P_w$$ _____ (14)

$$N = \frac{1}{m_{wbs}} * P_w \tag{15}$$

Applying the above equation for the case of a buildup test we have:

$$N = \frac{1}{m_{wbs}} * \Delta P_{ws} \tag{16}$$

Applying material balance to the time:

$$\Delta t = \Delta t_{wbs} + N \tag{17}$$

$$\Delta t_{wbs} = \Delta t - \frac{1}{m_{wbs}} \Delta p_{wbs} \tag{18}$$

Also, the rate due to wellbore storage in a buildup test is given as:

$$q_{ref} = q_{BU} + q_{sf} \tag{19}$$

$$q^* = 1 - \frac{q_{sf}}{q_{ref}} \tag{20}$$

$$q^* = 1 - q_{wbs} \tag{21}$$

The wellbore storage-based, material balance time function is expressed as:

$$\Delta t_{mb} = \frac{N}{1 - q_{wbs}} \tag{22}$$

Substituting equation (11) and (16) into (22)
Then the wellbore storage based rate-normalized pressure drop function becomes

$$\Delta p_s = \frac{1}{1 - \frac{1}{m_{wbs}} \frac{d}{d\Delta t} (\Delta p_{wbs})} (\Delta p_{wbs}) \cdots \tag{23}$$

Plot of rate-normalized pressure function versus the material balance time function shows that the material balance time function does correct the erroneous shift in the semi log straight-line obtained by rate normalization.

III. B- Deconvolution Formulation

Van Everdingen and Hurst (1953) introduced an exponential model for the sandface rate during the wellbore storage distortion period of a pressure transient test. The exponential formulation of the flowrate function is given as:

$$q_D(t_D) = 1 - e^{-\beta t_D} \cdots \tag{24}$$

Equation (23) is based on the empirical observations made by Van Everdingen and Hurst.
Recalling the Duhamel's convolution principle equation:

$$P_{WD} = \int_0^{t_D} q'_D(\tau) p_{SD}(t_D - \tau) d\tau \tag{25}$$

Laplace transform of integration function is given as follows: If

$$g(t) = \int_0^t f(\tau) d\tau \tag{26}$$

Then $$L(g)(z) = \frac{1}{Z} L(f)(z) \tag{27}$$

Where z is the Laplace space function.
Therefore, applying Laplace transformation to equation (24):

$$\bar{P}_{wD}(Z) = Z' \bar{q}_D(Z) \bar{P}_{SD}(Z) \underline{\hspace{4cm}}(28)$$

NOTE: $z' = \dfrac{1}{z}$

Rearranging for the equivalent constant rate pressure drop function, $\bar{P}_{SD}$ we obtain The Laplace transform of the rate profile, equation (15) is: Substituting equation (19) into equation (18), we obtain:

$$\bar{P}_{SD}(Z) = \bar{P}_{wD}(Z)(1 + \frac{Z}{\beta}) \underline{\hspace{3cm}}(29)$$

Taking the inverse Laplace transformation of this result yields the "beta" deconvolution formula:

$$\beta = \frac{1}{P(t_D) - P_{WD}} \frac{dp_{WD}(t_D)}{dt_D} \underline{\hspace{3cm}}(30)$$

To alleviate the issue of the exponential sandface flowrate, equation (18) to solve for the β-term. Solving equation (18) for the β-term, we have:

Multiplying through by the C_D – term, we have:

$$\beta C_D = \frac{1}{P_{sD}(t_D) - P_{wD}(t_D)} C_D \frac{dP_{wD}(t_D)}{dt_D} \dots \underline{\hspace{3cm}} (31)$$

Recalling the definition of the wellbore storage model, we have:

$$q_D(t_D) = 1 - C_D \frac{dp_{wD}(t_D)}{dt_D} \dots \underline{\hspace{3cm}} (32)$$

Assuming wellbore storage domination (i.e $q_D \approx 0$) at early times then equation (32) becomes:

$$C_D \frac{dp_{wD}(t_D)}{dt_D} \approx 1 \quad \text{(Early time)} \underline{\hspace{3cm}} (33)$$

Integrating by separating the variables in equation (23) above, we have:

$$P_{wD} \approx \frac{t_D}{C_D} \quad \text{(Early time)}. \underline{\hspace{3cm}}(34)$$

Substituting Equation (33) and (34) into equation (31); we obtain:

$$\text{(Early time)}\dots\dots\dots\dots\dots\dots \quad (35)$$

$$\beta C_D = \frac{1}{P_{sD}(t_D) - \dfrac{t_D}{C_D}}$$

Equation (35) has shown that one can "correlate" the βC_D-product with $\dfrac{t_D}{C_D}$-this observation becomes the basis for the use of these plotting functions to compare the β-deconvolution relations. The "master" plot of the β-deconvolution function for the case of a single well in an infinite-acting homogenous reservoir is derived using equation (20).

IV. Derivation Of The Coefficient For B-Deconvolution

From Van Everdingen and Hurst exponential rate model, we have:

$$q_D = 1 - e^{-\beta(t_D)t_D} \underline{\hspace{3cm}} \underline{\hspace{2cm}}(36)$$

Taking the time derivative of equation (36), we have

$$q'_D = \frac{dq_D}{dt_D} = b(t_D)e^{-\beta(t_D)t_D} \underline{\hspace{3cm}}(37)$$

Where the $b(t_D)$-term is defined as:

$$b(t_D) = \beta(t_D) + \beta'(t_D)t_D \underline{\hspace{3cm}}(28)$$

taking the time derivative, $q'_D(t_D) = \dfrac{dq_D}{dt_D} = -C_D \dfrac{d^2 P_{wD}}{dt^2} = -C_D p''_{wD} \dots \underline{\hspace{2cm}}(39)$

Equating equations (27) and (28) gives:

$$C_D p''_{wD}(t_D) = C_D \frac{d^2 p_{wD}}{dt_D^2} = -b(t_D)e^{-\beta(t_D)t_D} \dots \underline{\hspace{4cm}} \quad (40)$$

Equating equation (21) and (25), we have:

$$e^{-\beta(t_D)t_D} = C_D \frac{dp_{wD}}{dt_D} = C_D p'_{wD}(t_D) \underline{\hspace{5cm}} \quad (41)$$

Combining equation (28) and (29) and solving for $b(t_D)$

$$b(t_D) = -\frac{1}{t_D} \frac{p''_{wD}}{p'_{wD}} = \beta(t_D) + \beta'(t_D)t_D \underline{\hspace{4cm}} \quad (42)$$

Where the p''_{wD} and p'_{wD} terms are defined as:

$$P_{wDd} = t_D \frac{dp_{wD}}{dt_D} \underline{\hspace{5cm}} \quad (43)$$

$$P_{wDdd} = t^2_D \frac{d^2 p_{wD}}{dt^2_D} \underline{\hspace{5cm}} \quad (44)$$

Equation (41) can be used to determine β(t_D) and β'(t_D) — a graphical representation of the equation, where the intercept and slope values are β(t_D) and β'(t_D) respectively.

The value of β(t_D) and β'(t_D) can be approximated by numerical methods such as least square — which is the functional approach adopted here

V.   Discussion And Result:

A single-phase and single-rate pressure buildup test was conducted on a case study oil well- Xl. using the following reservoir parameters: B_O = 1.224 rb/stb, h = 55ft, ϕ = 0.06, r_w = 0.21ft, C_t =17.5 x 10^{-6} Psi^{-1}, μ_o = 0.65 cp, ρ_o =53.5 lbm/ft^3, q_f = 250stb/day, t_p=13,630 hours.

The material balance deconvolution technique performs extremely well, with minor discrepancies at the start of the data set. At the beginning of the data set deconvolved, the material balance shows an abnormal curve or deviation from the normal trend, thus, not a better tool for deconvolving during this time period.

However, after the very early time period, the material balance deconvolution method performs very well like every other deconvolution method and gives a better estimation of the reservoir parameters than any other deconvolution technique.   The 'beta' deconvolution method was also a good deconvolution method as shown in Figure 3 It has an advantage over the material balance during the very early time period and after which it is not a better deconvolution method than the material balance method as shown in Figure 4. However it gives an estimate of the reservoir parameters during the periods dominated by the wellbore storage effects, though not as accurate as the material balance method. Nevertheless, both can yield reservoir parameters at any time, provided the production rate varies exponentially during the shut-in period.
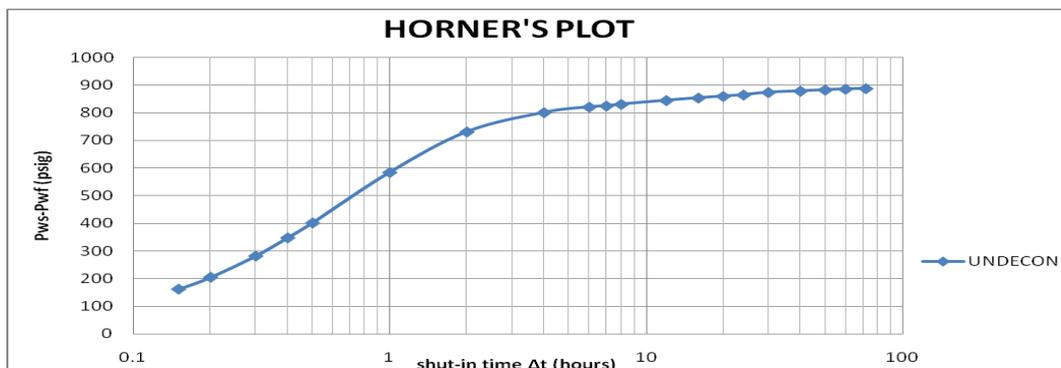

Figure 1: Horner's plot for the case study well-XI.

Table1 : Pressure buildup data from the case study well-XI

S/N	Δt	P_{ws}
1	0	3519
2	0.15	3680
3	0.2	3723
4	0.3	3800
5	0.4	3866
6	0.5	3920
7	1	4103
8	2	4250
9	4	4320
10	6	4340
11	7	4344
12	8	4350
13	12	4364
14	16	4373
15	20	4379
16	24	4384
17	30	4393
18	40	4398
19	50	4402
20	60	4405
21	72	4407

Table 2: Shut-in time and undeconvolved Pressure data

S/N	Δt	P_{ws}	ΔP_{ws}
1.	0	3519	0
2	0.15	3680	161
3	0.2	3723	204
4	0.3	3800	281
5	0.4	3866	347
6	0.5	3920	401
7	1	4103	584
8	2	4250	731
9	4	4320	801
10	6	4340	821
11	7	4344	825
12	8	4350	831
13	12	4364	845
14	16	4373	854
15	20	4379	860
16	24	4384	865
17	30	4393	874
18	40	4398	879
19	50	4402	883
20	60	4405	886
21	72	4407	888

Table 3: Material data deconvolution data

S/N	$P_{ws}(psig)$	$P_{ws}(psia)$	$\Delta t(hours)$	Δp_{ws}	$\frac{1}{m_{abs}}\Delta p_{ws}$	$\Delta t - \frac{1}{m_{abs}}\Delta p_{ws}$	$\frac{d}{d\Delta t}[\Delta p_{ws}]$	$\frac{1}{m_{abs}}\frac{d}{d\Delta t}[\Delta p_{ws}]$	$1-\frac{1}{m_{abs}}\frac{d}{d\Delta t}[\Delta p_{ws}]$	t_{mbs}	p_{mbs}
1	3519	3534	0	0	0	0	0	0	1	0	0
2	3680	3695	0.15	161	0.1607	-0.0107	509.250	0.509	0.491	-0.022	327.458
3	3723	3738	0.2	204	0.2037	-0.0037	400.000	0.400	0.600	-0.006	339.500
4	3800	3815	0.3	281	0.2807	0.0193	357.500	0.358	0.643	0.030	436.887
5	3866	3881	0.4	347	0.3467	0.0533	300.000	0.300	0.700	0.076	495.286
6	3920	3935	0.5	401	0.4007	0.0993	197.500	0.198	0.803	0.124	499.315
7	4103	4118	1	584	0.5837	0.4163	110.000	0.110	0.890	0.468	655.843
8	4250	4265	2	731	0.7307	1.2693	36.167	0.036	0.964	1.317	758.119
9	4320	4335	4	801	0.8007	3.1993	11.250	0.011	0.989	3.236	809.810
10	4340	4355	6	821	0.8207	5.1793	4.000	0.004	0.996	5.200	823.996
11	4344	4359	7	825	0.8247	6.1753	2.500	0.003	0.998	6.191	826.767
12	4350	4365	8	831	0.8307	7.1693	2.000	0.002	0.998	7.184	832.365
13	4364	4379	12	845	0.8447	11.1553	1.438	0.001	0.999	11.171	845.916
14	4373	4388	16	854	0.8537	15.1463	0.938	0.001	0.999	15.161	854.501
15	4379	4394	20	860	0.8597	19.1403	0.688	0.001	0.999	19.153	860.291
16	4384	4399	24	865	0.8647	23.1353	0.700	0.001	0.999	23.152	865.306
17	4393	4408	30	874	0.8737	29.1263	0.438	0.000	1.000	29.139	874.082
18	4398	4413	40	879	0.8787	39.1213	0.225	0.000	1.000	39.130	878.898
19	4402	4417	50	883	0.8827	49.1173	0.175	0.000	1.000	49.126	882.854
20	4405	4420	60	886	0.8857	59.1143	0.114	0.000	1.000	59.121	885.801
21	4407	4422	72	888	0.8877	71.1123	7.381	0.007	0.993	71.641	894.301
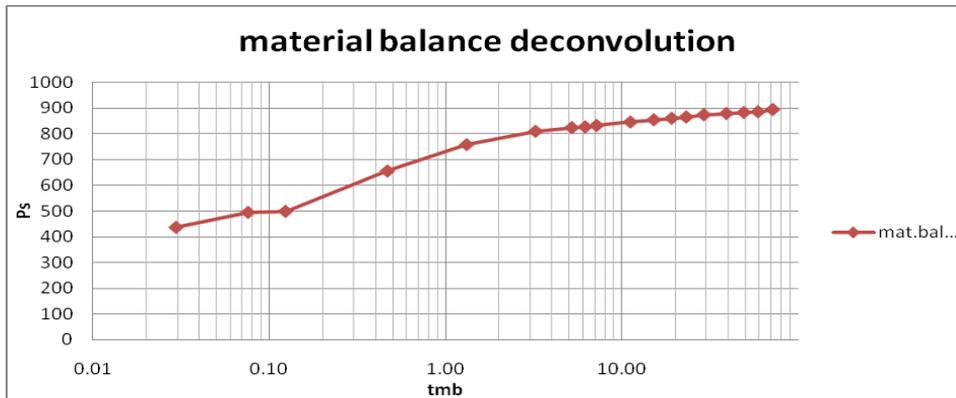
Figure 2: Material balance deconvolved data

Table 4: Beta deconvolution data

| S/N	Δt	P_{ws}	ΔP_{ws}	P_D	t_D	$\frac{\partial P_D}{\partial t_D}$	$\frac{1}{\beta}\frac{\partial P_D}{\partial t_D}$	P_{SD}
1	0	3519	0	0	0	0		0
2	0.15	3680	161	2.605118	0.110518	11.2003	0.00053	338.7342
3	0.2	3723	204	3.300895	0.147357	8.78455	0.000554	429.1883
4	0.3	3800	281	4.54682	0.221036	7.851191	0.000742	591.1832
5	0.4	3866	347	5.614757	0.294715	6.588412	0.000831	730.0264
6	0.5	3920	401	6.488523	0.368394	4.337371	0.000684	843.5969
7	1	4103	584	9.44962	0.736787	2.415751	0.000761	1134.046
8	2	4250	731	11.82821	1.473574	0.79427	0.000501	1419.445
9	4	4320	801	12.96087	2.947148	0.247065	0.000311	1555.341
10	6	4340	821	13.28448	4.420722	0.087845	0.000166	1594.158
11	7	4344	825	13.34921	5.157509	0.054903	0.000121	1601.919
12	8	4350	831	13.44629	5.894296	0.043923	0.000111	1613.568
13	12	4364	845	13.67282	8.841444	0.031569	0.000119	1640.753
14	16	4373	854	13.81845	11.78859	0.020589	0.000104	1658.227
15	20	4379	860	13.91554	14.73574	0.015098	9.52E-05	1669.876
16	24	4384	865	13.99644	17.68289	0.015373	0.000116	1679.587
17	30	4393	874	14.14207	22.10361	0.009608	9.09E-05	1697.059
18	40	4398	879	14.22297	29.47148	0.004941	6.23E-05	1706.764
19	50	4402	883	14.2877	36.83935	0.003843	6.06E-05	1714.531
20	60	4405	886	14.33624	44.20722	0.002496	4.72E-05	1720.354
21	72	4407	888	14.3686	53.04867	0.162148	0.00368	1724.674
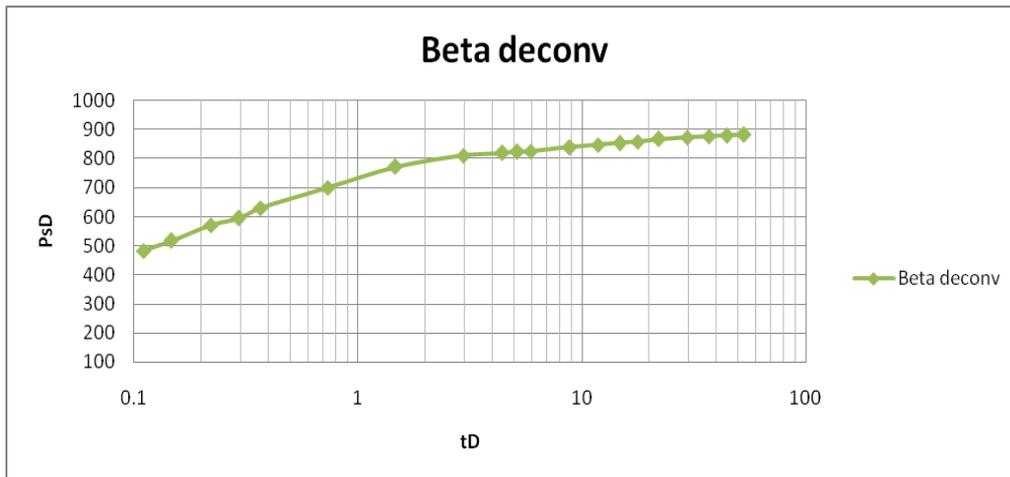
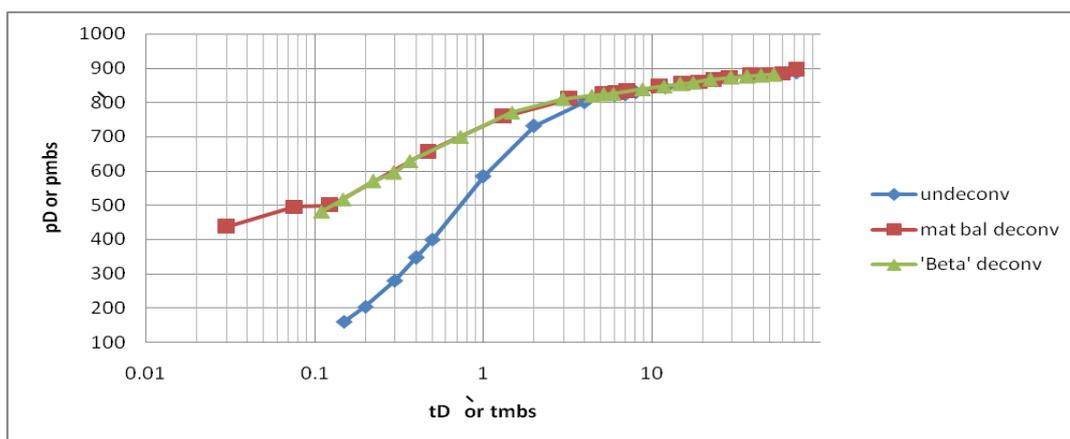**Figure 3: β-deconvolved data**

Figure 4: Comparison of the deconvolved and undeconvolved data

Table 5. Comparing the Undeconvolution and Deconvoluted Results

PARAMETER	UNDECONVOLUTED	DECONVOLUTED	
	(MTR)	MATERIAL BAL	BETA'
m(psi/cycle)	70	110	200
K(mD)	8.4	5.4	3.9
S	5.87	4.2	2.7

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